PG/IS/PHY/PH-1101/09

M.Sc 1st Semester Examination, 2009

PHYSICS

(Methods of Mathematical Physics)

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PAPER—PH- 1101 (A)

[*Marks* : 20]

Time : 1 hour

Answer **Q.No.1** and any *one* from the rest

1. Answer any *five* bits : 2x5

(*a*) Prove that a Hermitian operator is represented by a Hermitian matrix.

(*b*) Find orthogonal set from non-orthogonal set $\{1, x, x^2\}$ by Gram-Schmidt process.

(*c*) Find the inverse of the matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

(*d*) Evaluate

$$\int_0^\infty \sqrt{x}\, e^{-x^3}\, dx.$$

(*e*) Prove that the real and the imaginary parts of a complex analytic function are harmonic functions.

(*f*) Transform the following equation into an equivalent self-adjoint form :

$$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + xy = 0.$$

(*g*) Locate and classify the singularities of the function

$$f(z) = \exp\left(\frac{1}{z^2}\right).$$

(*h*) Consider the equation

$$Ly = 0, \text{ where } L \equiv \frac{d^2}{dx^2} + q(x)$$

If y_1 and y_2 are two linearly independent solutions of this equation, then show that their Wrouskian is a constant.

2. (*a*) Express

$$f(x) = x^3 + 2x^2 - x - 3$$

in terms of Legendre polynomials in the internal $-1 < x < 1$.

(*b*) Prove that

$$2x\, H_n(x) = 2n\, H_{n-1}(x) + H_{n+1}(x)$$

using generating function of Hermite polynomial.

(*c*) Find the value of

$$\int_{-1}^{+1} (1+x)\, P_n(x)\, dn \quad \text{for} \quad n>1.$$

3 + 4 + 3

3. (*a*) Prove that if two matrices commute, then they must have the same set of eigenvectors. Assume that the eigenvalues are non-degenerate.

(*b*) If $f(x)=0$ for $-1 \leq x<0$ and $f(x)=x$ for $0 \leq x \leq 1$, then obtain first three terms in the expansion of $f(x)$ in terms of Legendre polynomials.

(*c*) Evaluate the following integral using the residue theorem :

$$\int_{0}^{\infty} \frac{dx}{x^2+4}$$

Draw the contour and show the singular points clearly. 3 + 3 + 4

PAPER—PH- 1101 (B)

[*Marks* : 20]

Time : 1 hour

Answer **Q.No.1** and any *one* from the rest

1. Answer any *five* bits : 2×5

(*a*) With the help of Greek alphabet how can you express the 4th state of matter ?

(*b*) State the process by which plasma occur in nature.

(*c*) Draw the schematic circuit of Inductively coupled toroidal discharge for the study of the breakdown process in air.

(*d*) Show graphically, in classical view point, the phase space and volume element under the concepts of plasma kinetic theory.

(*e*) Write mathematical expression of electron temperature (T_e) in terms of electric field and pressure in plasma, and work out its physical significance.

(*f*) Discuss the distribution function in phase space.

(*g*) Discuss, with graphical presentation of current waveform with time for the operation of laboratory built exploding wire discharge method.

(*h*) Depending upon the degree of ionization name the catagories into which plasma can be classified.

2. Define thermal ionization. Deduce Saha's ionization formula and point out its applications. 10

3. What do you mean by ambipolar diffusion in plasma ? Deduce an expression for ambipolar diffusion coefficient. Show that when $T_e \simeq T_i$, the ambipolar diffusion coefficient is approximately twice the ion diffusion coefficient. 3 + 5 + 2