Chapter 7

Stage structure predator model with two preys*

Prey-predator model with stage structure of predator is considered in this chapter. Two types of prey are considered for immature and mature predator respectively in the formulated model. Consumption rate of prey by immature predator is described by Holling type II functional response whereas for mature predator is taken as Holling type III functional response. Logistic growth rates are chosen for both preys in this chapter. As this is a stage structure model, so immature predator transforms to mature predator after certain stage with a constant rate which is treated as bifurcation parameter. Different mortality rates are taken for both predators. Local and global stabilities are discussed to validity of the proposed model. Finally, a numerical simulation has been included to verify the analytic results and the system is analyzed through graphical illustrations. Conclusions of the findings and outlook of the chapter are depicted at last.

7.1 Introduction

Ecology is the scientific analysis with the study of interactions among organisms and their environments. Amount of biomass and number of population of particular organisms, as well as co-operation and competition between them,

*A part of this chapter has been communicated to the International Journal

within and among ecosystems are the main topics of interest by many ecologists. The relationship between two organisms, where one organism consumes to other is called prey-predator relationship. A predator is an organism that eats another organism. The prey is an organism which the predator eats. There are different types of predator-prey models in different environments. Depending on time in the species life cycle, feeding capacities of species are changed. Sometimes, resources of food are also different on different stages of species life cycle. For this reason, prey of predator is also changed based on life cycle of predator. In different stages of life cycle of an organism there are different food habits. Many researchers described about stage structure but they did not change feeding habit of predators. In ecology, change of feeding habit in different stages in life cycle of species is a common phenomenon. In this chapter, we concentrate this fact and to develop the proposed model.
Here, we consider an example on frog's life cycle, where tadpole (immature predator) food habit is algae and frog (mature predator) food habit is insect. Algae and insect lived in different environments. They also belong to different classes. So their growth rates and environmental carrying capacities are different and also they have no direct link in prey-predator relationship. Tadpole transforms into frog after certain stage but some tadpoles died of environmental difficulties. To more clarify this fact, a graphical representation is depicted in Figure 7.1. Considering this real-life phenomenon, we design the mathematical model in Section 7.3.

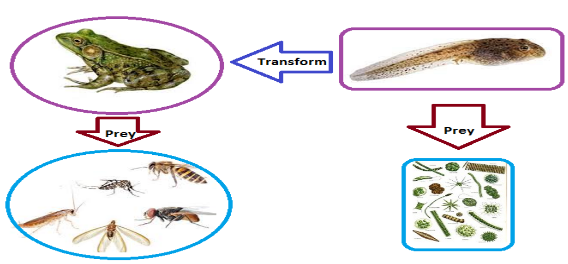

Figure 7.1: Graphical presentation of discussed phenomenon.

7.2 Notation

Table-7.2.1: Description of the parameters.

Parameter	Description of the parameters
x	Prey population for immature predator at time t
y	Prey population for mature predator at time t
z_1	Population of immature predator at time t
z_2	Population of mature predator at time t
r	Intrinsic growth rate of prey for immature predator
K	Environmental carrying capacity of the prey for immature predator
s	Intrinsic growth rate of prey for mature predator
L	Environmental carrying capacity of the prey for mature predator
m	Maximal immature predator per capita consumption rate
m_1	Maximal mature predator per capita consumption rate
a, b	Half saturation constants
d_1	Natural death rate of immature predator
d_2	Natural death rate of mature predator
β	Immature to mature predator transform rate

7.3 Mathematical model

At time t, assume that $x(t)$ denotes prey population for immature predator which is denoted by $z_1(t)$, and $y(t)$ denotes prey for mature predator which is considered by $z_2(t)$. Here algae is chosen as prey for immature predator and insect is considered as prey for mature predator. Depending on intake capacity of predator and other factor, it is assumed that the functional response of immature population will be Holling type II functional response and that of mature predator population will be Holling type III functional response. Then the system of equations becomes as follows:

$$\left.\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right) - m\frac{xz_1}{a+x} \\
\frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right) - m_1\frac{y^2 z_2}{b+y^2} \\
\frac{dz_1}{dt} &= n\frac{xz_1}{a+x} - \beta z_1 - d_1 z_1 \\
\frac{dz_2}{dt} &= \beta z_1 + n_1\frac{y^2 z_2}{b+y^2} - d_2 z_2
\end{aligned}\right\} \tag{7.1}$$

with initial conditions $x(0) \geq 0, y(0) \geq 0, z_1(0) \geq 0$ and $z_2(0) \geq 0$.
And the growth rates of immature and mature predators are denoted by n and n_1 respectively.

7.4 Equilibrium point, Positivity and Boundedness

Consider an interior equilibrium point is $P(\bar{x}, \bar{y}, \bar{z_1}, \bar{z_2})$ where $\bar{x}, \bar{y}, \bar{z_1}$ and $\bar{z_2}$ are the positive roots of the equation $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz_1}{dt} = \frac{dz_2}{dt} = 0$. From these equations, the following results are obtained.

$$\bar{z_2} = s\left(1 - \frac{\bar{y}}{L}\right)\frac{b + \bar{y}^2}{m_1\bar{y}} \tag{7.2}$$

$$\bar{z_1} = r\left(1 - \frac{\bar{x}}{K}\right)\frac{a + \bar{x}}{m} \tag{7.3}$$

$$sm\left(1 - \frac{\bar{y}}{L}\right)\frac{d_2 b - \bar{y}^2(n_1 - d_2)}{\bar{y}} = \beta m_1 r\left(1 - \frac{\bar{x}}{K}\right)(a + \bar{x}) \tag{7.4}$$

$$\bar{x} = \frac{a(\beta + d_1)}{n - d_1 - \beta} \tag{7.5}$$

So, the sufficient conditions for the system (7.1) with a positive interior equilibrium point are $n > d_1 + \beta$ and $n_1 > d_2$, i.e., growth rate of immature predator population will be greater than the sum of death rate of immature predator population and transform rate of immature to mature predator population; and also growth rate of mature predator population will be greater than the death rate of mature predator population.

Theorem 7.4.1. *All the solutions of the system* (7.1) *are always bounded.*

Proof: From the first equation of system (7.1), it is seen that carrying capacity of prey population of immature predator population is K and from the second equation of system (7.1), it is seen that the carrying capacity of prey population of mature predator population is L. For $\epsilon_1 > 0$, we have $x \leq K + \epsilon_1$ as $t \to \infty$ and for $\epsilon_2 > 0$, we have $y \leq L + \epsilon_2$ as $t \to \infty$.
Now let $W = \frac{n}{m}x + \frac{n_1}{m_1}y + z_1 + z_2$.

Then we get,

$$
\begin{aligned}
&\frac{dW}{dt} = \frac{n}{m}\frac{dx}{dt} + \frac{n_1}{m_1}\frac{dy}{dt} + \frac{dz_1}{dt} + \frac{dz_2}{dt}\\
&\frac{dW}{dt} = rx\frac{n}{m}\left(1-\frac{x}{K}\right) + sy\frac{n_1}{m_1}\left(1-\frac{y}{L}\right) - d_1z_1 - d_2z_2\\
&\frac{dW}{dt} \leq rx\frac{n}{m} + sy\frac{n_1}{m_1} - \rho z_1 - \rho z_2, \qquad \text{where} \quad \rho = \min\{d_1, d_2\}\\
i.e., \quad &\frac{dw}{dt} \leq -\rho W + (r+\rho)\frac{n}{m}x + (s+\rho)\frac{n_1}{m_1}y\\
i.e., \quad &\frac{dW}{dt} \leq -\rho W + \frac{n}{m}(r+\rho)(K+\epsilon_1) + \frac{n_1}{m_1}(s+\rho)(L+\epsilon_2)\\
i.e., \quad &\frac{dW}{dt} + \rho W \leq I, \qquad (7.6)\\
&where \quad I = \frac{n}{m}(r+\rho)(K+\epsilon_1) + \frac{n_1}{m_1}(s+\rho)(L+\epsilon_2)
\end{aligned}
$$

On integrating both sides of equation (7.6) between 0 to t, we obtain $0 < W \leq \frac{I}{\rho}(1-e^{-\rho t}) + e^{-\rho t}W[x(0), y(0), z_1(0), z_2(0)]$ as $t \to \infty$. Also, we have $0 < W \leq \frac{I}{\rho}$. From above analysis, we conclude that the solution space (x, y, z_1, z_2) is bounded in the specified region. Thus, the result follows the theorem.

7.5 Stability analysis

The stability criterion of the system is analyzed at the interior equilibrium point $P(\bar{x}, \bar{y}, \bar{z}_1, \bar{z}_2)$. The Jacobian matrix of the system (7.1) at the interior equilibrium point P is denoted by J and is defined as follows:

$$
J = \begin{pmatrix}
-\frac{r}{K}\bar{x} + m\frac{\bar{x}\bar{z}_1}{(a+\bar{x})^2} & 0 & -m\frac{\bar{x}}{a+\bar{x}} & 0\\
0 & m_1\bar{y}\bar{z}_2\frac{\bar{y}^2-b}{(b+\bar{y}^2)^2} - \frac{s}{L}\bar{y} & 0 & -m_1\frac{\bar{y}^2}{b+\bar{y}^2}\\
\frac{an\bar{z}_1}{(a+\bar{x})^2} & 0 & 0 & 0\\
0 & 2bn_1\frac{\bar{y}\bar{z}_2}{(b+\bar{y}^2)^2} & \beta & -\beta\frac{\bar{z}_1}{\bar{z}_2}
\end{pmatrix}.
$$

Now the characteristic equation of system (7.1) around its interior equilibrium is

$det(J - \lambda I) = 0$, i.e., $\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0$.

where I represents an identity matrix of order 4 and

$$
\begin{aligned}
A_1 &= \frac{r}{K}\bar{x} + \frac{s}{L}\bar{y} - m\frac{\bar{x}\bar{z}_1}{(a+\bar{x})^2} - m_1\bar{y}\bar{z}_2\frac{\bar{y}^2-b}{(b+\bar{y}^2)^2} - \beta\frac{\bar{z}_1}{\bar{z}_2},\\
A_2 &= \beta\frac{\bar{z}_1}{\bar{z}_2}\left(m_1\bar{y}\bar{z}_2\frac{\bar{y}^2-b}{(b+\bar{y}^2)^2} - \frac{s}{L}\bar{y}\right) + 2bm_1n_1\frac{\bar{y}^3\bar{z}_2}{(b+\bar{y}^2)^3} + amn\frac{\bar{x}\bar{z}_1}{(a+\bar{x})^3}\\
&\quad + \left(m\frac{\bar{x}\bar{z}_1}{(a+\bar{x})^2} - \frac{r}{K}\bar{x}\right)\left(m_1\bar{y}\bar{z}_2\frac{\bar{y}^2-b}{(b+\bar{y}^2)^2} + \beta\frac{\bar{z}_1}{\bar{z}_2} - \frac{s}{L}\bar{y}\right),\\
A_3 &= \left\{\beta\frac{\bar{z}_1}{\bar{z}_2}\left(\frac{m_1\bar{y}\bar{z}_2(\bar{y}^2-b)}{(b+\bar{y}^2)^2} - \frac{s\bar{y}}{L}\right) + \frac{2bm_1n_1\bar{y}^3\bar{z}_2}{(b+\bar{y}^2)^3}\right\}\left(\frac{r}{K}\bar{x} - m\frac{\bar{x}\bar{z}_1}{(a+\bar{x})^2}\right)\\
&\quad - amn\frac{\bar{x}\bar{z}_1}{(a+\bar{x})^3}\left(m_1\bar{y}\bar{z}_2\frac{\bar{y}^2-b}{(b+\bar{y}^2)^2} + \beta\frac{\bar{z}_1}{\bar{z}_2} - \frac{s}{L}\bar{y}\right),\\
A_4 &= amn\beta\frac{\bar{x}\bar{z}_1^2}{(a+\bar{x})^2\bar{z}_2}\left(m_1\bar{y}\bar{z}_2\frac{\bar{y}^2-b}{(b+\bar{y}^2)^2} - \frac{s}{L}\bar{y}\right) + \frac{2abmnm_1n_1\bar{x}\bar{y}^3\bar{z}_1\bar{z}_2}{(a+\bar{x})^3(b+\bar{y}^2)^3}
\end{aligned}
$$

Let us consider, $A = c_1 - \beta c_2, B = c_3 - \beta c_4, C = c_5 - \beta c_6, D = c_7 - \beta c_8$. Then we have $BC - AD = Q_1\beta^2 + Q_2\beta + Q_3 = \varphi(\beta)$ (say), where $Q_1 = c_4c_6 - c_2c_8, Q_2 = c_2c_7 + c_1c_8 - c_5c_4 - c_3c_6, Q_3 = c_3c_5 - c_1c_7$ and $det\begin{pmatrix} C & D & 0\\ A & B & C\\ 0 & 1 & A\end{pmatrix} = Q_4\beta^3 + Q_5\beta^2 + Q_6\beta + Q_7 = \psi(\beta)$ (say), where $Q_4 = c_2^2c_8 - c_2c_4c_6,\ Q_5 = c_2c_4c_5 + c_2c_3c_6 + c_1c_4c_6 - c_2^2c_7 - 2c_1c_2c_8 - c_6^2,\ Q_6 = c_1^2c_8 + 2c_1c_2c_7 + 2c_5c_6 - c_2c_3c_5 - c_1c_4c_5 - c_1c_3c_6,\ Q_7 = c_1c_3c_5 - c_1^2c_7 - c_5^2$.

Here all $Q_j,\ (j = 1, 2, ..., 7)$ are functions of β since the interior equilibrium depends on β. But for a known parameter set it is possible to find all the values of Q_j in terms of β. In that case, we assume that $\overline{\beta}$ is the common positive root of $\varphi(\beta) = 0$ and $\psi(\beta) = 0$. Now using Routh-Hurwitz criteria around the interior equilibrium point, we can state and prove the following theorem for the local asymptotic stability of the system (7.1).

Theorem 7.5.1. *Assuming all $Q_j\ \ (j = 1, 2, ..., 7)$, C, $BC - AD$ and $det\begin{pmatrix} C & D & 0\\ A & B & C\\ 0 & 1 & A\end{pmatrix}$ be positive. Then the equilibrium point $P(\bar{x}, \bar{y}, \bar{z_1}, \bar{z_2})$ of the system* (7.1) *is locally asymptotically stable.*

Proof: Using Routh-Hurwitz criterion, the conclusion becomes all eigenvalues of the system (7.1) around its interior equilibrium point $P(\bar{x}, \bar{y}, \bar{z_1}, \bar{z_2})$ has negative real part. Consequently the system will be locally asymptotically stable. This result evinces the proof of the theorem.

Lemma 6. *In theorem (7.5.1), we have proved that for a known parameter set we find all Q_j in terms of β. In this case if $\bar{\beta}$ is the only common positive root of $\varphi(\beta) = 0$ and $\psi(\beta) = 0$. Then for $\beta > \bar{\beta}$, $BC - AD$ and $det \begin{pmatrix} C & D & 0 \\ A & B & C \\ 0 & 1 & A \end{pmatrix}$ are positive. Again if $BC - AD$ and $det \begin{pmatrix} C & D & 0 \\ A & B & C \\ 0 & 1 & A \end{pmatrix}$ are negative then the system* (7.1) *must be unstable around $P(\bar{x}, \bar{y}, \bar{z}_1, \bar{z}_2)$. Also if $det \begin{pmatrix} C & D & 0 \\ A & B & C \\ 0 & 1 & A \end{pmatrix} = 0$ then the system* (7.1) *undergoes through a bifurcation. In the next theorem, we describe about Hopf bifurcation.*

Theorem 7.5.2. *The system* (7.1) *follows Hopf bifurcation about the point $P(\bar{x}, \bar{y}, \bar{z}_1, \bar{z}_2)$ for $\beta = \bar{\beta}$.*

Proof: For $\beta = \bar{\beta}$, we have $det \begin{pmatrix} C & D & 0 \\ A & B & C \\ 0 & 1 & A \end{pmatrix} = 0$ and then the eigenvalues of the system at $P(\bar{x}, \bar{y}, \bar{z}_1, \bar{z}_2)$ can be represented as $\lambda_{1,2} = \pm i\sqrt{C_1}$ and $\lambda_{3,4} = \pm i\sqrt{C_2}$.

Considering $\lambda_{1,2} = \phi_1(\beta) \pm i\phi_2(\beta)$ and $\lambda_{3,4} = \phi_3(\beta) \pm i\phi_4(\beta)$. Now it is obvious to show that $\frac{d\phi}{d\beta} \neq 0$ at the point $\beta = \bar{\beta}$ where ϕ represents ϕ_1 and ϕ_3. Again, we have $\phi(\bar{\beta}) = 0$. Therefore, it is clear that our system (7.1) follows a Hopf bifurcation at the interior equilibrium for the critical value of β, i.e., for $\beta = \bar{\beta}$, with the help of given conditions (103). So, the theorem is obvious.

Prey-predator models with constant parameters are often found to approach a steady state in which the species coexist in equilibrium. But if parameters used in the model are changed, other types of dynamical behavior may occur and the critical parameter values at which such transitions happen, are called bifurcation points. The purpose of this chapter is to determine the stability behavior of the system in presence of different density-dependent factors of the prey-predator interactions. To study the transition of the system with respect to the small changes in the density dependent factors, we consider, β

as bifurcation parameter and it represents the critical value or the bifurcating value of the concerned bifurcation parameter.

Theorem 7.5.3. *The system* (7.1) *will be globally asymptotically stable at an interior equilibrium point* $P(\bar{x}, \bar{y}, \bar{z_1}, \bar{z_2})$*, if the sufficient conditions that* $R(0) > 0$ *where* $R(x) = \frac{r}{K} - \frac{m\bar{z_1}}{(a+\bar{x})(a+x)}$ *holds.*

Proof: Let us choose a Lyapunov function which is defined as follows
$V(x, y, z_1, z_2) = \int_{\bar{x}}^{x} \frac{x-\bar{x}}{x} dx + p_1 \int_{\bar{y}}^{y} \frac{y-\bar{y}}{y} dy + p_2 \int_{\bar{z_1}}^{z_1} \frac{z_1-\bar{z_1}}{z_1} dz_1 + p_3 \int_{\bar{z_2}}^{z_2} \frac{z_2-\bar{z_2}}{z_2} dz_2$
where p_i $(i = 1, 2, 3)$ is suitable non negative constant, to be determined in the following subsequent steps.
Time derivative of the equation along the solution of the system (7.1) is given as:

$$\begin{aligned}
\frac{dV}{dt} &= \frac{x-\bar{x}}{x}\frac{dx}{dt} + p_1 \frac{y-\bar{y}}{y}\frac{dy}{dt} + p_2 \frac{z_1-\bar{z_1}}{z_1}\frac{dz_1}{dt} + p_3 \frac{z_2-\bar{z_2}}{z_2}\frac{dz_2}{dt} \\
&= (x-\bar{x})\left\{r\left(1-\frac{x}{K}\right) - m\frac{z_1}{a+x}\right\} + p_1(y-\bar{y})\left\{s\left(1-\frac{y}{L}\right) - m_1\frac{yz_2}{b+y^2}\right\} \\
&\quad + p_2(z_1-\bar{z_1})\left\{n\frac{x}{a+x} - \beta - d_1\right\} + p_3(z_2-\bar{z_2})\left\{\beta\frac{z_1}{z_2} - n_1\frac{y^2}{b+y^2} - d_2\right\}
\end{aligned}$$

Again at the interior equilibrium point $P(\bar{x}, \bar{y}, \bar{z_1}, \bar{z_2})$, $\dot{x} = \dot{y} = \dot{z_1} = \dot{z_2} = 0$ i.e.,
$r = \frac{r}{K}\bar{x} + \frac{m\bar{z_1}}{a+\bar{x}}$, $s = \frac{s}{L}\bar{y} + m_1\frac{\bar{y}\bar{z_2}}{b+\bar{y}^2}$, $\beta + d_1 = n\frac{\bar{x}}{a+\bar{x}}$, $-d_2 = n_1\frac{\bar{y}^2}{b+\bar{y}^2} - \beta\frac{\bar{z_1}}{\bar{z_2}}$.
Substituting these, we have

$$\begin{aligned}
\frac{dV}{dt} &= (x-\bar{x})\left\{-\frac{r}{K}(x-\bar{x}) + m\left(\frac{\bar{z_1}}{a+\bar{x}} - \frac{z_1}{a+x}\right)\right\} \\
&\quad + p_1(y-\bar{y})\left\{-\frac{s}{L}(y-\bar{y}) - m_1\left(\frac{yz_2}{b+y^2} - \frac{\bar{y}\bar{z_2}}{b+\bar{y}^2}\right)\right\} \\
&\quad + p_2(z_1-\bar{z_1})\left\{n\left(\frac{x}{a+x} - \frac{\bar{x}}{a+\bar{x}}\right)\right\} \\
&\quad + p_3(z_2-\bar{z_2})\left\{\beta\left(\frac{z_1}{z_2} - \frac{\bar{z_1}}{\bar{z_2}}\right) - n_1\left(\frac{y^2}{b+y^2} - \frac{\bar{y}^2}{b+\bar{y}^2}\right)\right\} \\
&= \left(-\frac{r}{K} + \frac{m\bar{z_1}}{(a+x)(a+\bar{x})}\right)(x-\bar{x})^2 - p_1\left(\frac{s}{L} + \frac{m_1 z_2(b-y\bar{y})}{(b+\bar{y}^2)(b+y^2)}\right)(y-\bar{y})^2 \\
&\quad - p_3\beta\frac{\bar{z_1}}{z_2\bar{z_2}}(z_2-\bar{z_2})^2 + \left(\frac{m}{a+x} + p_2\frac{an}{(a+\bar{x})(a+x)}\right)(x-\bar{x})(z_1-\bar{z_1})
\end{aligned}$$

$$-p_1\frac{m_1y}{(b+\bar{y}^2)}(y-\bar{y})(z_2-\bar{z_2})+p_3\frac{\beta}{z_2}(z_1-\bar{z_1})(z_2-\bar{z_2})$$
$$-p_3n_1b\frac{(y+\bar{y})}{(b+\bar{y}^2)(b+y^2)}(y-\bar{y})(z_2-\bar{z_2}) \qquad (7.7)$$

Assume that, $p_1 = 0$, $p_2 = \frac{m(a+\bar{x})}{an}$, $p_3 = 0$. Then (7.7) becomes as follows:

$$\frac{dV}{dt} = -\left(\frac{r}{K} - m\frac{\bar{z_1}}{(a+x)(a+\bar{x})}\right)(x-\bar{x})^2 \qquad (7.8)$$

Since, $R(0) > 0$ then from (7.8), we conclude that $\frac{dV}{dt} \leq 0$. So, the theorem holds.

7.6 Numerical Simulation

To describe the analytical results numerically, we take some hypothetical data. Here we use MATLAB 7.10 software to analyze numerical simulation. Now, we choose a parameter set $P_1(r = 1.5; s = 2.6; K = 20; L = 25; m_1 = 0.7; n_1 = 0.3; m = 2; n = 0.9; a = 2; d_2 = 0.2; d_1 = 0.1; b = 6)$ and $\beta = 0.7$ with initial point as $B(9, 7, 7, 3)$ and the graphical representation is shown in Figure 7.2. In Section 3, we have discussed about existence and boundedness of the interior equilibrium point $P(\bar{x}, \bar{y}, \bar{z_1}, \bar{z_2})$. From the stability analysis, we have seen that if $\beta > \bar{\beta}$ then the system is stable, and it $\beta < \bar{\beta}$ then the system is unstable. Using that parameter set with $\beta(= 0.7) > \bar{\beta}$, we draw Figure 7.2. From Figure 7.2, we have observed that the system is stable. Using that parameter set with $\beta(= 0.5) < \bar{\beta}$, we draw Figure 7.3. From Figure 7.3, we have seen that the system is unstable. For the existence of the system, the intrinsic growth rate of prey populations has an important role. From Figure 7.4, it is observed that, for fixed carrying capacity, the density of both prey population are inversely proportional and density of both predator population are directly proportional to the intrinsic growth rate of prey population of immature predator. From Figure 7.5, it is noticed that, for changing the growth rate of prey population of mature predator, prey population of immature predator as well as immature predator population will remain same but prey population of mature predator as well as mature predator is directly proportional. Carrying capacity of any system has an impact for an eco-friendly environment. When the other parameters remain same, the prey population of immature predator, immature predator population and mature predator population are directly proportional to the carrying capacity of the prey population of immature predator. Also, when the other parameters remain same, the prey population of mature predator is inversely proportional to the carrying capacity of the prey population of immature predator which is shown in Figure 7.6. Also from Figure 7.7, it is seen that both prey population directly proportional and both predator population inversely proportional to per capita consumption rate of immature predator population. From Figure 7.8, it is observed that the prey population of mature predator as well as mature predator population inversely proportional to the per capita consumption rate of mature predator but there is no impact of per capita consumption rate of mature predator on prey population of immature predator as well as immature predator population.

7.7 Chapter Summary

We have introduced a prey-predator model with stage structure of predator. Also we have considered that preys for immature predator and mature predator are different. Consumption rates of prey by the immature predator and mature predator have been described by suitable functions with an ecological phenomenon. Both preys obey logistic growth rate. Also, we have described the different growth rates and different carrying capacities for the different prey populations. In this context, this research work is significantly different in compare to other research works in this area. Immature predator transforms to mature predator in a constant rate, which is chosen as bifurcation parameter. Mortality rates of immature predator and mature predator are different. The local as well as global stability around at an interior equilibrium has been discussed. The problem has been illustrated with a numerical example. Also the proposed model has been analyzed with the geometrical figures. Global stability of the system has been shown by choosing a suitable Lyapunov function. Finally, we have presented some numerical simulations to verify the analytic results and the system has been analyzed through graphical illustrations.

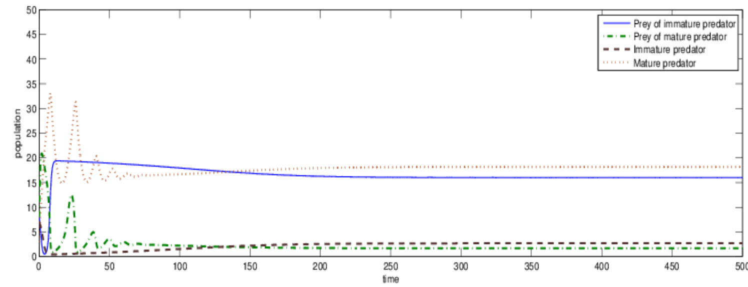

Figure 7.2: *Graphical representation of the system with parameter set* P_1 *and* $\beta = 0.7$.

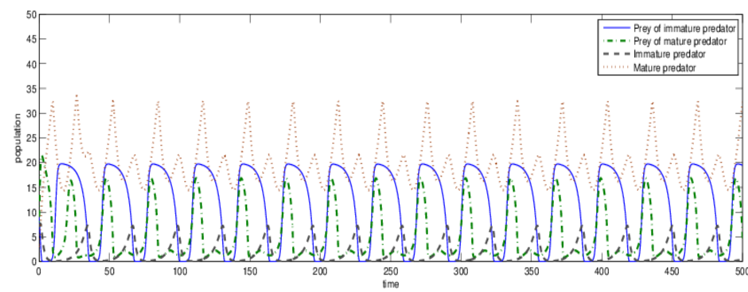

Figure 7.3: *Graphical representation of the system with parameter set* P_1 *and* $\beta = 0.5$.

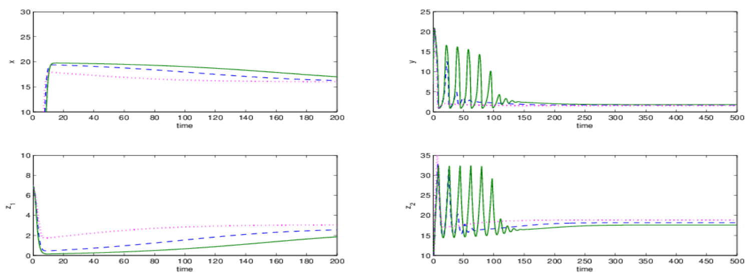

Figure 7.4: *Change of* x, y, z_1 *and* z_2 *of the system* (7.1) *with respect to change of intrinsic growth rate of prey population of immature predator population with parameter set* $\{s = 2.6; K = 20; L = 25; m_1 = 0.7; n_1 = 0.3; m = 2; n = 0.9; a = 2; d_2 = 0.2; d_1 = 0.1; b = 6\}$. *Here (—) line corresponds to* $r = 1.3$, *(- - -) line to* $r = 1.5$ *and* $(\cdots)$ *line to* $r = 1.7$.

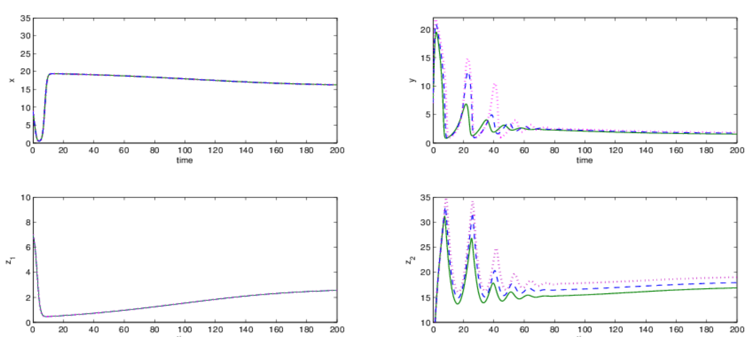

Figure 7.5: *Change of* x, y, z_1 *and* z_2 *of the system* (7.1) *with respect to change of intrinsic growth rate of prey population of mature predator population with parameter set* $\{r = 1.5; K = 20; L = 25; m_1 = 0.7; n_1 = 0.3; m = 2; n = 0.9; a = 2; d_2 = 0.2; d_1 = 0.1; b = 6\}$. *Here (—) line corresponds to* $s = 2.4$, *(- - -) line to* $s = 2.6$ *and* $(\cdots)$ *line to* $s = 2.8$.

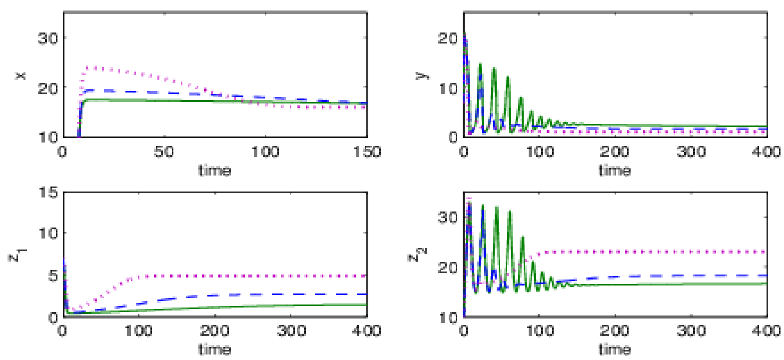

Figure 7.6: *Change of* x, y, z_1 *and* z_2 *of the system* (7.1) *with respect to change of carrying capacity of prey population of immature predator population with parameter set* $\{r = 1.5; s = 2.6; L = 25; m_1 = 0.7; n_1 = 0.3; m = 2; n = 0.9; a = 2; d_2 = 0.2; d_1 = 0.1; b = 6\}$. *Here (—) line corresponds to* $K = 18$, *(- - -) line to* $K = 20$ *and* ($\cdots$) *line to* $K = 25$.

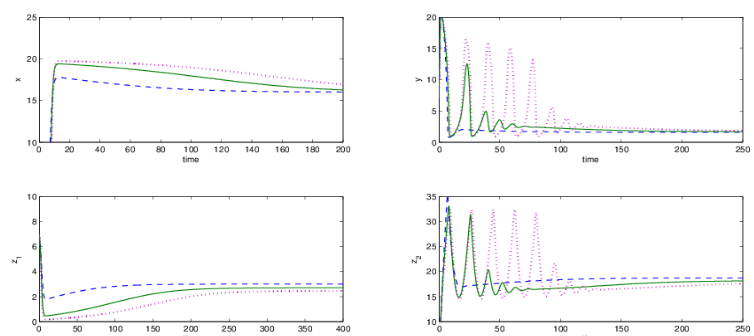

Figure 7.7: *Change of* x, y, z_1 *and* z_2 *with respect to change of* m *with parameter set* $\{r = 1.5; s = 2.6; K = 20; L = 25; m_1 = 0.7; n_1 = 0.3; n = 0.9; a = 2; d_2 = 0.2; d_1 = 0.1; b = 6\}$. *Here (- - -) line corresponds to* $m = 1.8$, *(—) line to* $m = 2$ *and* ($\cdots$) *line to* $m = 2.2$.

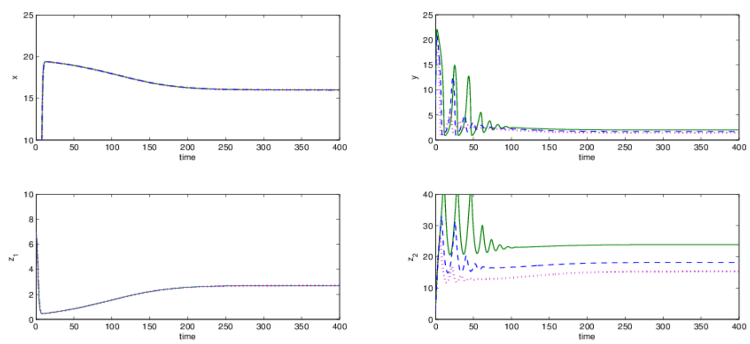

Figure 7.8: *Change of* x, y, z_1 *and* z_2 *with respect to change of* m_1 *with parameter set* $\{r = 1.5; s = 2.6; K = 20; L = 25; n_1 = 0.3; m = 2; n = 0.9; a = 2; d_2 = 0.2; d_1 = 0.1; b = 6\}$. *Here (—) line corresponds to* $m_1 = 0.5$, *(- - -) line to* $m_1 = 0.7$ *and (—) line to* $m_1 = 0.9$.