UG/3rd Sem/MATH(G)/T/19

2019

B.Sc.

3rd Semister Examination

MATHEMATICS (General)

Paper - SEC 1-T

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practiable. Illustrate the answers wherever necessary.

Theory of Equations

1. Answer any *five* questions : 5×2=10

(a) If α be a multiple root of order 3 of the equation $x^4+bx^2+cx+d=0$, then show that $\alpha=-\dfrac{8d}{3c}$.

(b) State Descarte's rule of signs.

9/224-1200

(c) If all the roots of the equation f(x) = 0 be non-zero real and ν, ν' are respectively the number of variations of signs in the sequence of coefficients of f(x) and f(−x) then prove that equation f(x) = 0 has ν positive roots and ν' negative roots.

(d) If α, β, γ be the roots of $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma \frac{\beta^2 + \gamma^2}{\beta + \gamma}$.

(e) Show that the equation $x^4 - 14x^2 + 24x + k = 0$ has four real and unequal roots if $-11 < k < -8$.

(f) If f(x) be a polynomial in x and a, b are unequal, show that the remainder in the division of f(x) by $(x-a)(x-b)$ is

$$\frac{(x-b)f(a)-(x-a)f(b)}{a-b}$$

(g) If $\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_n$ be the roots of the equation $x^n + nax + b = 0$, prove that

$$(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\ldots\ldots(\alpha_1-\alpha_n)=n\left(\alpha_1^{n-1}+a\right)$$

(h) Prove that the roots of the equation

$$\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3}=x \text{ are all real.}$$

2. Answer any *four* questions : 4×5=20

(a) Find the condition that the equation $x^4+px^3+qx^2+rx+s=0$ should have its roots $\alpha, \beta, \gamma, \delta$ connected by the relation $\alpha\beta+1=0$.

(b) Solve the equation $x^3+3x^2+6x+4=0$ by Cardan's method.

(c) Prove that the equation $x^3+x^2-5x-1=0$ has one positive root lying in (1, 2) and two negative roots lying in (–1, 0) and (–3, 2).

(d) If n be a prime and α is an imaginary root of $x^n-1=0$, then $1, \alpha, \alpha^2, \ldots\ldots\ldots, \alpha^{n-1}$ is a complete list of the roots of the equation $x^n-1=0$.

(e) Find the equation whose roots are squares of the differences of the roots of the equation

$$x^3+6x^2+9x+4=0$$

[Turn Over]

(f) Solve the equation $x^5 - 1 = 0$ and deduce the value of $\cos\frac{\pi}{5}$ and $\cos\frac{2\pi}{5}$.

3. Answer any *one* question : $1 \times 10 = 10$

(a) (i) Solve the equation $3x^3 + 5x^2 + 5x + 3 = 0$, given that it has three distinct roots of equal moduli.

(ii) The equation whose roots are the squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is found to be identical with this cubic. Prove that either (i) a = b = 0 or (ii) a = b = 3 or (iii) a, b are the roots of the equation $t^2 + t + 2 = 0$. 4+6

(b) (i) Prove that the equation $(x+1)^4 = a(x^4 + 1)$ is a reciprocal equation if $a \neq 1$ and solve it when a = –2.

(ii) If an equation with real coefficients has a complex root $\alpha + i\beta$, then prove that it has also the conjugate complex root $(\alpha - i\beta)$.

5+5

Logic and Sets

Group - A

1. Answer any *five* questions : 5×2=10

(a) Construct truth table of the following statement

$(p \wedge q) \wedge \sim (p \vee q)$

(b) Translate the following sentence into a statement formula

"It is raining but the sun is still shining"

(c) Let p(x) be the sentence x + 1 < 3. State whether p(x) is a predicate or not on the domain C (Set of complex numbers).

(d) What is tautology. Explain with example.

(e) $A = \left\{x \in U \mid x^2 - 5x + b = 0\right\}$, and

$B = \left\{x \in U \mid x^2 - 1 = 0\right\}$

Find A × B

(f) Construct an equivalence relation on set {1,2,3}.

(g) Define 'Partition of a set'.

[Turn Over]

(h) Prove De Morgan's law

$(A \cup B)^C = A^C \cap B^C$ using Venn diagram.

Group - B

2. Answer any *four* questions : 4×5=20

(a) Verify by truth tables

$p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$ 5

(b) Test whether the following set of assumptions is consistent "If the party is dull, Priya starts crying or Suman tells jokes. If Manas comes to the party, then either the party is dull or Priya starts crying. If Suman tells jokes then Priya doesn't start crying. Manas comes to the party if and only if Suman does not tell jokes. If Priya starts crying, then Suman tells jokes." 5

(c) Prove the followings

(i) $\sim \forall x p(x) \equiv \exists x \sim p(x)$ 2½

(ii) $\sim \exists x p(x) \equiv \forall x \sim p(x)$ 2½

(d) Prove the following results algebraically

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 3

(ii) $A \cap \varphi = \varphi$ 2

(e) ${}^{x}R_{y} : 2x+3y$ is divisible by 5. Show that the relation R is an equivalence relation. 4

Define Partial order relation. 1

(f) (i) Describe the following terms :

Difference between two sets and Symmetric difference of two sets. 1½+1½

(ii) Prove that

$$A \Delta B = \left(A \cap B^{C}\right) \cup \left(A^{C} \cap B\right) \qquad 2$$

Group - C

Answer any *one* question : 1×10=10

3. (a) Find the negation of each of the following quantified predicates

(i) $(\exists x \in D)(x+2=7)$

(ii) $(\forall x \in D)(x+3<10)$ where

D = {1, 2, 3, 4} 2+2

[Turn Over]

(b) Let N = the set of positive integers and D = {1, 2, 3, 4, 5} be the domains of x and y respectively. Find the truth set of the predicate $(\forall y \in D)(x+y<12)$ 3

(c) Test the validity of the statement "Every differentiable function is continuous and g is continuous. Therefore, g is differentiable". 3

4. (a) Prove that number of elements of a power set of set containing n elements is 2^n. 3

(b) Define relation and write different types of relation. 2

(c) Check whether $x \leq y$, $x, y \in z$ is equivalence or not. 3

(d) State De Morgan's law on set theory. 2

Boolean Algebra

1. Answer any *five* questions : $5\times2=10$

(a) Define a partially ordered set with example.

(b) Draw the Hasse diagram of the partially ordered set {2, 3, 4, 6} under the relation of divisibility.

(c) Let L be a lattice. Show that every singleton subset is a sublattics of L.

(d) If $(\beta, +, \bullet, ')$ is a Boolean algebra, then prove that (i) $a+1=1$ (ii) $a\cdot 0=0$ for all $a\in B$.

(e) In the Boolean algebra $(\beta, +, \bullet, ', 0, 1)$, show that the dual statement of the statement $a\cdot b\cdot c+a'+b'+c'=1$ for all a, b, $c\in B$ is true.

(f) Define modular lattice with an example.

(g) State the principle of duality in Boolean Algebra.

(h) Draw switching circuit which realize the Boolean expression : $xyz'+xy'z+x'y'z'$.

[*Turn Over*]

2. Answer any *four* questions : 4×5=20

(a) If a partially ordered set $(L, \leq)$ is a lattice then prove that it is also a lattice as an algebraic structure.

(b) Let L_1 and L_2 be two lattices and $L_1 \times L_2$ be the lattice under the product partial order. Show that the lattice $L_1 \times L_2$ is distributive if and only if the lattices L_1 and L_2 are distributive.

(c) Construct the circuit corresponding to the Boolean function

(i) $x \cdot (y' + z) + y \cdot z'$

(ii) $x \cdot y + x' \cdot (x + y + y')$ in the switching algebra.

(d) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design as simple a circuit as you can which will allow current to pass when and only when a proposal is approved.

(e) Define Disjunctive Normal Form (DNF). Express the Boolean expression $(x+y)(x+y')(x'+z)$ in DNF in the variables x, z. Also express it in DNF in the variables x, y, z. 1+3+1

(f) Let $(A, \vee, \wedge)$ be an algebraic system defined by a lattice $(A, \leq)$. Prove that both the join and meet operations are associative.

3. Answer any *one* question : 1×10=10

(a) (i) In the Boolean algebra $(B, +, \cdot, ')$, express the Boolean expression f(x, y, z) = $\left((xy)'z\right)'\left((x'+z)(y'+z')\right)'$ in its complete sum-of-product form. 4

(ii) In a Boolean algebra $(B, +, \bullet, ')$ express the Boolean expression

$f(x, y, z) = (x'+y)z + y'$ in its complete DNF form. 4

(iii) Show that a Boolean algebra of three elements does not exist. 2

[Turn Over]

(b) (i) If n be a positive integer and D_n denotes the set of all divisors of n, consider the partial order 'divides' in D_n. Then draw the Hasse diagrams for D_6, D_{24}, D_{30}. 6

(ii) Define complete lattice. Prove that the set D of all factors of 12 under divisibility forms a lattice. 1+3
