Total Pages - 6 UG/3rd Sem/MATH(G)/T/19

2019

B.Sc.

3rd Semister Examination

MATHEMATICS (General)

Paper - DSC 1A-T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practiable.
Illustrate the answers wherever necessary.

1. Answer any *ten* questions : 10×2=20

(a) Prove that the equation

$(x-1)^3+(x-2)^3+(x-3)^3+(x-4)^3=0$ has only real root.

(b) Divide the number 10 into two parts such that the sum of their cubes is the least possible.

(c) Find the asymptotes of the curve $y=xe^{1/x}$.

[Turn Over]

(d) If $f'(x) = (x-a)^4 (x-b)^7$, then show that f has neither a minimum nor a maximum at 'a'.

(e) Find the points where the function $f(x) = \dfrac{1}{\log|x|}$ is discontinuous.

(f) Let $f(x) = \begin{cases} x^2 & , \text{ when x is rational} \\ 0 & , \text{ when x is irrational} \end{cases}$

Show that $f'(o) = 0$.

(g) Is Rolle's theorem applicable to $f(x) = \dfrac{1}{2-x^2}$ on [–1, 1] ? Justify your answer.

(h) Determine the degree of homogeneity of

$$f(x, y) = \tan^{-1}\left(\frac{y}{x}\right) + \sin^{-1}\left(\frac{x}{y}\right)$$

(i) Show that in any plane curve

$$\frac{Subtangent}{Subnormal} = \left(\frac{\text{length of tangent}}{\text{length of normal}}\right)^2$$

(j) Find the radius of curvature at the point (S, ψ) on the curve $S = 8a^2 \sin^2 \frac{\psi}{6}$.

(k) Two horses start a race at the same time and finish in a tie. Show that at some time during the race they have the same speed.

(l) Find the point(s) of discontinuity of $f(x) = (-1)^{[x]}, x \in \mathbb{R}$.

(m) Give an example of a quadratic function of the form $f(x) = x^2 + bx + c$ whose tangent line is $y = 3x + 1$ at the point. (0, 1).

(n) Find the value of ξ using Cauchy's MVT for $f(x) = \sin x, g(x) = \cos x$ in $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

(o) Find Taylor Series for the function

$f(x) = e^{-6x}$ about x = –4.

[*Turn Over*]

2. Answer any *four* questions : $4\times5=20$

(a) If lx + my = n touches the curve

$$\left(\frac{x}{a}\right)^{p}+\left(\frac{y}{b}\right)^{p}=1.$$ Then prove that

$$(al)^{p/(p-1)}+(bm)^{\frac{p}{p-1}}=n^{\frac{p}{p-1}}$$

(b) Prove that the locus of the extremity of the polar subtangent of the curve

$$\frac{1}{r}+f(\theta)=0 \text{ is } \frac{1}{r}=f'\left(\frac{\pi}{2}+\theta\right).$$

(c) If $u=F(y-z,z-x,x-y)$, prove that

$$u_x+u_y+u_z=0$$

(d) Find the asymptotes of

$$4x^3-3xy^2-y^3+2x^2-xy-y^2-1=0$$

(e) If a function $f:\mathbb{R}\to\mathbb{R}$ satisfies $|f(x)-f(y)|\le|x-y|^2$, $\forall x, y\in\mathbb{R}$, show that f is constant everywhere in $\mathbb{R}$.

(f) Sketch the graph $f(x)=3x^4-8x^3+10$ after answering the following questions :

(i) Where is the graph increasing and where is decreasing ?

(ii) Where are the local minima and maxima ?

(iii) Where are the point(s) of inflections ?

3. Answer any *two* questions : 2×10=20

(a) (i) Trace the curve : $y^2(a+x)=x^2(3a-x)$

(ii) If $v=\log\left(x^3+y^3+z^3-3xyz\right)$, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v=-\frac{3}{(x+y+z)^2}.$$

(b) (i) Show that $\sqrt{x}$ and $x^{5/2}$ cannot be in Maclaurin's infinite series. 5

[Turn Over]

(ii) Find geometrically the pedal equation of an ellipse with respect to a focus. 5

(c) (i) If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, prove that

$$\left(x\frac{\delta}{\delta_x} + y\frac{\delta}{\delta_y}\right)^2 u = 0$$ 5

(ii) If $y^{\frac{1}{2019}} + y^{-\frac{1}{2019}} = 2x$, prove that

$$\left(x^2 - 1\right)y_{n+2} + (2n+1)xy_{n+1} + \left(n^2 - 2019^2\right)y_n = 0$$

5
