2018

CBCS

1st Semester

MATHEMATICS

PAPER—DSC1AT

(General)

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Differential Calculas

Group—A

Answer *all* questions

1. Answer any *ten* questions : 10×2

(a) Let $g:\mathbb{R}\to\mathbb{R}$ be defined such that

$$g(y)=\begin{cases}0, & \text{if } y\neq 0\\ 1, & \text{if } y=0\end{cases}$$

Explain why the limit $\lim_{y\to\infty} g(y)$ exists. What is the value of $\lim_{y\to\infty} g(y)$?

(Turn Over)

(b) Is mean value theorem valid for $f(x) = x^2 + 3x + 2$ in $1 \leq x \leq 2$? Find c, if the theorem be applicable.

(c) Determine the degree of the homogeneous function

$x \cos\left(\frac{y}{x}\right)$

(d) Check, if the following curve has a symmetry about the x-axis.

$$y^2 = (x-1)(x-2)^2$$

(e) Find derivative of $f(x) = x \log|x| - x$.

(f) What is the necessary condition for the Maclaurin expansion to be true for a function ?

(g) Find the coefficient of x^2 in the Taylor Series about $x = 0$ for $f(x) = e^{-x^2}$.

(h) If $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\frac{\partial x}{\partial r} \neq \frac{1}{\frac{\partial r}{\partial x}} \text{ and } \frac{\partial x}{\partial \theta} \neq \frac{1}{\frac{\partial \theta}{\partial x}}$$

(i) Is $\mathop{\mathrm{Lt}}_{x \to 0} \mathop{\mathrm{Lt}}_{y \to 0} f(x,y) = \mathop{\mathrm{Lt}}_{y \to 0} \mathop{\mathrm{Lt}}_{x \to 0} f(x,y)$? Support your answer by an example.

(j) Give geometrical interpretation of Rolle's theorem.

(k) Find the radius of curvature of the parabola $y^2=4x$ at the vertex.

(l) Write the n-th derivative of $(ax+b)^m$ for $m>n$.

(m) Show that the function $f(x) = x - [x]$ has discontinuity when $0 < x < 2$. Determine the discontinuity points and their natures.

(n) Examine the differentiability of the function $f(x) = |x| + |x-1|$ at $x = 0$ and $x = 1$ where f is a real valued function defined on $(-1, 2)$.

(o) State Maclaurin's theorm with Lagrange's form of remainder.

Group—B

2. Answer any *four* questions : 4×5

(a) Your friend is confused. The function $f : x \to x^{2/3}$ takes on the same values $x = -1$ and $x = 1$. So he concludes according to Rolle's theorem there should be a point C in the open interval $(-1, 1)$ where $f'(c)=0$. Find out the point C for your friend.

(b) The slope of the curve $6y^3 = px^2 + q$ at $(2, -2)$ is $\frac{1}{6}$. Find the values of p and q.

(c) If $y = \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}$, $|x|<1$, show that

$$\left(1-x^2\right) y_{n+2} - (2n+3)\, x y_{n+1} - (n+1)^2\, y_n = 0.$$

(d) Find the condition that the conics $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ shall cut orthogonally.

(e) State and prove Cauchy's mean value theorm.

(f) A function $f : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ $f(x+y) = f(x) + f(y)$ $\forall x, y \in \mathbb{R}$. If $f(1) = k$, prove that $f(x) = kx \ \forall \ x \in \mathbb{R}$

Group—C

1. Answer any *two* questions : 2×10

(a) (i) State the Lagrauge's Mean Value Theorem. Verify the theorem for $f(x)=(x-3)(x-6)(x-9)$ on $[3,5]$. 2+4

(ii) If $y = \tan^{-1} x$, then deduce that

$$\left(1+x^2\right) y_{n+1} + 2nxy_n + n(n-1)y_n = 0 .$$ 4

(b) (i) Is $u(x,y) = ax^2 + 2hxy + by^2$ a homogeneous function ? Verify Euler's theorem for u. 2+4

(ii) Examine whether $x^{1/x}$ possesses a maximum or a minimum and determine the same. 4

(c) (i) Trace out the curve cycloid

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$$ 5

(ii) Prove that the sum of intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant.

(d) (i) State and prove Taylor's theorem with lagrange's form of remainder. 6

(ii) Show that $f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4}, & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$

possesses first order partial derivatives at (0,0) yet it is not differentiable at (0,0). 6
