Chapter 5

Multi-objective transportation-location problem with variable carbon emission under neutrosophic environment[1]

This chapter introduces an optimization model that integrates FLP and TP under a multi-objective environment. This study delineates the stated formulation in which we need to seek the locations of facilities in the Euclidean plane, and the amounts of transported products so that the total transportation cost, transportation time, and carbon emission cost from existing sites to facilities will be minimized. In fact, variable carbon emission under carbon tax, cap and trade regulation is considered due to the locations of facilities and the amounts of transported flow. Thereafter, a hybrid approach is improved based on an alternating locate-allocate heuristic and the neutrosophic compromise programming to obtain the non-dominated solution. Additionally, the performance of our findings is evaluated by an application example. Furthermore, a sensitivity analysis is incorporated to explore the resiliency of the designed model. Finally, the chapter ends with the conclusions.

5.1 Introduction

Several industries locate a pre-assigned number of facilities in order to determine a transportation way for optimizing the objective functions simultaneously. FLP and TP are the core components of a tactical transportation planning system. Determining the best locations for the facilities (i.e., plants, depots, warehouses, offices, fire stations, railway stations, etc.) and minimizing the total transportation cost from existing sites to facilities can significantly affect transportation planning system. A fast-flowing of transportation emerges tremendous amounts of carbon, which is the fundamental explanation for global warming. To control carbon emanations, the government endorses several policies among all TCTP is widely accepted. Under TCTP, the companies are firstly allowed some emission cap with the usual

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tax basis from the government, and subsequently, they can also trade (i.e., buy or sell) the emission cap in the carbon trading market. The motivation of this study is to design a strategic green transportation network to reduce CO_2 emission in the atmosphere. Due to this fact, here, we flourish a formulation by integrating FLP, TP and carbon emission under TCTP in the light of a multi-objective optimization environment. Therefore, we refer to the proposed problem as *multi-objective transportation-location problem* (MOT-LP). In MOT-LP, one has to ask the locations of facilities in the Euclidean plane and the amounts of transported goods simultaneously with three objective functions. However, in real-life circumstances, the unpredictability occurs due to a lack of proper information. In fact, this type of mathematical formulation is difficult to tackle by traditional approaches. To overcome this situation, the neutrosophic set is formulated based on logic in which elements are represented by three degrees, explicitly, truth degree, indeterminacy degree and falsity degree. For more clarifications of the neutrosophic environment, we refer to the Chapter 1.

The main contributions of this chapter are as follows:

- An integrated nonlinear optimization model based on FLP and MOTP is introduced.
- The model finds the decision regarding the assignment from multiple existing facilities to multiple potential facilities in the continuous planner surface with a hyperbolic approximation of Euclidean distance.
- The total transportation cost, total transportation time and total carbon emission cost are considered.
- The impact of variable carbon emission under TCTP due to transportation is also incorporated, a major contribution in the modern age.
- An improved hybrid approach is followed to find the optimal solution of MOT-LP.
- The nature of the obtained optimal solution is also studied.

5.2 Mathematical description

In this section, we first define the proposed problem, i.e., MOT-LP. Thereafter, the mathematical formulation is introduced on the following premises and notations. Moreover, the connection between MOT-LP and MOTP, and some basic definitions are presented.

5.2.1 Problem background

Here, a practical logistics problem is inspected from an economical and environmental point of view. Our proposed problem deals with a transportation network that consists of multiple existing sites or sources, potential facility sites or demand points, and products are transported from existing sites to potential facility sites. The main aim is to minimize the total transportation cost, time, and carbon emission cost under TCTP by locating the potential facility sites simultaneously. Besides the transportation cost and time, the following postures are also handled in our model: *(i)* variable carbon emission under TCTP, *(ii)* weights of conveyances which affect the transportation cost and carbon emission cost, *(iii)* weights of obstacles in the path which are reflected in transportation time, *(iv)* selling cost as a reward to reduce carbon emission, and *(v)* penalty cost to avoid unnecessary carbon discharges. Fig. 5.1 illustrates the structure of the MOT-LP network. Assume that there are three existing sites S_1, S_2 and S_3, and four potential facility sites D_1, D_2, D_3 and D_4. In fact, the supply and demand of the corresponding sites are also known. Moreover, the locations of S_1, S_2 and S_3 are provided. But, the locations of D_1, D_2, D_3 and D_4 are not known in the Euclidean plane. Consequently, the dotted lines denote the product flow by conveyances (i.e., T_1, T_2, T_3 and T_4) from S_1, S_2 and S_3 to D_1, D_2, D_3 and D_4, respectively. Furthermore, the obstacle is designated by B_1. In this situation, the DM has to seek the optimal locations of the potential facility sites with mentioned objective functions.

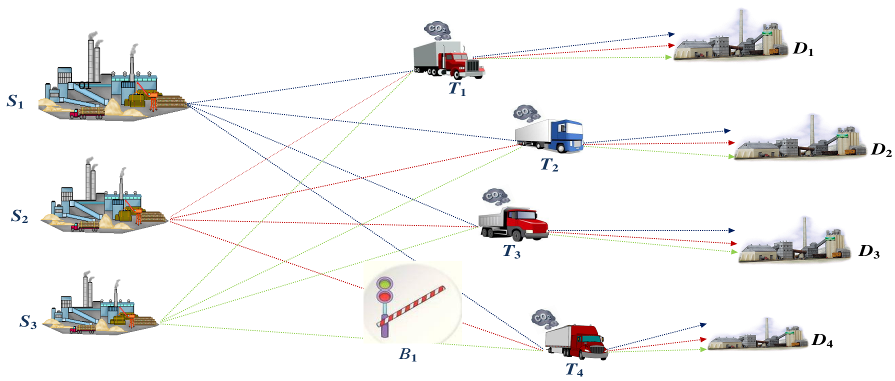


Fig. 5.1: Network for MOT-LP.

5.2.2 Notations and Assumptions

The following notations and assumptions are employed to formulate the model

m : number of existing facility sites.

p : number of potential facility sites.

k : number of objective functions.

a_i : availability at i-th existing facility site $(i = 1,2,\ldots,m)$.

b_j : demand at j-th potential facility site $(j = 1,2,\ldots,p)$.

e_i : in a location problem, the DM may put more important of the existing facility site, expressed as weight. Therefore, with each i-th existing site, we associate a weight e_i.

t_{ij} : there may be some obstacles (e.g., railway level crossing, bridge crossing, broken-down, etc.) of the path from i-th site to j-th site which are affected the transportation time. These will be designated as t_{ij}.

δ_{ij} : there may be used different type of conveyances to transport the goods from i-th site to j-th site. Depend on their machine performance, we assign the weight δ_{ij}.

α : tax for each unit product that emits carbon.

β : carbon trading (buying) cost per unit item.

γ : carbon trading (selling) cost per unit item.

C : emission cap (i.e., limited capacity of carbon emission permit).

P_c : penalty cost per unit emitted in excess of the cap.

(u_i, v_i): coordinates of the i-th existing facility site $(i = 1, 2, \ldots, m)$.

(x_j, y_j): coordinates of the j-th potential facility site $(j = 1, 2, \ldots, p)$.

w_{ij} : amount of flow to be transported from i-th existing facility site to j-th potential facility site.

W : $\{(w_{ij}) : \ i = 1, 2, \ldots m; \ j = 1, 2, \ldots, p\}$.

W^B : $\{(w^B_{ij}) : i = 1, 2, \ldots m; \ j = 1, 2, \ldots, p\}$, the optimal feasible solution.

F : $\mathbb{R}^{2p} \times W$, where $(x, y) \in \mathbb{R}^{2p}$ and $w \in W$, the feasible set.

S : neutrosophic set.

T_r : truth membership.

I_n : indeterminacy membership.

F_a : falsity membership.

L_k : lower value of the k-th objective function.

U_k : upper value of the k-th objective function.

ϕ : transportation cost function per unit flow from an existing facility site to a potential facility site depends on weight of conveyances.

ψ : time function per unit item from an existing facility site to a potential facility site depends on obstacle of path.

φ : average carbon emission function per unit product from an existing facility site to a potential facility site depends on weight of conveyances.

- Type of transportation cost function is a hyperbolic approximation of Euclidean distance in two-dimensional space $\left(\phi(u_i, v_i; x_j, y_j) = \sqrt{(u_i - x_j)^2 + (v_i - y_j)^2 + \delta_{ij}}\right)$.
- Transportation time function is a hyperbolic approximation of Euclidean distance in two-dimensional space $\left(\psi(u_i, v_i; x_j, y_j) = \sqrt{(u_i - x_j)^2 + (v_i - y_j)^2 + t_{ij}}\right)$.
- The carbon emission function is a hyperbolic approximation of Euclidean distance in two-dimensional space $\left(\varphi(u_i, v_i; x_j, y_j) = \sqrt{(u_i - x_j)^2 + (v_i - y_j)^2 + \delta_{ij}}\right)$.
- Facilities are capacitated.
- No relationship exists between potential facility sites.
- The opening costs of new potential facility sites are ignored.
- The solution space is continuous.
- The parameters are deterministic.
- The potential facility sites are located in the Euclidean plane.
- The potential facility sites are assumed as points.
- Transportation cost, transportation time, and average carbon emission are directly proportional to the amount of transported goods.

5.2.3 Model identification

Herein, a mathematical formulation is incorporated in light of the FLP and MOTP. In fact, this model asks transportation amounts and optimal locations for the facilities simultaneously. The mathematical formulation of MOT-LP under TCTP can be stated as follows:

Model 5.1

$$\text{minimize} \quad Z_{1(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij} \phi(u_i, v_i; x_j, y_j) \tag{5.1}$$

$$\text{minimize} \quad Z_{2(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij} \psi(u_i, v_i; x_j, y_j) \tag{5.2}$$

$$\text{minimize} \quad Z_{3(x,y,w)} = \alpha \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j) + P_c \beta \left(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j) - C \right)^{+} - \gamma \left(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j) \right)^{+} \tag{5.3}$$

$$\text{subject to} \quad \sum_{j=1}^{p} w_{ij} \leq a_i \quad (i = 1,2,\ldots,m), \tag{5.4}$$

$$\sum_{i=1}^{m} w_{ij} \geq b_j \quad (j = 1,2,\ldots,p), \tag{5.5}$$

$$w_{ij} \geq 0 \;\; \forall \;\; i,j, \tag{5.6}$$

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{p} b_j. \tag{5.7}$$

Where $\left(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j)\right)^{+} = \max\left(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j), 0\right)$
$= \begin{cases} C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j) & \text{if } C \geq \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j), \\ 0 & \text{otherwise.} \end{cases}$;
$\left(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j) - C\right)^{+} = \max\left(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j) - C, 0\right)$
$= \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j) - C & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j) \geq C, \\ 0 & \text{otherwise.} \end{cases}$

The objective function (5.1) aims to seek optimal locations for p-facilities, which minimizes the total transportation cost. The objective function (5.2) indicates to minimize the total transportation time by determining the optimal locations for p-facilities. The objective function (5.3) intents to minimize the total carbon emission cost under TCTP by locating the optimal locations for the p-facilities. Constraints (5.4) enforce that the total amounts of each existing site which cannot surpass its availability. Constraints (5.5) ensure that the total items of each potential site fulfill its desired demand. Constraints (5.6) are non-negativity conditions. Finally, constraints (5.7) suggest the feasibility condition.

The objective function (5.3) demonstrates that, depending on the cap, there are two feasible regions. Case 1 occurs when $C \geq \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j)$. And the second one is occurred when $C \leq \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j)$.

Case 1: This case can be represented by the following model:

Model 5.1.1

$$\begin{aligned} &\text{minimize} \quad Z_{1(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\phi(u_i,v_i;x_j,y_j) \\ &\text{minimize} \quad Z_{2(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\psi(u_i,v_i;x_j,y_j) \\ &\text{minimize} \quad Z_{3(x,y,w)} = \alpha\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j) - \gamma\Big(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j)\Big) \end{aligned} \tag{5.8}$$

subject to the constraints (5.4) to (5.7),

$$C \geq \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i,v_i;x_j,y_j). \tag{5.9}$$

Case 2: The following model is described for case 2:

Model 5.1.2

$$\begin{aligned}
\text{minimize} \quad & Z_{1(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij} \phi(u_i, v_i; x_j, y_j) \\
\text{minimize} \quad & Z_{2(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij} \psi(u_i, v_i; x_j, y_j) \\
\text{minimize} \quad & Z_{3(x,y,w)} = \alpha \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j) + P_c \beta \Big(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j) - C \Big)
\end{aligned} \tag{5.10}$$

subject to the constraints (5.4) to (5.7),

$$C \leq \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j). \tag{5.11}$$

5.2.4 Connection between MOT-LP and MOTP

The functions (i.e., ϕ, ψ and φ) are only dependent on the locations of the potential facility sites. In fact, if we fix the location of potential sites by finding optimal locations, then the functions should be converted into constant functions. Consequently, we designate (x_j^*, y_j^*) for the optimal locations, $e_i \phi(u_i, v_i; x_j^*, y_j^*) = c'_{ij}$ for the unit transportation cost from i-th source to j-th demand point, $e_i \psi(u_i, v_i; x_j^*, y_j^*) = t'_{ij}$ as the unit transportation time from i-th source to j-th demand point, and $\varphi(u_i, v_i; x_j^*, y_j^*) = d'_{ij}$ for the unit carbon emission for transportation of product flow from i-th source to j-th destination. Henceforth, Model 5.1 is read as follows:

Model 5.2

$$\text{minimize} \quad Z_1(w) = \sum_{i=1}^{m}\sum_{j=1}^{p} c'_{ij} w_{ij} \tag{5.12}$$

$$\text{minimize} \quad Z_2(w) = \sum_{i=1}^{m}\sum_{j=1}^{p} t'_{ij} w_{ij} \tag{5.13}$$

$$\text{minimize} \quad Z_3(w) = (\alpha + \gamma + P_c \beta) \sum_{i=1}^{m}\sum_{j=1}^{p} d'_{ij} w_{ij} - (\gamma + P_c \beta) C \tag{5.14}$$

subject to the constraints (5.4) to (5.7),

which is the well-known form of an MOTP.

5.2.5 Basic definitions

Here, some basic definitions related to the solution method of MOT-LP are presented.

Definition 5.1 *(**Ideal solution**): An ideal solution of MOT-LP is the one which minimizes each of the objective function simultaneously, i.e.,* $Z_k(x^*, y^*, w^*) = \min_{(x,y,w) \in F} Z_k(x, y, w)$, $k = 1, 2, 3$.

Definition 5.2 ***(Anti-ideal solution)****: The anti-ideal solution of MOT-LP is* $Z_k(x^A, y^A, w^A) = \max_{(x,y,w)\in F} Z_k(x,y,w)$*,* $k = 1,2,3$.

Definition 5.3 ***(Non-dominated solution)****: A solution* $(x^N, y^N, w^N) \in F$ *yields a non -dominated solution (otherwise called Pareto-optimal solution, efficient or non-inferior solution) of Model 5.1 iff there is no other solution* $(x,y,w) \in F$ *such that*

$$Z_k(x,y,w) \le Z_k(x^N, y^N, w^N) \ \text{for } k = 1,2,3, \text{and}$$
$$Z_k(x,y,w) < Z_k(x^N, y^N, w^N) \ \text{for at least one } k.$$

Definition 5.4 ***(Compromise solution)****: A non-dominated solution* $(x^N, y^N, w^N) \in F$ *is said to be the compromise solution of MOT-LP iff* $\mathbf{Z}(x^N, y^N, w^N) \le \wedge_{(x,y,w)\in F} \mathbf{Z}(x,y,w)$*, where* $\wedge$ *indicates the minimum.*

The ideal, anti-ideal, non-dominated and compromise solutions are described graphically in Fig. 5.2.

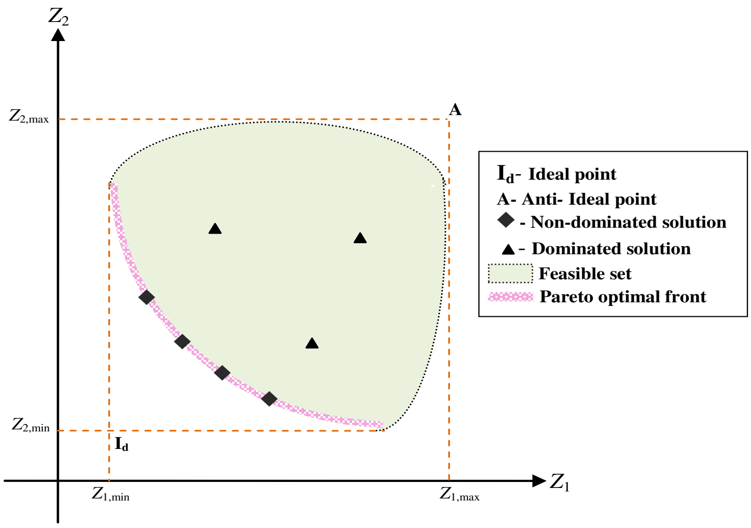


Fig. 5.2: The solution concept of a multi-objective optimization.

Definition 5.5 ***(Neutrosophic set [154])****: Let D be a universal set and* $s \in D$*. A neutrosophic set S in D is defined by three membership functions respectively, truth* $T_r(s)$*, indeterminacy* $I_n(s)$ *and falsity* $F_a(s)$*, and denoted by* $S = \{(s, T_r(s), I_n(s), F_a(s)) : s \in D\}$*, where*

(i) $T_r(s) : D \to [0,1]$, $I_n(s) : D \to [0,1]$ *and* $F_a(s) : D \to [0,1]$,

(ii) $0 \le \sup(T_r(s)) + \sup(I_n(s)) + \sup(F_a(s)) \le 3$.

5.3 Methodology

In this section, a hybrid approach is presented to solve the proposed MOT-LP. Thereafter, the advantages and disadvantages of the stated approach are also discussed.

5.3.1 Hybrid approach

Herein, a hybrid approach is developed based on an alternating Loc-Alloc heuristic [29], and an NCP [129]. Our hybrid approach comprises two parts. In the first part, three single objective T-LPs are solved by an alternating Loc-Alloc heuristic, and in the second part, the compromise non-dominated solution for MOT-LP is received by an NCP.

Alternating Loc-Alloc heuristic: The proposed heuristic is again divided into two parts. In Part 1, the heuristic seeks the initial locations, and in Part 2, it finds the optimum locations. Here, at first, the locations are placed for p-facilities from m-existing sites. If $p < m$, we generate all possible combinations of the m-existing sites taken p at a time. For each combination, the existing sites are to be considered as potential facility sites, and other existing sites are designated depending on which potential facility sites have the smallest distance. Finally, all designated distances are summed up. In fact, this phenomenon is repeated for all combinations. Therefore, the final initial potential locations for three distance functions are the combinations with the minimum sum of distances. With these final allocations, the distances between p-facilities and m-existing locations for three distance functions are easily computed. When $p = m$, the case is trivial and we easily get the distances between them. However, if $p > m$, we introduce a new heuristic concept to resolve this issue. Initially, we choose the m facility allocations as m existing sites randomly and allocate the remaining $(p-m)$ facilities in some Euclidean points with large coordinates such that the distances of those coordinates become very large numbers from facilities. Then, we easily compute the distances between p-facilities and m-existing sites and a large positive number is assigned for such distances which cannot be calculated. Now, it is already assumed that the distances are connected with cost, time and carbon emission functions per unit commodity from the i-th site to the j-th site. We take these distances as the cost, time and carbon emission coefficients. Then the problem converts into three classical transportation problems. Utilizing the initial potential location (x_j^I, y_j^I), we solve these problems individually:

Model 5.3

$$\begin{aligned} \text{minimize} \quad & Z_{1(w)} = \sum_{i=1}^{m} \sum_{j=1}^{p} e_i w_{ij} \phi(u_i, v_i; x_j^I, y_j^I) \\ \text{subject to} \quad & \text{the constraints (5.4) to (5.7).} \end{aligned}$$

Model 5.4

$$\begin{aligned} \text{minimize} \quad & Z_{2(w)} = \sum_{i=1}^{m} \sum_{j=1}^{p} e_i w_{ij} \psi(u_i, v_i; x_j^I, y_j^I) \\ \text{subject to} \quad & \text{the constraints (5.4) to (5.7).} \end{aligned}$$

Model 5.5

$$\begin{aligned} \text{minimize} \quad & Z_{3(w)} = \alpha \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j^I, y_j^I) + P_c\beta\Big(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j^I, y_j^I) - C\Big)^+ \\ & -\gamma\Big(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j^I, y_j^I)\Big)^+ \\ \text{subject to} \quad & \text{the constraints (5.4) to (5.7).} \end{aligned}$$

From Model 5.3 and Model 5.4, we can easily find optimal feasible solutions (W^B). But, to find the optimal feasible solution of Model 5.5, we split the model into two parts as Model 5.5.1 and Model 5.5.2, respectively. They are given as follows:

Model 5.5.1

$$\begin{aligned} \text{minimize} \quad & Z_{3(w)} = (\alpha + \gamma)\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j^I, y_j^I) - \gamma C \\ \text{subject to} \quad & \text{the constraints (5.4) to (5.7),} \\ & C \geq \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j^I, y_j^I). \end{aligned}$$

Model 5.5.2

$$\begin{aligned} \text{minimize} \quad & Z_{3(w)} = (\alpha + P_c\beta)\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j^I, y_j^I) - P_c\beta C \\ \text{subject to} \quad & \text{the constraints (5.4) to (5.7),} \\ & C \leq \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j^I, y_j^I). \end{aligned}$$

Thereafter, we solve Model 5.5.1 & Model 5.5.2 to extract the feasible solutions which lead the optimal solutions of these models. Then, we compare the solutions to find the optimal solution for Model 5.5. However, if one of them (Model 5.5.1 and Model 5.5.2) has the feasible solution (and the other has no feasible solution) then the optimal solution of the corresponding model is the optimal solution of Model 5.5. The iterative formula (see Appendix A.3) are used to minimize objective functions. Hence, $(x, y, w)^{(l)}$ is the local optimal (ideal) solution for the l-th single objective T-LP, where $l = 1, 2, 3$.

NCP: Here, a payoff table with entries $Z_{lk} := Z_k\big((x, y, w)^{(l)}\big)$, $l, k = 1, 2, 3$ are calculated for non-dominated solution of MOT-LP. Afterwards, the upper (U_k) and lower (L_k) bounds for each objective function are estimated as follows: $U_k = \max\{Z_{1k}, Z_{2k}, Z_{3k}\}$ and $L_k = Z_{kk}$, $k = 1, 2, 3$. Consequently, the upper and lower values for the neutrosophic environment are computed as

$$\begin{aligned} & U_k^{T_r} = U_k, \;\; L_k^{T_r} = L_k, \quad \text{for truth membership,} \\ & U_k^{I_n} = L_k^{T_r} + q'_k(U_k^{T_r} - L_k^{T_r}), \;\; L_k^{I_n} = L_k^{T_r}, \quad \text{for indeterminacy membership,} \\ & U_k^{F_a} = U_k^{T_r}, \;\; L_k^{F_a} = L_k^{T_r} + q_k(U_k^{T_r} - L_k^{T_r}), \quad \text{for falsity membership.} \end{aligned}$$

Where q_k and q'_k are tolerance variables, choose by the decision maker for falsity and indeterminacy membership functions. The membership functions for neutrosophic environment can be constructed as follows:

$$T_{rk}(Z_k(x,y,w)) = \begin{cases} 1 & Z_k(x,y,w) \le L_k^{T_r}, \\ 1 - \frac{Z_k(x,y,w) - L_k^{T_r}}{U_k^{T_r} - L_k^{T_r}} & L_k^{T_r} \le Z_k(x,y,w) \le U_k^{T_r}, \\ 0 & Z_k(x,y,w) \ge U_k^{T_r}. \end{cases}$$

$$I_{nk}(Z_k(x,y,w)) = \begin{cases} 1 & Z_k(x,y,w) \le L_k^{I_n}, \\ 1 - \frac{Z_k(x,y,w) - L_k^{I_n}}{U_k^{I_n} - L_k^{I_n}} & L_k^{I_n} \le Z_k(x,y,w) \le U_k^{I_n}, \\ 0 & Z_k(x,y,w) \ge U_k^{I_n}. \end{cases}$$

$$F_{ak}(Z_k(x,y,w)) = \begin{cases} 1 & Z_k(x,y,w) \ge U_k^{F_a}, \\ 1 - \frac{U_k^{F_a} - Z_k(x,y,w)}{U_k^{F_a} - L_k^{F_a}} & L_k^{F_a} \le Z_k(x,y,w) \le U_k^{F_a}, \\ 0 & Z_k(x,y,w) \le L_k^{F_a}. \end{cases}$$

As the objective functions are conflicting in nature, hence, $U_k^{T_r} = L_k^{T_r}$, $U_k^{I_n} = L_k^{I_n}$ and $U_k^{F_a} - L_k^{F_a}$ are not possible for any (x_k^*, y_k^*, w_k^*) $(k = 1,2,3)$. The neutrosophic model for MOT-LP can be stated as follows:

Model 5.6 (For Case 1)

$$\begin{aligned} \text{maximize} \quad & \theta \\ \text{minimize} \quad & \mu \\ \text{maximize} \quad & \nu \\ \text{subject to} \quad & T_{rk}(Z_k(x,y,w)) \ge \theta,\ \ F_{ak}(Z_k(x,y,w)) \le \mu,\ \ I_{nk}(Z_k(x,y,w)) \ge \nu, \\ & \text{the constraints (5.4) to (5.7)}, \\ & C \ge \sum_{i=1}^{m} \sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j), \\ & \theta \ge \mu, \theta \ge \nu, \theta + \mu + \nu \le 3, \\ & 0 \le q_k \le U_k - L_k, \\ & 0 \le q'_k \le U_k - L_k, \\ & \theta, \mu, \nu \in [0,1], k = 1,2,3. \end{aligned}$$

Where θ, μ and ν represent the global degree of satisfaction, indeterminacy and dissatisfaction of a solution, respectively.

Model 5.7 (For Case 2)

$$\begin{aligned} \text{maximize} \quad & \theta \\ \text{minimize} \quad & \mu \\ \text{maximize} \quad & \nu \\ \text{subject to} \quad & T_{rk}(Z_k(x,y,w)) \ge \theta,\ \ F_{ak}(Z_k(x,y,w)) \le \mu,\ \ I_{nk}(Z_k(x,y,w)) \ge \nu, \\ & \text{the constraints (5.4) to (5.7)}, \\ & C \le \sum_{i=1}^{m} \sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j), \\ & \theta \ge \mu, \theta \ge \nu, \theta + \mu + \nu \le 3, \\ & 0 \le q_k \le U_k - L_k, \end{aligned}$$

$$0 \leq q'_k \leq U_k - L_k,$$
$$\theta, \mu, \nu \in [0,1], k = 1,2,3.$$

Thereafter, the simplified neutrosophic model of MOT-LP can be represented to derive the compromise non-dominated solution as follows:

Model 5.8 (For Case 1)

$$\begin{aligned}
\text{maximize} \quad & \theta - \mu + \nu \\
\text{subject to} \quad & Z_k(x,y,w) + (U_k^{T_r} - L_k^{T_r})\theta \leq U_k^{T_r} \\
& Z_k(x,y,w) + (U_k^{I_n} - L_k^{I_n})\nu \leq U_k^{I_n} \\
& Z_k(x,y,w) - (U_k^{F_a} - L_k^{F_a})\mu \leq L_k^{F_a} \\
& \text{the constraints (5.4) to (5.7)}, \\
& C \geq \sum_{i=1}^{m} \sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j), \\
& \theta \geq \mu, \theta \geq \nu, \theta + \mu + \nu \leq 3, \\
& 0 \leq q_k \leq U_k - L_k, \\
& 0 \leq q'_k \leq U_k - L_k, \\
& \theta, \mu, \nu \in [0,1], k = 1,2,3.
\end{aligned}$$

Model 5.9 (For Case 2)

$$\begin{aligned}
\text{maximize} \quad & \theta - \mu + \nu \\
\text{subject to} \quad & Z_k(x,y,w) + (U_k^{T_r} - L_k^{T_r})\theta \leq U_k^{T_r} \\
& Z_k(x,y,w) + (U_k^{I_n} - L_k^{I_n})\nu \leq U_k^{I_n} \\
& Z_k(x,y,w) - (U_k^{F_a} - L_k^{F_a})\mu \leq L_k^{F_a} \\
& \text{the constraints (5.4) to (5.7)}, \\
& C \leq \sum_{i=1}^{m} \sum_{j=1}^{p} w_{ij} \varphi(u_i, v_i; x_j, y_j), \\
& \theta \geq \mu, \theta \geq \nu, \theta + \mu + \nu \leq 3, \\
& 0 \leq q_k \leq U_k - L_k, \\
& 0 \leq q'_k \leq U_k - L_k, \\
& \theta, \mu, \nu \in [0,1], k = 1,2,3.
\end{aligned}$$

5.3.2 Advantages of the proposed approach

In this subsection, we explore the main advantages of our hybrid approach.

- The main advantage of the hybrid approach is to give a general structure for dealing with the indeterminacy uncertainties in available data. Moreover, it does not require trade-offs or complicated parameters or any other reference directions from the DM. In fact, the employing of the approach guarantees a solution that maximizes the global degree of satisfaction and dissatisfaction, and minimizes indeterminacy level, and truly, it is a non-dominated optimal solution.

- The information about the data of MOT-LP is not precisely defined, the mathematical formulation of our approach has the capability to manipulate vague ideas like the number of objective functions and constraints.
- The stated approach provides a simple mathematical structure which makes easier for understanding and using. In fact, it always gives a compromise solution within a relatively short computational time for small scale entries.

5.3.3 Disadvantages of our approach

The main limitation of our approach is that it cannot deal with the fixed-charge cost for route selection or vehicle. If the fixed-charge cost is incurred, then the continuous structure of the problem will be lost. Furthermore, we have used the C++ programming language for the iterations and optimization solver for the NCP. Therefore, if an algorithm is specially designed for this complex structure might yields result faster, and is certainly necessary to solve large scale instances.

5.4 Analysis of non-dominated solution

Here, we first demonstrate that if (x^*,y^*,w^*) is a non-dominated solution of MOT-LP, then (x^*,y^*) is a non-dominated solution of the unconstrained multi-objective FLPs of Eqs. (5.1), (5.2) and (5.8) or Eqs. (5.1), (5.2) and (5.10), where $w = w^*$.

Lemma 5.1 *Let (x^*,y^*,w^*) is a non-dominated solution of MOT-LP of Eqs. (5.1), (5.2) and (5.8). Then (x^*,y^*) is a non-dominated solution of the multi-objective FLP:*

$$\text{minimize} \quad Z_{1(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w^*_{ij}\phi(u_i,v_i;x_j,y_j)$$

$$\text{minimize} \quad Z_{2(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w^*_{ij}\psi(u_i,v_i;x_j,y_j)$$

$$\text{minimize} \quad Z_{3(x,y)} = (\alpha+\gamma)\sum_{i=1}^{m}\sum_{j=1}^{p} w^*_{ij}\varphi(u_i,v_i;x_j,y_j) - \gamma C$$

Proof. This lemma can be proved by the method of indirect proof (i.e., contradiction). Let $(\bar{x},\bar{y})$ be a solution such that $Z_k(\bar{x},\bar{y},w^*) \leq Z_k(x^*,y^*,w^*)$ for $k(=1,2,3)$, and $Z_k(\bar{x},\bar{y},w^*) < Z_k(x^*,y^*,w^*)$ for at least one k. Again $(\bar{x},\bar{y},w^*)$ is a feasible solution of the problem, then there is a contradiction to a non-dominated solution of (x^*,y^*,w^*). This ends the lemma.□

Lemma 5.2 *Let (x^*,y^*,w^*) is a non-dominated solution of MOT-LP of Eqs. (5.1), (5.2) and (5.10). Then (x^*,y^*) is a non-dominated solution of the multi-objective FLP:*

$$\begin{aligned} \text{minimize} \quad & Z_{1(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}^{*}\phi(u_i,v_i;x_j,y_j) \\ \text{minimize} \quad & Z_{2(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}^{*}\psi(u_i,v_i;x_j,y_j) \\ \text{minimize} \quad & Z_{3(x,y)} = (\alpha+P_c\beta)\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}^{*}\varphi(u_i,v_i;x_j,y_j) - P_c\beta C \end{aligned}$$

Proof. It can be easily proved by similar way. □

Lemma 5.3 *Let $(x^*,y^*,w^*,\theta^*,\mu^*,\nu^*)$ be an optimal solution of Model 5.8, then it should be also a non-dominated solution (x^*,y^*,w^*) of Model 5.1.1.*

Proof. Let the contrary be true. Then there is a solution $(\bar{x},\bar{y},\bar{w}) \in F$ such that $Z_k(\bar{x},\bar{y},\bar{w}) < Z_k(x^*,y^*,w^*)\ \forall\, k,\ k=1,2,3$. Now θ^*, μ^* and ν^* are the optimal values of Model 5.8, then

$$\begin{aligned} & Z_k(\bar{x},\bar{y},\bar{w}) + (U_k^{T_r}-L_k^{T_r})\theta^* < Z_k(x^*,y^*,w^*) + (U_k^{T_r}-L_k^{T_r})\theta^* \le U_k^{T_r}, \quad k=1,2,3, \\ & Z_k(\bar{x},\bar{y},\bar{w}) + (U_k^{I_n}-L_k^{I_n})\nu^* < Z_k(x^*,y^*,w^*) + (U_k^{I_n}-L_k^{I_n})\nu^* \le U_k^{I_n}, \quad k=1,2,3, \\ & Z_k(\bar{x},\bar{y},\bar{w}) - (U_k^{F_a}-L_k^{F_a})\mu^* < Z_k(x^*,y^*,w^*) - (U_k^{F_a}-L_k^{F_a})\mu^* \le L_k^{F_a}, \quad k=1,2,3. \end{aligned}$$

Henceforth, there exist $\theta > \theta^*$, $\mu > \mu^*$, $\nu > \nu^*$ and an $l \in \{1,2,3\}$ such that

$$\begin{aligned} & Z_l(\bar{x},\bar{y},\bar{w}) + (U_l^{T_r}-L_l^{T_r})\theta = U_l^{T}, \\ & Z_k(\bar{x},\bar{y},\bar{w}) + (U_k^{T_r}-L_k^{T_r})\theta \le U_k^{T}, k \neq l, \\ & Z_l(\bar{x},\bar{y},\bar{w}) + (U_l^{I_n}-L_l^{I_n})\nu = U_l^{I_n}, \\ & Z_k(\bar{x},\bar{y},\bar{w}) + (U_k^{I_n}-L_k^{I_n})\nu \le U_k^{I_n}, k \neq l, \\ & Z_l(\bar{x},\bar{y},\bar{w}) - (U_l^{F_a}-L_l^{F_a})\mu = L_l^{F_a}, \\ & Z_k(\bar{x},\bar{y},\bar{w}) - (U_k^{F_a}-L_k^{F_a})\mu \le L_k^{F_a}, k \neq l, \end{aligned}$$

which contradict that $(x^*,y^*,w^*,\theta^*,\mu^*,\nu^*)$ is an optimal solution of Model 5.8. This completes the proof of lemma. □

Lemma 5.4 *Let $(x^*,y^*,w^*,\theta^*,\mu^*,\nu^*)$ be an optimal solution of Model 5.9, then it should be also a non-dominated solution (x^*,y^*,w^*) of Model 5.1.2.*

Proof. The proof is left to the reader. □

5.5 Experimental example

Herein, a real-life based example is presented to validate our model and methodology. In the example, an industrial association wishes to start a few new firms with the aim of minimizing the total transportation cost, time and carbon emission cost under tax, cap and trade policy. The association has 4 existing firms: S_1, S_2, S_3 and S_4, and they want to establish 3 new firms: D_1, D_2 and D_3. They transport goods by conveyances. In fact, we consider the weights of the conveyances depend on the machine performance which is reflected in transportation and carbon emission cost. Thereafter, the obstacles of the paths are also appraised in transportation time to become the problem more realistic. Under TCTP, the industrial association is allocated a carbon emission cap C. When they emit less (more) than the cap C, then they can sell (buy) the extra permit in (from) the carbon trading market. Moreover, if the firms emit more than the cap, then they have to pay extra cost as a penalty to reduce carbon emission. Hypothetical data of real-life scenarios are created. Here, we take carbon tax $\alpha = 0.3$, carbon buying cost from the trade market $\beta = 0.5$, carbon selling cost $\gamma = 0.7$ and penalty cost $P_c = 0.9$. The availability of S_1, S_2, S_3 and S_4 and the demand of the firms D_1, D_2, and D_3 are known. Moreover, the locations and the weights of the plants S_1, S_2, S_3 and S_4 are also provided. Table 5.1 represents the locations and weights of the existing firms. The supply, demand and non-negative weights are given in Table 5.2.

Table 5.1: Locations and weights of the existing firms.

	Position (u_i, v_i)	Weight (e_i)
S_1	$(5, 10)$	0.3
S_2	$(2, 25)$	0.1
S_3	$(17, 25)$	0.4
S_4	$(20, 5)$	0.2

Table 5.2: Pay-off table (t_{ij}, δ_{ij}).

	D_1	D_2	D_3	Supply (a_i)
S_1	$(0.0, 0.2)$	$(0.7, 0.0)$	$(0.3, 0.5)$	60
S_2	$(0.1, 0.3)$	$(0.2, 0.4)$	$(0.7, 0.0)$	40
S_3	$(0.3, 0.5)$	$(0.5, 0.4)$	$(0.2, 0.1)$	30
S_4	$(0.0, 0.0)$	$(0.6, 0.6)$	$(0.4, 0.1)$	25
Demand (b_j)	50	70	35	

Two special cases are considered for TCTP.
Case 5.1: First, when the carbon cap is $C = 800$.

5.5.1 Performance of the hybrid approach

The steps for solving the proposed MOT-LP are as follows:

Step 1. At first, three initial potential locations are picked up from Table 5.2 for three plants. Thereafter, four *cases* are appeared which are shown in Tables 5.3 to 5.6.

Table 5.3: Case 5.1.1.

	Position	Weight
D_1	$(5,10)$	0.3
D_2	$(2,25)$	0.1
D_3	$(17,25)$	0.4

Table 5.4: Case 5.1.2.

	Position	Weight
D_1	$(2,25)$	0.1
D_2	$(17,25)$	0.4
D_3	$(20,5)$	0.2

Table 5.5: Case 5.1.3.

	Position	Weight
D_1	$(17,25)$	0.4
D_2	$(20,5)$	0.2
D_3	$(5,10)$	0.3

Table 5.6: Case 5.1.4.

	Position	Weight
D_1	$(20,5)$	0.2
D_2	$(5,10)$	0.3
D_3	$(2,25)$	0.1

Step 2. Now, the distances (i.e., for each individual distance function) are estimated among assigned initial potential locations and the rest site for each case. Thereafter, the smallest distance is chosen for each individual distance function from the said four cases. The final initial potential locations are Case 5.1.4 for the first distance function, Case 5.1.3 for the 2nd distance function, and Case 5.1.4 for the 3rd distance function.

Step 3. The distances between existing and initial locations of plants are calculated, and the distances are considered as a cost, time and average carbon emission coefficients. The coefficients are as follows:

Cost coefficients (c'_{ij}):

$c'_{11} = 4.745$, $c'_{12} = 0$, $c'_{13} = 4.595$, $c'_{21} = 2.691$, $c'_{22} = 1.531$, $c'_{23} = 0$, $c'_{31} = 8.094$, $c'_{32} = 7.688$, $c'_{33} = 6.001$, $c'_{41} = 0$, $c'_{42} = 3.166$, $c'_{43} = 5.382$.

Time coefficients (t'_{ij}):

$t'_{11} = 5.763$, $t'_{12} = 4.749$, $t'_{13} = 0.164$, $t'_{21} = 1.500$, $t'_{22} = 2.691$, $t'_{23} = 1.532$, $t'_{31} = 0.218$, $t'_{32} = 8.094$, $t'_{33} = 7.685$, $t'_{41} = 4.047$, $t'_{42} = 0.155$, $t'_{43} = 3.165$.

Carbon emission coefficients (d'_{ij}):

$d'_{11} = 15.817$, $d'_{12} = 0$, $d'_{13} = 15.313$, $d'_{21} = 26.912$, $d'_{22} = 15.310$, $d'_{23} = 0$, $d'_{31} = 20.236$, $d'_{32} = 19.219$, $d'_{33} = 15.003$, $d'_{41} = 0$, $d'_{42} = 15.830$, $d'_{43} = 26.909$.

Step 4. Thereafter, LINGO 17.0 iterative scheme is employed to obtain the individual optimal feasible solution as follows:

For **Model 5.3**:

$w_{12} = 60$, $w_{22} = 10$, $w_{23} = 30$, $w_{31} = 25$, $w_{33} = 5$, $w_{41} = 25$ with all other $w_{ij} = 0$ and $Z_1 = 247.676$.

For **Model 5.4**:

$w_{12} = 25$, $w_{13} = 35$, $w_{21} = 20$, $w_{22} = 20$, $w_{31} = 30$, $w_{42} = 25$ with all other $w_{ij} = 0$ and $Z_2 = 218.751$.

For **Model 5.5**:

For Model 5.5.1:

$w_{12} = 60$, $w_{22} = 5$, $w_{23} = 35$, $w_{31} = 25$, $w_{32} = 5$, $w_{41} = 25$ with all other $w_{ij} = 0$, and $Z_3 = 118.545$.

For Model 5.5.2:

$w_{11} = 2.548$, $w_{12} = 57.451$, $w_{22} = 12.548$, $w_{23} = 27.452$, $w_{31} = 22.452$, $w_{33} = 7.548$, $w_{41} = 25$ with all other $w_{ij} = 0$, and $Z_3 = 240$. Therefore, the optimal solution of Model 5.5 is the optimal solution corresponding to Model 5.5.1.

Step 5. The C++ programming language is explored for executing our model as a single objective function to get the individual optimal potential locations for the plants. The respective optimal potential locations are as follows:

$(x_1,y_1)^{(1)} = (17.065, 24.568)$, $(x_2,y_2)^{(1)} = (5.000, 10.000)$, $(x_3,y_3)^{(1)} = (2.458, 25.000)$, $(x_1,y_1)^{(2)} = (16.907, 25.000)$, $(x_2,y_2)^{(2)} = (5.613, 10.040)$, $(x_3,y_3)^{(2)} = (5.000, 10.000)$, $(x_1,y_1)^{(3)} = (18.512, 14.918)$, $(x_2,y_2)^{(3)} = (5.000, 10.000)$ and $(x_3,y_3)^{(3)} = (2.000, 25.000)$.

Step 6. Using the obtained solutions, we compute the upper and lower values for each objective function and they are as follows:

$U_1 = \max\{153.021, 157.550, 268.432\}$, $L_1 = 153.021$;

$U_2 = \max\{168.263, 157.136, 283.457\}$, $L_2 = 157.136$;

$U_3 = \max\{195.02, 489.503, 118.856\}$, $L_3 = 118.856$.

Step 7. Upper and lower bounds based on the NCP are calculated for each objective function:

For $Z_1(x,y,w)$:

$U_1^{T_r} = U_1 = 268.432$, $L_1^{T_r} = L_1 = 153.021$,

$U_1^{I_n} = L_1^{T_r} + q_1'(U_1^{T_r} - L_1^{T_r}) = 153.021 + 115.411q_1'$, $L_1^{I_n} = L_1^{T_r} = 153.021$,

$U_1^{F_a} = U_1^{T_r} = 268.432$, $L_1^{F_a} = L_1^{T_r} + q_1(U_1^{T_r} - L_1^{T_r}) = 153.021 + 115.411q_1$.

For $Z_2(x,y,w)$:

$U_2^{T_r} = U_2 = 283.457$, $L_2^{T_r} = L_2 = 157.136$,

$U_2^{I_n} = L_2^{T_r} + q_2'(U_2^{T_r} - L_2^{T_r}) = 157.361 + 126.321q_2'$, $L_2^{I_n} = L_2^{T_r} = 157.361$,

$U_2^{F_a} = U_2^{T_r} = 283.457$, $L_2^{F_a} = L_2^{T_r} + q_2(U_2^{T_r} - L_2^{T_r}) = 157.361 + 126.321q_2$.

For $Z_3(x,y,w)$:

$U_3^{T_r} = U_3 = 489.503$, $L_3^{T_r} = L_3 = 118.856$,

$U_3^{I_n} = L_3^{T_r} + q_3'(U_3^{T_r} - L_3^{T_r}) = 118.856 + 370.647q_3'$, $L_3^{I_n} = L_3^{T_r} = 118.856$,

$U_3^{F_a} = U_3^{T_r} = 489.503$, $L_3^{F_a} = L_3^{T_r} + q_3(U_3^{T_r} - L_3^{T_r}) = 118.856 + 370.647q_3$.

Step 8. Using the LINGO 17.0 iterative scheme, we solve the simplified neutrosophic model (Model 5.8). The optimal compromise solution of the above MOT-LP is as follows: $w_{11} = 50$, $w_{12} = 10$, $w_{13} = 0$, $w_{21} = 0$, $w_{22} = 5$, $w_{23} = 35$, $w_{31} = 0$, $w_{32} = 30$, $w_{33} = 0$, $w_{41} = 0$, $w_{42} = 25$, $w_{43} = 0$, $q_1 = q_3 = 0.3$, $q_2 = 0.21$, $q_1' = q_3' = 1$, $q_2' = 0.80$, $\theta = 0.699$, $\mu = 0$, $\nu = 0.699$, $(x_1,y_1) = (5,10)$, $(x_2,y_2) = (16.380, 22.108)$, $(x_3,y_3) = (2,25)$, $Z_1 = 187.713$, $Z_2 = 184.191$, $Z_3 = 230.269$.

Case 5.2: Furthermore, consider the carbon cap as $C = 675$. Here, Steps 1 to 4 are exactly

same with Case 5.1. Since, the feasible solution of Model 5.5.1 does not exist, therefore, we take the solution of Model 5.5.2 as Model 5.5. The optimal solution of Model 5.5 is as follows:
$w_{12} = 60$, $w_{22} = 5$, $w_{23} = 35$, $w_{31} = 25$, $w_{32} = 5$, $w_{41} = 25$ with all other $w_{ij} = 0$, and $Z_3 = 205.159$.

Step 5. Using C++ programming, we obtain the individual optimal potential locations for the plants. The respective optimal potential locations are as follows:
$(x_1,y_1)^{(1)} = (17.065, 24.568)$, $(x_2,y_2)^{(1)} = (5.000, 10.000)$, $(x_3,y_3)^{(1)} = (2.458, 25.000)$,
$(x_1,y_1)^{(2)} = (16.907, 25.000)$, $(x_2,y_2)^{(2)} = (5.613, 10.040)$, $(x_3,y_3)^{(2)} = (5.000, 10.000)$,
$(x_1,y_1)^{(3)} = (18.512, 14.918)$, $(x_2,y_2)^{(3)} = (5.000, 10.000)$ and $(x_3,y_3)^{(3)} = (2.000, 25.000)$.

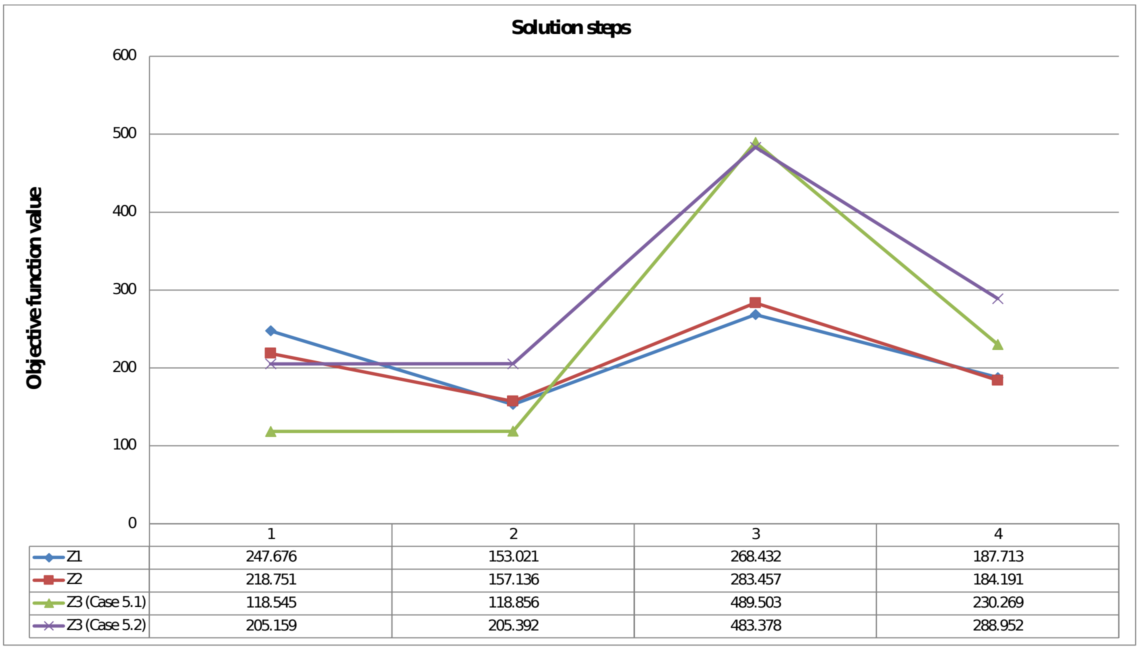
Step 6. Thereafter, we calculate upper and lower bounds and they are as follows:
$U_1 = \max\{153.021, 157.550, 268.432\}$, $L_1 = 153.021$;
$U_2 = \max\{168.263, 157.136, 283.457\}$, $L_2 = 157.136$;
$U_3 = \max\{262.515, 483.378, 205.392\}$, $L_3 = 205.392$.

Then, using Step 7, we obtain the optimal compromise solution of the proposed problem. And the solution is as follows: $w_{11} = 50$, $w_{12} = 10$, $w_{13} = 0$, $w_{21} = 0$, $w_{22} = 5$, $w_{23} = 35$, $w_{31} = 0$, $w_{32} = 30$, $w_{33} = 0$, $w_{41} = 0$, $w_{42} = 25$, $w_{43} = 0$, $q_1 = 1.239$, $q_2 = q_3 = q'_1 = q'_2 = q'_3 = 1.234$, $\theta = 0.699$, $\mu = 0$, $\nu = 0.699$, $(x_1,y_1) = (5,10)$, $(x_2,y_2) = (16.380, 22.108)$, $(x_3,y_3) = (2,25)$, $Z_1 = 187.713$, $Z_2 = 184.191$, $Z_3 = 288.952$.

5.6 Computational results and discussion

An application example is provided to analyze the proposed model with the help of the hybrid approach. The approach first finds the initial locations, optimal feasible solutions, optimal locations, ideal solutions (individual minimum), and anti-ideal solutions (individual maximum), and then we determine the upper and lower bounds for truth, indeterminacy, and falsity. Thereafter, the neutrosophic models for two cases of MOT-LP are formulated to derive optimal compromise solutions. The obtained results of the example show that the total transportation cost, delivery time and the optimal locations of the firms are same in both cases. But under TCTP, the total carbon emission cost is varied. In fact, we analyze that when the cap is larger than the total emission, the carbon emission cost decreases as they sell the extra permit in the trade market. For that reason, the industrial organizations will make more profit. Again when the cap is less than a threshold, the carbon emission cost increases as they have to buy carbon emission permits from others, as well as they have to pay also the penalty cost which minimized the profit of firms. Thereafter, TCTP can affect to adjust the carbon emissions due to transportation at the same time to the green environment. The steps of the objective functions and optimal facilities locations for the given example are depicted in Figures 5.3 to 5.4. The alternating Loc-Alloc heuristic approach is coded in C++ and conducted using a code-block compiler, and a NCP is coded in the LINGO 17.0

iterative scheme on a Lenovo z580 computer with 2.50 GHz Intel (R) Core (TM) i5-3210M CPU with 4 GB RAM. The computational results are compared with Linux terminal on a computer with Intel(R) Core (TM) i3-4130 CPU @3.40 GHz with 4 GB RAM.



	1	2	3	4
Z1	247.676	153.021	268.432	187.713
Z2	218.751	157.136	283.457	184.191
Z3 (Case 5.1)	118.545	118.856	489.503	230.269
Z3 (Case 5.2)	205.159	205.392	483.378	288.952

Fig. 5.3: Graphical representation of the proposed hybrid approach for example.

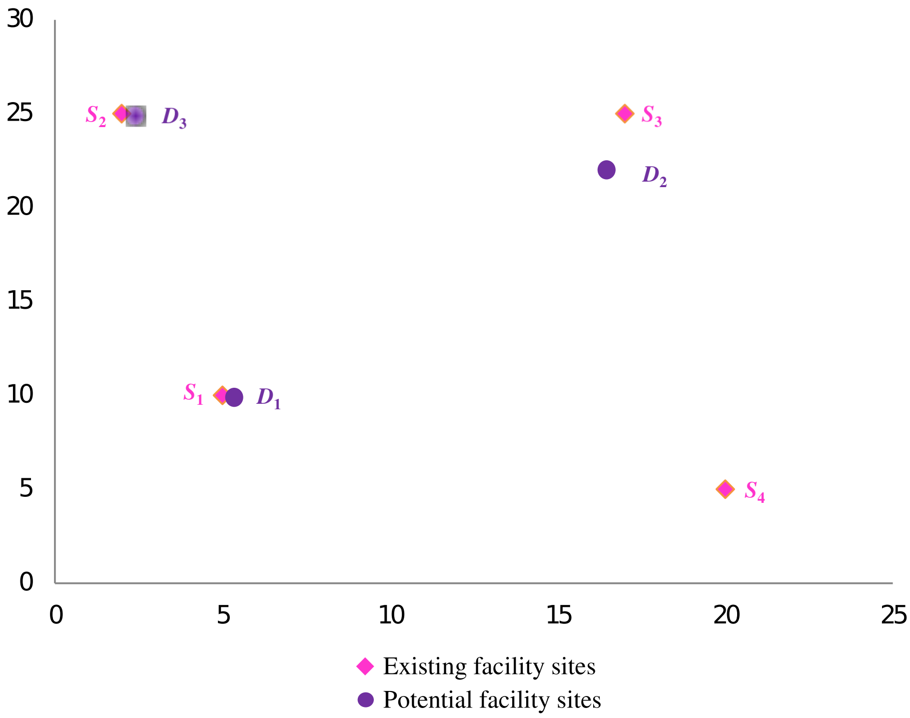


Fig. 5.4: The existing and optimal potential facilities in example.

5.7 Sensitivity analysis

In this section, we investigate the resiliency of optimal compromise solutions in MOT-LP by varying the parameters. For MOT-LP, the difficulty arises when the range of parameters are chosen after small changes for which the optimal solution remains optimal. In fact, the complexity increases when the number of variables and constraints are in large size. Due to this reason, a simple procedure is already carried out in Chapter 3 (see Section 3.5) to analyze the sensitivity of parameters. Here, the same steps (Steps 1- 4) are repeated to obtain the validity ranges of the parameters in MOT-LP.

Sensitivity analysis for supply and demand parameters:
Let a_i be changed to a_i^* $(i = 1,2,3,4)$ and b_j be altered to b_j^* $(j = 1,2,3)$. Utilizing the step-wise procedure from above, the values of a_i^* and b_j^* are easily calculated, which shown in Tables 5.7 to 5.8. Note that the range of the other parameters in MOT-LP are resolved in a similar way.

Table 5.7: The range of supply and demand parameters for Case 5.1.

Real values of a_i and b_j	Changing values of a_i and b_j
$a_1 = 60$	$60 \leq a_1^* \leq 118.5$
$a_2 = 40$	$40 \leq a_2^* \leq 65.3$
$a_3 = 30$	$30 \leq a_3^* < \infty$
$a_4 = 25$	$25 \leq a_4^* < \infty$
$b_1 = 50$	$10.2 \leq b_1^* \leq 50$
$b_2 = 70$	$39.1 \leq b_2^* \leq 70$
$b_3 = 35$	$0.1 \leq b_3^* \leq 35$

Table 5.8: The range of supply and demand parameters for Case 5.2.

Real values of a_i and b_j	Changing values of a_i and b_j
$a_1 = 60$	$60 \leq a_1^* \leq 94.9$
$a_2 = 40$	$40 \leq a_2^* \leq 64.9$
$a_3 = 30$	$30 \leq a_3^* \leq 50$
$a_4 = 25$	$25 \leq a_4^* < \infty$
$b_1 = 50$	$22.5 \leq b_1^* \leq 50$
$b_2 = 70$	$54 \leq b_2^* \leq 70$
$b_3 = 35$	$-\infty < b_3^* \leq 35$

5.8 Managerial insights

The fact that MOT-LP is an especially application-based region, makes it essential to receive deep insights into the characteristics of optimal solutions. Herein, we gather information about the optimal solutions derived when employing Model 5.1 into two sub-problems. Observing the outcomes, the management's discretion can easily pick the optimal solution between two sub-problems. A brief discussion of the effect of carbon emission under TCTP is depicted. From that discussion, the managements can easily decide the optimal potential facility sites so that they can easily reach the sites with minimum transportation cost, transportation time and carbon emission. There is an analysis of carbon tax, cap and trade policy in emission, from that the managements can decide which case emits the least amount of carbon emission. As a result, they can balance between their profits and green environment, which may gain reputation in the global market. On the other hand, the machine performance of the conveyances is displayed in transportation cost and emission. In case the machine performance is good, then the total transportation cost along with carbon emission will be reduced. For that reason, the managements can easily choose which type of conveyances are better for transporting goods. Again, the time for obstacles of the paths is also considered in transportation time, so that the managements can calculate the more accurate transportation time which improves their services to the customers. A sensitivity analysis is provided to show which range of the parameter is more appropriate for the managements. Finally, we can say that the mentioned formulation will be effective for the managements to seek optimal potential sites to transport with minimum cost, time and carbon emission.

5.9 Conclusion

This study has been presented a practical formulation for planning and transportation system with the objectives of minimizing the total transportation cost, total transportation time, and total carbon emission cost under TCTP on the entire transportation chain, and at the same time it also asks the potential facility sites along with the amounts of transported goods simultaneously. To the best of the knowledge, the problem of designing MOT-LP, considering variable carbon emission under TCTP, has not been studied before. Additionally, we have improved a hybrid approach to solve the proposed problem in an effective way. The stated formulation and improved hybrid approach have been tested by a real-life based example. Thereafter, the effect of variable carbon emissions under TCTP is investigated by two special cases. In fact, we explore the optimal decision to reduce carbon emission for companies under TCTP. Therefore, the nature of the obtained compromise solution is analyzed by four lemmas. Lastly, the sensitivity analysis has been given to check the resiliency of the parameters in the MOT-LP. Moreover, our formulation can be utilized in other industrial applications like the manufacturing of plants, green supply chain model, production-inventory system, financial and further applications.