Chapter 8

Conclusions and Future Research Directions

This chapter displays the overall concluding parts of the research studies which are described in the **Chapter 2** to **Chapter 7**. At the end, scopes of the research works are explored which may be a pattern to assist the ways of future research works.

8.1   Conclusions

From the very beginning of game theory numerous in-depth studies and researches have been made regarding its broad application in different types of decision making problems, both from abstract point of view and application point of view. Several literature surveys are done on Game theories in cooperative, non-cooperative fields of game-situations. This thesis promotes several uncertain environments like triangular intuitionistic fuzzy and its type-2 version, linguistic neutrosophic in single-valued and interval-valued, hesitant triangular intuitionistic fuzzy, hesitant interval-valued intuitionistic fuzzy-linguistic term set and single-valued triangular neutrosophic to deal with the vague/imprecise parameters in Game theories arising in real-world decision making problems.

The game theory has its wide applications in competitive systems, business planning and strategic management. In real-world applications, the elements of the payoff matrix in game problems may not be known precisely due to uncertain factors. If the payoff elements are considered as crisp values rather than uncertain variable, some helpful information interpreting the most accuracy of the object may be erased. That's why discussions on the matrix and bi-matrix games under uncertain environments are necessary. Since the payoff elements are expressed by uncertain variables like fuzzy, intuitionistic fuzzy, hesitant fuzzy, linguistic term set, linguistic fuzzy, and neutrosophic linguistic fuzzy rather than crisp values, more information is provided to take right decision for decision maker.

The results of considered problems are solved using simplex method, matrix method, robust ranking method, TOPSIS method, interval analysis. In addition, examples are given to illustrate the usefulness of the theories under developed concepts and their wider applications in marketing-shopping problems, water management problem, medical diagnosis problem, human trafficking problem, strike-policy problem, production planning problem.

Using our robust ranking technique we have shown that the government is more aggressive to minimize the demands of strike supporters and again using the same technique, with great interest, we have also shown that in Online Shopping-Marketing Problem, Cash On Delivery is more effective way to increase the sales-amount than Net-Banking or Debit-Card payment options from company's view-point.

Considered with the water management problem we may conclude that the use of different ranking functions and type-2 intuitionistic fuzzy environment, may be of the form of triangular or trapezoidal or hexagonal or interval, can derive reliable solutions for many ecological-environmental problems.

This study significantly shows that discussion on matrix game theory under neutrosophic environment has a significant effect in real-life problems like medical diagnosis, where treatment mostly depends upon liguistic behaviour. Applications in a variety of areas, for example, energy, environment, risk management, reliability, logistics, supply chain management, transportation, location, health-care, etc. may be done by building decision strategies against the related constraints using neutrosophic sets, logic and game theory as further research works.

In this thesis, we have attempted to explore the potentiality of the neuro-fuzzy systems in modeling game phenomenon and to access its behavioural structures through ANN and logic-gate switching circuits, where we have tried to explain fuzzy matrix game using max fuzzy neuron and min fuzzy neuron in hybrid fuzzy neural network. The analysis of the results indicates that the rendition of FGSCANN model in game theory would be significantly improved if the input data are transformed into the normal or real domain prior to model formulation. The results of the proposed study highly encourage the researchers with a suggestion that ANN is viable for modeling daily life problems in the light of game theory.

In our work, we have attempted to solve the Prisoners' Dilemma game in human trafficking problem depending on the ergonomics in information process and organisation management in OR. Considering the problem with its advantages and disadvantages, the proposed method is different in every regard of game theory due to the collaboration of hesitant interval-valued intuitionistic fuzzy-linguistic term set with TOPSIS simultaneously. Our work shows a new approach towards the solution of the very big real-life problem of human trafficking through the essence of Prisoners' Dilemma.

We have proposed neutrosophic environment to solve bi-matrix game. For this purpose, we have considered neutrosophic characteristics, i.e., degree of acceptance, degree of rejection and degree of indeterminacy to judge the object's behaviour. In bi-matrix game model, we have used the proposed de-neutrosophic approach as a ranking approach.

Shortly to say, this thesis is designed to implement various types of uncertainty in games by two-person zero-sum and non-zero-sum formats through some new approaches in mathematical models, solution-procedures towards real-life problematic situations. And, this thesis claims some new outcomes against the problems, associated in each chapter.

8.2   Future research directions

There are many avenues of future work arising from this thesis. It deserves important points which significantly explore the future research works. The points are as follows:

(i) The applications of uncertainty in games and game in uncertain environments are the most important areas of today's research field in game theory. Researches can be done based on degree of uncertain environments through game. Researchers can develop new types of uncertain environment which may be widely applied in several decision-making problems.

(ii) The concept of uncertain programming approach on payoff matrix of game will open the new dimension of game theory.

(iii) The uncertainty can be assumed as strategy/ies in the new area of the game theory.

(iv) In this thesis, for an example, water management is treated through game. One can choice reservoir water management, waste water management, industrial water management, etc., as new areas of research.

(v) The matrix and bi-matrix games having payoff elements with different uncertain variables, can be considered, with different methodologies.

(vi) The researchers can challenge the problem on uncertain extensive game and uncertain coalition game.

(vii) The researchers can choice the problems on matrix and bi-matrix games under the environment of uncertain multilevel programming through Stackelberg-Nash equilibrium solution to the expected value on bi-level programming.

(viii) The proposed methodology of the thesis can be further extended if it has multiple payoffs with specific goal in game theory.

(ix) The studies of two-person zero-sum game and two-person non-zero-sum game in uncertainty can be studied through some extended version of interval environment like, rough-interval, soft-interval, soft-rough-hesitant-interval, etc.

(x) Significant research works can be executed over some classical non-zero-sum games, e.g., Battle of the Sexes, Chicken Game, Cournot Game, Hotelling's Model and Bertrand Game in uncertain environments applying new methods of solutions.

(xi) Construction of a bi-objective programming problem associated with intuitionistic fuzzy linguistic environment in matrix and bi-matrix games can be done and solved as new areas of researches.

(xii) Analysis of the solutions for matrix games and bi-matrix games as multi-objective optimization problems can be considered as new direction of future research works.

(xiii) Game theory can be applied in the fields of pattern recognition, artificial intelligence as future research works.

(xiv) Extension of hesitant fuzzy matrix games as well as bi-matrix games can be considered in Pythagorean fuzzy environment.

(xv) Finally, one can develop the mathematical tools for practical problems on engineering, economics, investment planning, operations research and other real-life problems using matrix and bi-matrix games under different uncertain environments.