Chapter 7

Two-person non-zero-sum game in neutrosophic environment*

In this chapter, we consider two-person non-zero-sum game under neutrosophic environment. Single valued triangular neutrosophic numbers liberally assume the indeterminacy in choice of elements based upon decision makers' intuition, assumption, judgement, behaviour, evaluation and decision. Here, a new ranking approach is based on the (α, β, γ)-cut of single valued triangular neutrosophic number and is applied on two-person non-zero-sum game theory by validating real-life problem.

7.1 Motivation

In **Chapter 6**, we discuss on two-person non-zero-sum game in HIVIFLTS based environment. Here, we assume two-person non-zero-sum game situation in neutrosophic environment (this neutrosophic environment are discussed earlier in zero-sum game phenomenon in **Chapter 4**). So, we can say that the construction of this chapter is done under the motivational thoughts of **Chapter 4** and **Chapter 6**.

7.2 Introduction

In this chapter, we consider two-person non-zero-sum game theories through two persons or players. The game model is considered with single-valued triangular neutrosophic numbers as uncertain payoff elements. The problem of uncertainty is transformed to certainty using cut-set approach of neutrosophic set. This de-neutrosophic technique is used to solve the proposed bi-matrix game. Main contributions of our chapter are:

- Solution of two-person non-zero-sum game under neutrosophic environment;
- Applying a newly proposed ranking approach on single valued triangular neutrosophic numbers;

*A part of this chapter has been communicated to an International Journal.

- Discussion on real-life shopping-marketing management problem under neutrosophic environment and game theory.

7.3 Basic Concepts

In this section, triangular fuzzy number, triangular intuitionistic fuzzy set, single-valued neutrosophic set and their properties are discussed. A fuzzy set, defined in the real line, is called fuzzy number if it is convex, normalized and its membership function is piecewise continuous on the real line. Among the various shapes and structures of fuzzy number, triangular fuzzy number is highly popular due to its easy accessibility and compatibility in several types of operations in reality. We recall a bit of definition of TIFN **(Definition 3.3.2, Chapter 3)** as an extended version of triangular fuzzy number. Now, we define the corresponding cut-set definitions.

Definition 7.3.1 *α-cut set, β-cut set and (α,β)-cut set of TIFN are defined below, as:*

(i) *A α-cut set of a TIFN $\hat{\varphi} = \langle(\underline{\varphi}, \varphi, \overline{\varphi});\ \epsilon_{\hat{\varphi}},\ \rho_{\hat{\varphi}}\rangle$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as: $\hat{\varphi}_\alpha = \{x : \phi_{\hat{\varphi}}(x) \geq \alpha\} = \left[\underline{\varphi} + \frac{\alpha}{\epsilon_{\hat{\varphi}}}(\varphi - \underline{\varphi}), \overline{\varphi} - \frac{\alpha}{\epsilon_{\hat{\varphi}}}(\overline{\varphi} - \varphi)\right]$.*

(ii) *A β-cut set of a TIFN $\hat{\varphi} = \langle(\underline{\varphi}, \varphi, \overline{\varphi});\ \epsilon_{\hat{\varphi}},\ \rho_{\hat{\varphi}}\rangle$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as: $\hat{\varphi}_\beta = \{x : \Phi_{\hat{\varphi}}(x) \leq \beta\} = \left[\frac{(1-\beta)\varphi + (\beta-\rho_{\hat{\varphi}})\underline{\varphi}}{1-\rho_{\hat{\varphi}}}, \frac{(1-\beta)\varphi + (\beta-\rho_{\hat{\varphi}})\overline{\varphi}}{1-\rho_{\hat{\varphi}}}\right]$.*

(iii) *A (α,β)-cut set of a TIFN $\hat{\varphi} = \langle(\underline{\varphi}, \varphi, \overline{\varphi});\ \epsilon_{\hat{\varphi}},\ \rho_{\hat{\varphi}}\rangle$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as: $\hat{\varphi}_{(\alpha,\beta)} = \{x : \phi_{\hat{\varphi}}(x) \geq \alpha, \Phi_{\hat{\varphi}}(x) \leq \beta\}$.*

We consider the definition of SVNS, neutrosophic normality and neutrosophic convexity from **Chapter 4 (Definitions 4.3.2, 4.3.3 and 4.3.4)**.

Definition 7.3.2 (α,β,γ)***-cut set of SVNS:*** *An (α,β,γ)-cut set of a SVNS $\acute{A}$, a crisp subset over the set of real numbers $\mathbb{R}$, is defined as: $\acute{A}_{(\alpha,\beta,\gamma)} = \{x : T_{\acute{A}}(x) \geq \alpha, I_{\acute{A}}(x) \leq \beta, F_{\acute{A}}(x) \leq \gamma\}$, with $0 \leq \alpha \leq 1,\ 0 \leq \beta \leq 1,\ 0 \leq \gamma \leq 1$, and $0 \leq \alpha + \beta + \gamma \leq 3$.*

7.3.1 Single-valued triangular neutrosophic numbers

A single-valued triangular neutrosophic number (SVTNN) is a particular type of single-valued neutrosophic numbers. Here we define single-valued triangular neutrosophic number and the corresponding arithmetic operations.

Definition 7.3.3 ***Single-valued triangular neutrosophic number:*** *A SVTNN $\breve{A} = \{\langle x, T_{\breve{A}}(x), I_{\breve{A}}(x), F_{\breve{A}}(x)\rangle : x \in X\}$ with the set of parameters $c_{11} \leq b_{11} \leq a_{11} \leq c_{21} \leq b_{21} \leq a_{21} \leq a_{31} \leq b_{31} \leq c_{31}$, is defined as: $\breve{A} = \langle(a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31})\rangle$ in the set of real numbers $\mathbb{R}$. The truth membership, the indeterminacy membership and the falsity membership* *degree of $\breve{A}$ can be obtained from the following.*

$$T_{\breve{A}}(x) = \begin{cases} \left(\frac{x-a_{11}}{a_{21}-a_{11}}\right), & \text{if } a_{11} \le x < a_{21}, \\ 1, & \text{if } x = a_{21}, \\ \left(\frac{a_{31}-x}{a_{31}-a_{21}}\right), & \text{if } a_{21} < x \le a_{31}, \\ 0, & \text{otherwise}, \end{cases} \tag{7.1}$$

$$I_{\breve{A}}(x) = \begin{cases} \left(\frac{b_{21}-x}{b_{21}-b_{11}}\right), & \text{if } b_{11} \le x < b_{21}, \\ 0, & \text{if } x = b_{21}, \\ \left(\frac{x-b_{21}}{b_{31}-b_{21}}\right), & \text{if } b_{21} < x \le b_{31}, \\ 1, & \text{otherwise} \end{cases} \tag{7.2}$$

$$\& \; F_{\breve{A}}(x) = \begin{cases} \left(\frac{c_{21}-x}{c_{21}-c_{11}}\right), & \text{if } c_{11} \le x < c_{21}, \\ 0, & \text{if } x = c_{21}, \\ \left(\frac{x-c_{21}}{c_{31}-c_{21}}\right), & \text{if } c_{21} < x \le c_{31}, \\ 1, & \text{otherwise.} \end{cases} \tag{7.3}$$

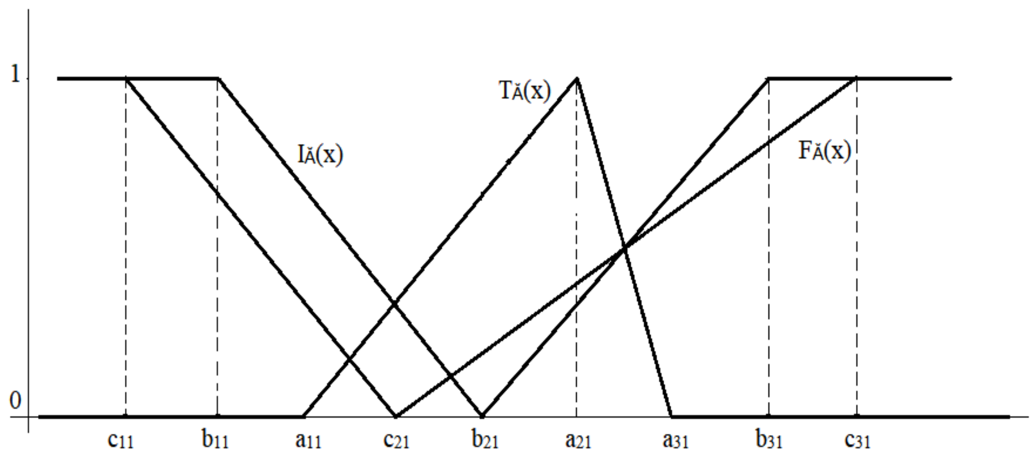

Figure 7.1: Single valued triangular neutrosophic number.

In Fig. 7.1, non-decreasing part of $T_{\breve{A}}$ represents the left side of truth function of $\breve{A}$. Similarly, non-increasing parts of each functions $I_{\breve{A}}$ and $F_{\breve{A}}$ represent the left sides of indeterminacy and falsity membership functions of $\breve{A}$ respectively. Similarly, the three right side parts of truth, indeterminacy and falsity membership functions of $\breve{A}$ are defined.

Definition 7.3.4 ***Operations and properties on SVTNNs:*** *Consider two SVTNNs $\breve{A} = \langle(a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31})\rangle$ and $\breve{B} = \langle(a_{12}, a_{22}, a_{32}), (b_{12}, b_{22}, b_{32}), (c_{12}, c_{22}, c_{32})\rangle$ in the set of real numbers $\mathbb{R}$, with the truth membership $T_{\breve{A}}$, the indeterminacy membership $I_{\breve{A}}$, the falsity membership $F_{\breve{A}}$ of $\breve{A}$ and the truth membership $T_{\breve{B}}$, the indeterminacy membership $I_{\breve{B}}$ and the falsity membership $F_{\breve{B}}$ of $\breve{B}$. Let $\lambda > 0, \ \lambda_1 > 0, \ \lambda_2 > 0$. Then the following operations* **(i)-(iv)** *and properties* **(v)-(vii)** *are obtained as:*

(i) $\breve{A} \oplus \breve{B} = \langle (a_{11} + a_{12} - a_{11}a_{12}, a_{21} + a_{22} - a_{21}a_{22}, a_{31} + a_{32} - a_{31}a_{32}), (b_{11}b_{12}, b_{21}b_{22}, b_{31}b_{32}), (c_{11}c_{12}, c_{21}c_{22}, c_{31}c_{32}) \rangle$;

(ii) $\breve{A} \otimes \breve{B} = \langle (a_{11}a_{12}, a_{21}a_{22}, a_{31}a_{32}), (b_{11} + b_{12} - b_{11}b_{12}, b_{21} + b_{22} - b_{21}b_{22}, b_{31} + b_{32} - b_{31}b_{32}), (c_{11} + c_{12} - c_{11}c_{12}, c_{21} + c_{22} - c_{21}c_{22}, c_{31} + c_{32} - c_{31}c_{32}) \rangle$;

(iii) $\lambda \breve{A} = \langle (1 - (1 - a_{11})^{\lambda}, 1 - (1 - a_{21})^{\lambda}, 1 - (1 - a_{31})^{\lambda}), (b_{11}^{\lambda}, b_{21}^{\lambda}, b_{31}^{\lambda}), (c_{11}^{\lambda}, c_{21}^{\lambda}, c_{31}^{\lambda}) \rangle$;

(iv) $\breve{A}^{\lambda} = \langle (a_{11}^{\lambda}, a_{21}^{\lambda}, a_{31}^{\lambda}), (1 - (1 - b_{11})^{\lambda}, 1 - (1 - b_{21})^{\lambda}, 1 - (1 - b_{31})^{\lambda}), (c_{11}^{\lambda}, 1 - (1 - c_{21})^{\lambda}, 1 - (1 - c_{31})^{\lambda}) \rangle$;

(v) $\breve{A} \oplus \breve{B} = \breve{B} \oplus \breve{A}$; $\breve{A} \otimes \breve{B} = \breve{B} \otimes \breve{A}$;

(vi) $\lambda(\breve{A} \oplus \breve{B}) = \lambda \breve{A} \oplus \lambda \breve{B}$; $(\breve{A} \otimes \breve{B})^{\lambda} = \breve{A}^{\lambda} \otimes \breve{B}^{\lambda}$;

(vii) $\lambda_1 \breve{A} \oplus \lambda_2 \breve{A} = (\lambda_1 + \lambda_2)\breve{A}$; $\breve{A}^{\lambda_1} \oplus \breve{A}^{\lambda_2} = \breve{A}^{(\lambda_1 + \lambda_2)}$.

Definition 7.3.5 *α-cut set, β-cut set, γ-cut set and (α, β, γ)-cut set of a SVTNN $\breve{A} = \langle (a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31}) \rangle$ in the set of real numbers $\mathbb{R}$, with the truth membership $T_{\breve{A}}$, the indeterminacy membership $I_{\breve{A}}$ and the falsity membership $F_{\breve{A}}$ of $\breve{A}$ can be obtained from the following.*

(i) *A α-cut set of a SVTNN $\breve{A} = \langle (a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31}) \rangle$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as:*
$\breve{A}_{\alpha} = \{x : T_{\breve{A}}(x) \geq \alpha\} = [L^{\alpha}(\breve{A}), R^{\alpha}(\breve{A})] = [a_{11} + \alpha(a_{21} - a_{11}), a_{31} - \alpha(a_{31} - a_{21})]$.

(ii) *A β-cut set of a SVTNN $\breve{A} = \langle (a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31}) \rangle$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as:*
$\breve{A}_{\beta} = \{x : I_{\breve{A}}(x) \leq \beta\} = [L^{\beta}(\breve{A}), R^{\beta}(\breve{A})] = [b_{11} + \beta(b_{21} - b_{11}), b_{21} + \beta(b_{31} - b_{21})]$.

(iii) *A γ-cut set of a SVTNN $\breve{A} = \langle (a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31}) \rangle$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as:*
$\breve{A}_{\gamma} = \{x : F_{\breve{A}}(x) \leq \gamma\} = [L^{\gamma}(\breve{A}), R^{\gamma}(\breve{A})] = [c_{11} + \gamma(c_{21} - c_{11}), c_{21} + \gamma(c_{31} - c_{21})]$.

(iv) *A (α, β, γ)-cut set of a SVTNN $\breve{A} = \langle (a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31}) \rangle$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as:*
$\breve{A}_{(\alpha,\beta,\gamma)} = \{x : T_{\breve{A}}(x) \geq \alpha, I_{\breve{A}}(x) \leq \beta, F_{\breve{A}}(x) \leq \gamma\}$
$= \langle [L^{\alpha}(\breve{A}), R^{\alpha}(\breve{A})], [L^{\beta}(\breve{A}), R^{\beta}(\breve{A})], [L^{\gamma}(\breve{A}), R^{\gamma}(\breve{A})] \rangle$
$= \langle [a_{11} + \alpha(a_{21} - a_{11}), a_{31} - \alpha(a_{31} - a_{21})], [b_{11} + \beta(b_{21} - b_{11}), b_{21} + \beta(b_{31} - b_{21})], [c_{11} + \gamma(c_{21} - c_{11}), c_{21} + \gamma(c_{31} - c_{21})] \rangle$ *with* $0 \leq \alpha \leq 1, 0 \leq \beta \leq 1, 0 \leq \gamma \leq 1$, *and* $0 \leq \alpha + \beta + \gamma \leq 3$.

Definition 7.3.6 *Sum of two SVTNNs $\breve{A}$ and $\breve{B}$ is again a SVTNN. Therefore, α-cut set, β-cut set and γ-cut set of SVTNN $(\breve{A} + \breve{B})$ in the set of real numbers $\mathbb{R}$, with the truth membership $T_{\breve{A}+\breve{B}}$, the indeterminacy membership $I_{\breve{A}+\breve{B}}$ and the falsity membership $F_{\breve{A}+\breve{B}}$ can be obtained from the following:*

(i) *A α-cut set of a SVTNN $\breve{A} + \breve{B}$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as:*
$(\breve{A} + \breve{B})_{\alpha} = \{x : T_{\breve{A}+\breve{B}}(x) \geq \alpha\} = [L^{\alpha}(\breve{A} + \breve{B}), R^{\alpha}(\breve{A} + \breve{B})]$;

(ii) *A β-cut set of a SVTNN $\breve{A}+\breve{B}$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as:*
$(\breve{A}+\breve{B})_\beta = \{x : I_{\breve{A}+\breve{B}}(x) \le \beta\} = [L^\beta(\breve{A}+\breve{B}), R^\beta(\breve{A}+\breve{B})]$;

(iii) *A γ-cut set of a SVTNN $\breve{A}+\breve{B}$ is a crisp subset of set of real numbers $\mathbb{R}$, defined as:*
$(\breve{A}+\breve{B})_\gamma = \{x : F_{\breve{A}+\breve{B}}(x) \le \gamma\} = [L^\gamma(\breve{A}+\breve{B}), R^\gamma(\breve{A}+\breve{B})]$.

Definition 7.3.7 *(α,β,γ)-cut set of a SVTNN $\breve{A} = \langle (a_{11},a_{21},a_{31}),(b_{11},b_{21},b_{31}),(c_{11},c_{21},c_{31})\rangle$ in the set of real numbers $\mathbb{R}$, with the truth membership $T_{\breve{A}}$, the indeterminacy membership $I_{\breve{A}}$ and the falsity membership $F_{\breve{A}}$ of $\breve{A}$ satisfy the following relations:*

(i) $\frac{dL^\alpha(\breve{A})}{d\alpha} > 0, \frac{dR^\alpha(\breve{A})}{d\alpha} < 0, \forall\, \alpha \in [0,1] \Rightarrow L^1(\breve{A}) \ge R^1(\breve{A})$.

(ii) $\frac{dL^\beta(\breve{A})}{d\beta} < 0, \frac{dR^\beta(\breve{A})}{d\beta} > 0, \forall\, \beta \in [0,1] \Rightarrow L^0(\breve{A}) \le R^0(\breve{A})$.

(iii) $\frac{dL^\gamma(\breve{A})}{d\gamma} < 0, \frac{dR^\gamma(\breve{A})}{d\gamma} > 0, \forall\, \gamma \in [0,1] \Rightarrow L^0(\breve{A}) \le R^0(\breve{A})$.

These clarify the stability analysis of cut-sets.

7.3.2 Values and Ambiguities indices for SVTNNs

Here, we define the values and ambiguities indices according to the cut-sets on SVTNN. Let α-cut set, β-cut set and γ-cut set of a SVTNN $\breve{A} = \langle (a_{11},a_{21},a_{31}),(b_{11},b_{21},b_{31}),(c_{11},c_{21},c_{31})\rangle$ in the set of real numbers $\mathbb{R}$, with the truth membership $T_{\breve{A}}$, the indeterminacy membership $I_{\breve{A}}$ and the falsity membership $F_{\breve{A}}$ are given, respectively, as: $\breve{A}_\alpha = [L^\alpha(\breve{A}), R^\alpha(\breve{A})]$, $\breve{A}_\beta = [L^\beta(\breve{A}), R^\beta(\breve{A})]$, $\breve{A}_\gamma = [L^\gamma(\breve{A}), R^\gamma(\breve{A})]$. Therefore, the value and ambiguity indices are calculated as:

Value indices:

(1) For truth-membership, the values of SVTNN $\breve{A}$ for α-cut is denoted by $V_T(\breve{A})$ and is defined as: $V_T(\breve{A}) = \int_0^1 \Big(L^\alpha(\breve{A}) + R^\alpha(\breve{A})\Big) f(\alpha) d\alpha$.

(2) For indeterminacy-membership, the values of SVTNN $\breve{A}$ for β-cut is denoted by $V_I(\breve{A})$ and is defined as: $V_I(\breve{A}) = \int_0^1 \Big(L^\beta(\breve{A}) + R^\beta(\breve{A})\Big) g(\beta) d\beta$.

(3) For falsity-membership, the values of SVTNN $\breve{A}$ for γ-cut is denoted by $V_F(\breve{A})$ and is defined as: $V_F(\breve{A}) = \int_0^1 \Big(L^\gamma(\breve{A}) + R^\gamma(\breve{A})\Big) h(\gamma) d\gamma$.

Ambiguity indices:

(4) For truth-membership, the ambiguities of SVTNN $\breve{A}$ for α-cut is denoted by $A_T(\breve{A})$ and is defined as: $A_T(\breve{A}) = \int_0^1 \Big(R^\alpha(\breve{A}) - L^\alpha(\breve{A})\Big) f(\alpha) d\alpha$.

(5) For indeterminacy-membership, the ambiguities of SVTNN $\breve{A}$ for β-cut is denoted by $A_I(\breve{A})$ and is defined as: $A_I(\breve{A}) = \int_0^1 \Big(R^\beta(\breve{A}) - L^\beta(\breve{A})\Big) g(\beta) d\beta$.

(6) For falsity-membership, the ambiguities of SVTNN $\breve{A}$ for γ-cut is denoted by $A_F(\breve{A})$ and is defined as: $A_F(\breve{A}) = \int_0^1 \Big(R^\gamma(\breve{A}) - L^\gamma(\breve{A})\Big)h(\gamma)d\gamma$.

Here the weighting functions $f(\alpha),\ g(\beta),\ h(\gamma)$ can be considered according as the nature of the decision making problem. Consider $f(\alpha) = \alpha,\ g(\beta) = 1 - \beta,\ h(\gamma) = 1 - \gamma$.
Again $f(\alpha)$ is non-negative, monotonic and non-decreasing function on the interval $[0, 1]$, satisfying the conditions $f(0) = 0,\ f(1) = 1$. Also, $g(\beta)$ is a non-negative, monotonic and non-increasing function on the interval $[0, 1]$ satisfying the conditions $g(0) = 1,\ g(1) = 0$; and $h(\gamma)$ is also non-negative, monotonic and non-increasing function on the interval $[0, 1]$ satisfying the conditions $h(0) = 1,\ h(1) = 0$.

7.4 Ranking approach on SVTNNs

In this section, the ranking method of SVTNNs based on values and ambiguities are discussed. Here, the corresponding properties of ranking approach are also discussed. Let $\breve{A}$ be a SVTNN, i.e., $\breve{A} = \langle (a_{11}, a_{21}, a_{31}), (b_{11}, b_{21}, b_{31}), (c_{11}, c_{21}, c_{31})\rangle$. Then the value indices and ambiguity indices, using the descriptions from Section 7.3.2, are evaluated as follows:

$$\begin{aligned} V_T(\breve{A}) &= \frac{a_{11} + 4a_{21} + a_{31}}{6}, \quad A_T(\breve{A}) = \frac{a_{31} - a_{11}}{6} \\ V_I(\breve{A}) &= \frac{b_{11} + 6b_{21} + b_{31}}{6}, \quad A_I(\breve{A}) = \frac{b_{31} - b_{11}}{6} \\ V_F(\breve{A}) &= \frac{c_{11} + 6c_{21} + c_{31}}{6}, \quad A_F(\breve{A}) = \frac{c_{31} - c_{11}}{6} \end{aligned} \tag{7.4}$$

Definition 7.4.1 *The weighted value-ambiguity index for SVTNN is defined as:*
$R_{w_1,w_2,w_3}(\breve{A}) = \big[w_1V_T(\breve{A}) + (1 - w_1)A_T(\breve{A})\big] + \big[w_2V_I(\breve{A}) + (1 - w_2)A_I(\breve{A})\big] + \big[w_3V_F(\breve{A}) + (1 - w_3)A_F(\breve{A})\big],\ w_1, w_2, w_3 \in [0, 1]$.

Theorem 7.4.1 R_{w_1,w_2,w_3} *is linear.*

Proof: Assume $\breve{M}$ and $\breve{N}$ be two SVTNNs. Then to prove R_{w_1,w_2,w_3} as linear, we have to show that, $R_{w_1,w_2,w_3}(\breve{M} + \kappa\breve{N}) = R_{w_1,w_2,w_3}(\breve{M}) + \kappa R_{w_1,w_2,w_3}(\breve{N})$.
Here,

$$\begin{aligned} R_{w_1,w_2,w_3}(\breve{M}) = &\{w_1V_T(\breve{M}) + (1 - w_1)A_T(\breve{M})\} + \{w_2V_I(\breve{M}) + (1 - w_2)A_I(\breve{M})\} \\ &+ \{w_3V_F(\breve{M}) + (1 - w_3)A_F(\breve{M})\}, \text{ and} \\ R_{w_1,w_2,w_3}(\breve{N}) = &\{w_1V_T(\breve{N}) + (1 - w_1)A_T(\breve{N})\} + \{w_2V_I(\breve{N}) + (1 - w_2)A_I(\breve{N})\} \\ &+ \{w_3V_F(\breve{N}) + (1 - w_3)A_F(\breve{N})\}. \end{aligned}$$

Also, $\kappa \in \mathbb{R}$ be a real number. According to **Definition 7.3.1**, α-cut set of $(\breve{M} + \kappa\breve{N})$ is (α-cut of $\breve{M}$ + α-cut of $\kappa\breve{N}$); and α-cut of $\kappa\breve{N}$ is κ(α-cut of $\breve{N}$).
Now,

$$\begin{aligned} &R_{w_1,w_2,w_3}(\breve{M} + \kappa\breve{N}) \\ &= \{w_1V_T(\breve{M} + \kappa\breve{N}) + (1 - w_1)A_T(\breve{M} + \kappa\breve{N})\} + \{w_2V_I(\breve{M} + \kappa\breve{N}) + (1 - w_2)A_I(\breve{M} + \kappa\breve{N})\} \\ &\quad + \{w_3V_F(\breve{M} + \kappa\breve{N}) + (1 - w_3)A_F(\breve{M} + \kappa\breve{N})\} \end{aligned}$$

$$= \{w_1(V_T(\breve{M}) + V_T\kappa(\breve{N})) + (1-w_1)(A_T(\breve{M}) + A_T\kappa(\breve{N}))\} + \{w_2(V_I(\breve{M}) + V_I\kappa(\breve{N})) + (1-w_2)(A_I(\breve{M}) + A_I\kappa(\breve{N}))\} + \{w_3(V_F(\breve{M}) + V_F\kappa(\breve{N})) + (1-w_3)(A_F(\breve{M}) + A_F\kappa(\breve{N}))\}$$
$$= \{w_1(V_T(\breve{M}) + \kappa V_T(\breve{N})) + (1-w_1)(A_T(\breve{M}) + \kappa A_T(\breve{N}))\} + \{w_2(V_I(\breve{M}) + \kappa V_I(\breve{N})) + (1-w_2)(A_I(\breve{M}) + \kappa A_I(\breve{N}))\} + \{w_3(V_F(\breve{M}) + \kappa V_F(\breve{N})) + (1-w_3)(A_F(\breve{M}) + \kappa A_F(\breve{N}))\}$$
$$= \{w_1 V_T(\breve{M}) + \kappa w_1 V_T(\breve{N}) + (1-w_1)A_T(\breve{M}) + \kappa(1-w_1)A_T(\breve{N})\} + \{w_2 V_I(\breve{M}) + \kappa w_2 V_I(\breve{N}) + (1-w_2)A_I(\breve{M}) + \kappa(1-w_2)A_I(\breve{N})\} + \{w_3 V_F(\breve{M}) + \kappa w_3 V_F(\breve{N}) + (1-w_3)A_F(\breve{M}) + \kappa(1-w_3)A_F(\breve{N})\}$$
$$= \{w_1 V_T(\breve{M}) + (1-w_1)A_T(\breve{M}) + w_2 V_I(\breve{M}) + (1-w_2)A_I(\breve{M}) + w_3 V_F(\breve{M}) + (1-w_3)A_F(\breve{M})\} + \{\kappa w_1 V_T(\breve{N}) + \kappa(1-w_1)A_T(\breve{N}) + \kappa w_2 V_I(\breve{N}) + \kappa(1-w_2)A_I(\breve{N}) + \kappa w_3 V_F(\breve{N}) + \kappa(1-w_3)A_F(\breve{N})\}$$
$$= \{w_1 V_T(\breve{M}) + (1-w_1)A_T(\breve{M}) + w_2 V_I(\breve{M}) + (1-w_2)A_I(\breve{M}) + w_3 V_F(\breve{M}) + (1-w_3)A_F(\breve{M})\} + \kappa\{w_1 V_T(\breve{N}) + (1-w_1)A_T(\breve{N}) + w_2 V_I(\breve{N}) + (1-w_2)A_I(\breve{N}) + w_3 V_F(\breve{N}) + (1-w_3)A_F(\breve{N})\}$$
$$= R_{w_1,w_2,w_3}(\breve{M}) + \kappa R_{w_1,w_2,w_3}(\breve{N})$$

Thus the theorem is proved. □

Property 7.4.1 *Consider $\breve{M}$ and $\breve{N}$ be two SVTNNs and the weighted value-ambiguity index function for SVTNN be R_{w_1,w_2,w_3}. Then, we have the following relations:*
(i) $R_{w_1,w_2,w_3}(\breve{M}) \succ R_{w_1,w_2,w_3}(\breve{N}) \Rightarrow \breve{M} \succ \breve{N}$;
(ii) $R_{w_1,w_2,w_3}(\breve{M}) \prec R_{w_1,w_2,w_3}(\breve{N}) \Rightarrow \breve{M} \prec \breve{N}$;
(iii) $R_{w_1,w_2,w_3}(\breve{M}) = R_{w_1,w_2,w_3}(\breve{N}) \Rightarrow \breve{M} = \breve{N}$.

7.5 Mathematical Model

Bi-matrix game in neutrosophic environment: Consider a bi-matrix game with two minimizing players, player I (PI) and player II (PII). The set of pure strategies S_1 and S_2, respectively, and that of mixed strategies Y and Z for PI and PII, respectively, are defined as:

$$S_1 = \{\alpha_1, \alpha_2, \ldots, \alpha_p\}, S_2 = \{\beta_1, \beta_2, \ldots, \beta_q\}, \tag{7.5}$$

$$Y = \{y = (y_1, y_2, \ldots, y_p)^T : \sum_{i=1}^{p} y_i = 1,\ y_i \geq 0, \forall\, i\}, \tag{7.6}$$

$$Z = \{z = (z_1, z_2, \ldots, z_q)^T : \sum_{j=1}^{q} z_j = 1,\ z_j \geq 0, \forall\, j\}, \tag{7.7}$$

where y_i $(i = 1, 2, \ldots, p)$ and z_j $(j = 1, 2, \ldots, q)$ are probabilities in which PI and PII choose their pure strategies $\alpha_i \in S_1$ $(i = 1, 2, \ldots, p)$ and $\beta_j \in S_2$ $(j = 1, 2, \ldots, q)$ respectively and the game is expressed as $G \equiv (Y, Z; A, B)$. Here, payoff matrices for PI and PII are depicted respectively as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1q} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2q} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & a_{p3} & \ldots & a_{pq} \end{bmatrix} \text{and } B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \ldots & b_{1q} \\ b_{21} & b_{22} & b_{23} & \ldots & b_{2q} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{p1} & b_{p2} & b_{p3} & \ldots & b_{pq} \end{bmatrix}.$$

If PI chooses any mixed strategy $y \in Y$ and PII chooses any mixed strategy $z \in Z$, then the expected payoffs of PI and PII are defined as $E_{PI}(y, z) = y^T Az$ and $E_{PII}(y, z) = y^T Bz$, respectively. *Nash equilibrium* is a solution concept in non-cooperative game theory involving two or more players in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. When each player chooses a strategy and no player can benefit by changing strategies while the other players keep their strategies unchanged, then the current set of strategy choices and the corresponding payoffs constitute *Nash equilibrium*. The Pareto optimal solution is guaranteed by Nash existence theorem [113].

Definition 7.5.1 ***Pareto optimal solution*** *In a bi-matrix game* (Y, Z, A, B)*, assume that there is a pair* $(y^*, z^*) \in Y \times Z$*. A strategy* $y^* \in Y$ *is the Pareto optimal strategy for PI if there exists no* $y \in Y$ *such that* $y^T Az^* \geq {y^*}^T Az^*$*. Similarly, a strategy* $z^* \in Z$ *is the Pareto optimal strategy for PII if there is no* $z \in Z$ *such that* ${y^*}^T Bz^* \leq {y^*}^T Bz$*. Here* $u^* = {y^*}^T Az^*$ *and* $v^* = {y^*}^T Bz^*$ *are called optimal values of PI and PII, respectively. Here* $\leq, \ =, \ \geq$ *have usual meaning in neutrosophic environment.*

Theorem 7.5.1 *Assume* $(Y, Z; A, B)$ *be any bi-matrix game in neutrosophic environment. Now,* (y^*, z^*, u^*, v^*) *is a Pareto optimal solution of the bi-matrix game* $(Y, Z; A, B)$ *if and only if it is a solution of the following programming problem as:*

$$\begin{aligned} \textit{maximize} \quad & \left[y^T(A+B)z - u - v\right] \\ \textit{subject to} \quad & Az \leq ue_p, \\ & By \leq ve_q, \\ & y^T e_p = 1, \\ & z^T e_q = 1, \\ & y \geq 0, z \geq 0, e_p = (1, 1, \ldots, 1), e_q = (1, 1, \ldots, 1). \end{aligned} \tag{7.8}$$

Furthermore, (y^*, z^*, u^*, v^*) *is a solution of the Eq. (7.8), then the relation* ${y^*}^T(A+B)z^* - u^* - v^* = 0$ *holds.*

Proof: According to the constraints of the programming problem (given by Eq. (7.8)), $y^T Az \leq -y^T ue_p$, $z^T By \leq -z^T ve_q$, $y^T e_p = 1$, $z^T e_q = 1$; We have $y^T Az \leq -u$ and $z^T By \leq -v$. Therefore, ${y^*}^T(A+B)z^* - u^* - v^* \leq 0$. According to **Definition 7.5.1**, the Pareto optimal solution of the neutrosophic bi-matrix game can be obtained by solving the following mathematical model:

$$\begin{aligned} \text{maximize} \quad & \left[y^T Az^* + {y^*}^T Bz\right] \\ \text{subject to} \quad & y^T e_p = 1, \\ & z^T e_q = 1, \\ & y \geq 0, z \geq 0 \end{aligned} \tag{7.9}$$

Let $u = \max_{y \in Y} y^T Az^*$ and $v = \max_{z \in Z} {y^*}^T Bz$. The constraint $u > y^T Az^* > y^T Az$ is also true for all $y \geq 0$. So we have $ue_p \geq Az$. Similarly, for $v > {y^*}^T Bz > y^T Bz, \forall z \geq 0$, we have $ve_q \geq B^T y$.

Then the Eq. (7.9) can be transformed into the following programming model in neutrosophic environment:

$$\begin{aligned}
\text{maximize} \quad & \left[(y^T Az - u) + (y^T Bz - v)\right] \\
\text{subject to} \quad & Az \leq ue_p, \\
& By \leq ve_q, \\
& y^T e_p = 1, \\
& z^T e_q = 1, \\
& y \geq 0, z \geq 0, e_p = (1, 1, \ldots, 1), e_q = (1, 1, \ldots, 1).
\end{aligned} \tag{7.10}$$

Thus the theorem is proved. □

Now we apply the ranking method of single-valued triangular neutrosophic number, as proposed in Section 7.4, the game problem in neutrosophic environment (given by Eq. (7.10)) is converted into the parameterized non-linear programming model as follow:

$$\begin{aligned}
\text{maximize} \quad & \Big[\sum_{j=1}^{q}\sum_{i=1}^{p} y_i \Big(R_{w_1,w_2,w_3}(\breve{A}_{ij}) + R_{w_1,w_2,w_3}(\breve{B}_{ij})\Big) z_j \\
& -R_{w_1,w_2,w_3}(u) - R_{w_1,w_2,w_3}(v)\Big] \\
\text{subject to} \quad & \sum_{j=1}^{q} R_{w_1,w_2,w_3}(\breve{A}_{ij}) z_j \leq R_{w_1,w_2,w_3}(u), \ i = 1(1)p, \\
& \sum_{i=1}^{p} R_{w_1,w_2,w_3}(\breve{B}_{ij})^T z_j \leq R_{w_1,w_2,w_3}(v), \ j = 1(1)q, \\
& \sum_{i=1}^{p} y_i = 1, y_i \geq 0, \ i = 1(1)p, \\
& \sum_{j=1}^{q} z_j = 1, z_j \geq 0, \ j = 1(1)q.
\end{aligned} \tag{7.11}$$

If $(y^*, z^*, R_{w_1,w_2,w_3}(u^*), R_{w_1,w_2,w_3}(v^*))$ is an optimal solution of the parameterized non-linear programming model, then (y^*, z^*) is a Pareto-optimal strategy of the bi-matrix game in single-valued triangular neutrosophic environment and $R_{w_1,w_2,w_3}(u^*), R_{w_1,w_2,w_3}(v^*)$ are the Pareto-optimal values of players I and II, respectively.

7.6 Numerical Simulation

In order to illustrate the accessibility and effectiveness of the proposed ranking approach to the bi-matrix game, an example is considered from mobile selling shopping-marketing management problem. We consider here two mobile producing companies, say C_1 and C_2 (i.e., player I and player II). They wish to make a decision as to how to capture the mobile-market share by their products. One uses Android technology while the other is configured by Windows. Due to uncertain and imprecise information from the view points of sellers, customers, reviewers, markets etc., we cannot use simple fuzzy set or its any extended forms to represent the payoff for any one of the selling strategies. Thus it seems to be more close to real-life phenomenon. Here we assume C_1 and C_2 as rational, i.e., they choose their optimal strategies to maximize their own profits without any cooperation. Suppose C_1 has two strategies to sell its products as, α_1: features in mobile, and α_2: reasonable price. Similarly, C_2 has two strategies as, β_1: customer care, and β_2: features in mobile. Thus we construct the decision matrix (bi-matrix game), which in separated form can be expressed as $\breve{A}$ and $\breve{B}$, with the set of pure strategies $\alpha_1,\ \alpha_2,\ \beta_1$ and β_2, as:

$$\breve{A} = \begin{pmatrix} \langle(0.50, 0.65, 0.80), (0.10, 0.15, 0.30), (0.10, 0.20, 0.30)\rangle & \langle(0.10, 0.20, 0.30), (0.20, 0.30, 0.40), (0.40, 0.50, 0.70)\rangle \\ \langle(0.30, 0.45, 0.50), (0.10, 0.20, 0.40), (0.10, 0.20, 0.30)\rangle & \langle(0.20, 0.35, 0.50), (0.10, 0.25, 0.30), (0.20, 0.30, 0.40)\rangle \end{pmatrix},$$

and,

$$\breve{B} = \begin{pmatrix} \langle(0.30, 0.40, 0.50), (0.10, 0.20, 0.30), (0.20, 0.25, 0.40)\rangle & \langle(0.40, 0.50, 0.70), (0.20, 0.30, 0.50), (0.10, 0.20, 0.30)\rangle \\ \langle(0.20, 0.30, 0.35), (0.10, 0.10, 0.10), (0.60, 0.70, 0.80)\rangle & \langle(0.40, 0.50, 0.60), (0.20, 0.30, 0.40), (0.20, 0.30, 0.40)\rangle \end{pmatrix}.$$

Here $\langle(0.50, 0.65, 0.80), (0.10, 0.15, 0.30), (0.10, 0.20, 0.30)\rangle$ from $\breve{A}$ represents that company C_1 is able to convince customers that its products are better than those of others positively by 65% with lower limit 50% to upper limit 80%, unable to convince customers by 20% within 10% to 30%. The company C_1 has indeterminacy about convince of customers by 15% having upper limit 30% from lower limit 10%. These situations occur only when player I adopts α_1 and player II adopts β_1. Similarly others elements of $\breve{A}$ and $\breve{B}$ are explained. All these explanations are based on numbers of sold items. Now the mobile companies try to determine the ranges of the expected profits. Thus, we want to compute the neutrosophic values of the mobile companies in the bi-matrix game.
Applying the proposed ranking approach to the bi-matrix game, we have,

$$R_{w_1,w_2,w_3}(\breve{A})$$

$$= \begin{pmatrix} R_{w_1,w_2,w_3}(\breve{A}_{11}) & R_{w_1,w_2,w_3}(\breve{A}_{12}) \\ R_{w_1,w_2,w_3}(\breve{A}_{21}) & R_{w_1,w_2,w_3}(\breve{A}_{22}) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{6}(3.60w_1 + 1.10w_2 + 1.40w_3 + 0.70) & \frac{1}{6}(w_1 + 2.20w_2 + 3.80w_3 + 0.70) \\ \frac{1}{6}(2.40w_1 + 1.40w_2 + 1.40w_3 + 0.70) & \frac{1}{6}(1.80w_1 + 1.70w_2 + 2.20w_3 + 0.70) \end{pmatrix},$$

and $R_{w_1,w_2,w_3}(\breve{B})$

$$= \begin{pmatrix} R_{w_1,w_2,w_3}(\breve{B}_{11}) & R_{w_1,w_2,w_3}(\breve{B}_{12}) \\ R_{w_1,w_2,w_3}(\breve{B}_{21}) & R_{w_1,w_2,w_3}(\breve{B}_{22}) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{6}(2.20w_1 + 1.40w_2 + 1.90w_3 + 0.60) & \frac{1}{6}(2.80w_1 + 2.20w_2 + 1.40w_3 + 0.80) \\ \frac{1}{6}(1.60w_1 + 0.80w_2 + 5.40w_3 + 0.35) & \frac{1}{6}(2.80w_1 + 2.20w_2 + 2.20w_3 + 0.60) \end{pmatrix}.$$

Therefore, we achieve,
$R_{w_1,w_2,w_3}(\breve{A}+\breve{B})$

$$= \begin{pmatrix} \frac{1}{6}(5.80w_1+2.50w_2+3.30w_3+1.30) & \frac{1}{6}(3.80w_1+4.40w_2+5.20w_3+1.50) \\ \frac{1}{6}(4.00w_1+2.20w_2+6.80w_3+1.05) & \frac{1}{6}(4.60w_1+3.90w_2+4.40w_3+1.30) \end{pmatrix}.$$

Again we apply the ranking approach on SVTNN, proposed in Section 7.4, and get the converted parameterized non-linear programming model as follows:

$$\begin{aligned}
\text{maximize} \quad & \Big[\frac{1}{6}(5.80w_1+2.50w_2+3.30w_3+1.30)y_1z_1 \\
& +\frac{1}{6}(3.80w_1+4.40w_2+5.20w_3+1.50)y_1z_2 \\
& +\frac{1}{6}(4.00w_1+2.20w_2+6.80w_3+1.05)y_2z_1 \\
& +\frac{1}{6}(4.60w_1+3.90w_2+4.40w_3+1.30)y_2z_2 \\
& -R_{w_1,w_2,w_3}(u)-R_{w_1,w_2,w_3}(v)\Big] \\
\text{subject to} \quad & \frac{1}{6}(3.60w_1+1.10w_2+1.40w_3+0.70)z_1 \\
& +\frac{1}{6}(1.00w_1+2.20w_2+3.80w_3+0.70)z_2 \\
& \leq R_{w_1,w_2,w_3}(u), \\
& \frac{1}{6}(2.40w_1+1.40w_2+1.40w_3+0.70)z_1 \\
& +\frac{1}{6}(1.80w_1+1.70w_2+2.20w_3+0.70)z_2 \\
& \leq R_{w_1,w_2,w_3}(u), \\
& \frac{1}{6}(2.20w_1+1.40w_2+1.90w_3+0.60)y_1 \\
& +\frac{1}{6}(1.60w_1+0.80w_2+5.40w_3+0.35)y_2 \\
& \leq R_{w_1,w_2,w_3}(v), \\
& \frac{1}{6}(2.80w_1+2.20w_2+1.40w_3+0.80)y_1 \\
& +\frac{1}{6}(2.80w_1+2.20w_2+2.20w_3+0.60)y_2 \\
& \leq R_{w_1,w_2,w_3}(v), \\
& y_1+y_2=1, \\
& z_1+z_2=1, \\
& y_1,y_2\geq 0, \\
& z_1,z_2\geq 0.
\end{aligned} \tag{7.12}$$

Using Lingo iterative scheme, we obtain the solutions (from set of Eqs. (7.12)), as depicted in Table 7.1.

Table 7.1: Pareto optimal strategies and value-ambiguity based optimal values

(w_1, w_2, w_3)	player I(y^*)	$R_{w_1,w_2,w_3}(u^*)$	player II(z^*)	$R_{w_1,w_2,w_3}(v^*)$
(0.3, 0.4, 0.5)	(1, 0)	0.6048	(0, 1)	0.5152
(0.3, 0.6, 0.5)	(1, 0)	0.6752	(0, 1)	0.5856
(0.5, 0.5, 0.5)	(1, 0)	0.6720	(0, 1)	0.6400
(0.5, 0.4, 0.7)	(1, 0)	0.7584	(0, 1)	0.6496
(0.55, 0.4, 0.7)	(1, 0)	0.7664	(0, 1)	0.6720
(0.555, 0.4, 0.7)	(1, 0)	0.7672	(0, 1)	0.6742
(0.5555, 0.4, 0.7)	(1, 0)	0.7672	(0, 1)	0.6744
(0.6, 0.5, 0.5)	(1, 0)	0.6880	(0, 1)	0.6848
(0.6, 0.6, 0.6)	(1, 0)	0.7232	(0, 1)	0.7200
(0.6, 0.6, 0.7)	(1, 0)	0.8448	(0, 1)	0.7648
(0.7, 0.6, 0.7)	(1, 0)	0.8608	(0, 1)	0.8096
(0.7, 0.7, 0.7)	(1, 0)	0.8960	(0, 1)	0.8448
(0.8, 0.7, 0.7)	(1, 0)	0.9120	(0, 1)	0.8896
(0.8, 0.8, 0.7)	(1, 0)	0.9472	(0, 1)	0.9248
(0.8, 0.8, 0.8)	(1, 0)	1.0080	(0, 1)	0.9472
(0.9, 0.8, 0.8)	(1, 0)	1.0240	(0, 1)	0.9920
(0.9, 0.9, 0.9)	(1, 0)	1.1200	(0, 1)	1.0496
(1.0, 0.9, 0.9)	(1, 0)	1.1360	(0, 1)	1.0944
(1.0, 1.0, 0.9)	(1, 0)	1.1712	(0, 1)	1.1296
(1.0, 1.0, 0.95)	(1, 0)	1.2016	(0, 1)	1.1408
(1.0, 1.0, 1.0)	(1, 0)	1.2320	(0, 1)	1.1520

From Table 7.1, we get the optimal solutions as: $(y^*, z^*, R_{w_1,w_2,w_3}(u^*), R_{w_1,w_2,w_3}(v^*))$, where $y^* = (1,\ 0)$ and $z^* = (0,\ 1)$; $R_{w_1,w_2,w_3}(u^*) = 1.2320$ and $R_{w_1,w_2,w_3}(v^*) = 1.1520$. The increment in w_1 from 0.5 to 0.5555 has a prominent value-increase in $R_{w_1,w_2,w_3}(u^*)$ and $R_{w_1,w_2,w_3}(v^*)$, shown fourth row to seventh row; similar cases arise for w_2 and w_3 in eleventh-twelfth rows and in fourteenth-fifteenth rows, respectively. These can be generalized as the increment of game-value due to increment of weights allowed.

7.7 Conclusion

In this chapter, we have proposed neutrosophic environment to solve bi-matrix game. For this purpose, we have considered neutrosophic characteristics, i.e., degree of acceptance, degree of rejection and degree of indeterminacy to judge the object's behaviour. In bi-matrix game model, we have used the proposed de-neutrosophic approach as a ranking approach. Though the elements, used in the problem, are neutrosophic in nature, the obtained result of the value of bi-matrix game is expressed in percentage form. This is an important contribution of this study. Also, if weights are assigned properly with their strategies in neutrosophic environment towards optimality, it can be easily interpreted that increase of weights imply better strategies and better game value. The development of such mathematical models used to stimulate mobile selling is a growing area in mobile marketing.

Applications in a variety of areas, for example, energy, environment, risk management, reliability, logistics, supply chain management, transportation, location, health-care, etc. may be done by building decision strategies against the related constraints using neutrosophic sets, logic and

game theory as further research works.
Table 7.2 surveys a comparative study of our proposed work with some existing literature. This

Table 7.2: Comparative study with other literatures.

Literature	**Game type**	**Domain representation**	**Computational methods**	**Equilibrium solution environment**	**Real problems oriented**
[34]	Matrix	Neutrosophic	Secondary data analysis	Real numbers	Yes
[101]	Bimatrix	Bi-rough	Birough programming approach	Stochastic data	Yes
[118]	Bimatrix	Rough	Rough set approach	Real numbers	Yes
[119]	Bimatrix	Bi-fuzzy	Graphical methods	N/A	Yes
Our proposed work	Bimatrix	Triangular neutrosophic	Ranking on neutroshic and Game theory approach	intuitionistic interval numbers	Yes

study significantly shows that discussion on bimatrix game theory under neutrosophic environment have a significant effect in real-life problems.