Chapter 6

Two-person non-zero-sum game in hesitant fuzzy-linguistic term set *

A game is a real-life situation involving a set of players, may be two or more. When the situation is conceived or concluded as if one gains other looses and gain of one equals loss of other, the game is called zero-sum game. But this situation does not come to happen always in reality, outcome of the game is not linguistically zero. This type of game is called *non-zero-sum game*. In this chapter, Prisoners' Dilemma game, a non-zero-sum game, is discussed through hesitant interval-valued intuitionistic fuzzy-linguistic term set based environment, where linguistic terms in interval are expressed by linguistic semantics first, and then corresponding indices are used. Finally, Nash equilibrium is derived, and the achieved results establish a close contact with reality using TOPSIS and Dominance property of matrix game theory with an example.

6.1 Motivation

This chapter is motivated by Prisoners' Dilemma (PD) game of non-zero-sum game theory. This chapter can be seen as an extension of PD game in uncertain environment with an example taken from daily newspaper's headings- 'human trafficking'. We discuss here the situation when suspects are caught, their interrogation are done, and their terms under custody through two-person non-zero-sum game phenomena.

6.2 Introduction

Sometimes two-person zero-sum games are not able to describe the problematic situation and that's why two-person non-zero-sum game phenomena are assumed to depict the problems. Extensive works, both in certain and uncertain environments, have been carried out on two-person non-zero-sum games out of which Prisoners' Dilemma, from various aspects and in many fields starting from mathematical sciences to biological sciences, plays an important role as cited examples of this game. Here we have adopted the TOPSIS method and linguistic variables to solve

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the Prisoners' Dilemma. Human trafficking has been discussed also in several works from mathematical point of view. But in literature, no work on human trafficking has so far been done using two-person non-zero-sum game. In this respect, this chapter might be considered as a novel presentation. Here, we combine four types of uncertain environments, e.g., hesitant, linguistic fuzzy, interval-valued, and intuitionistic environments.

6.3 Basic Concepts

This section of our work is presented and evaluated with the properties of intuitionistic fuzzy set, hesitant fuzzy set, linguistic term set, hesitant fuzzy-linguistic term set and hesitant intuitionistic fuzzy-linguistic term set (HIFLTS).

6.3.1 IFS, HFS, LTS, HFLTS, HIFLTS

Here, we propose the hesitant interval-valued intuitionistic fuzzy-linguistic term set (HIVIFLTS) and due to this fact, this section reviews the main concepts necessarily related to basic definitions and operations of IFS, HFS, LTS, HFLTS, HIFLTS. From **Chapter 2**, we recall the definition of IFS, IFN. Based on the **Definition 2.3.1** of IFS, we consider some properties on IFS as below:

Property 6.3.1 *[77] Assume A_{IF} and B_{IF} be two IFSs over the universe X with an arbitrary real number $\lambda > 0$; then the following properties hold:*

(i) $A_{IF} \subseteq B_{IF}$ *if and only if* $\mu_{A_{IF}}(x) \leq \mu_{B_{IF}}(x)$ *and* $\nu_{A_{IF}}(x) \geq \nu_{B_{IF}}(x)$*, for every* $x \in X$*;*

(ii) $A_{IF} = B_{IF}$ *if and only if* $\mu_{A_{IF}}(x) = \mu_{B_{IF}}(x)$ *and* $\nu_{A_{IF}}(x) = \nu_{B_{IF}}(x)$*, for every* $x \in X$*;*

(iii) $A_{IF}^c = \{\langle x, \nu_{A_{IF}}(x), \mu_{A_{IF}}(x)\rangle : x \in X\}$*;*

(iv) $A_{IF} \cup B_{IF} = \{\langle x, \mu_{A_{IF}}(x) \vee \mu_{B_{IF}}(x), \nu_{A_{IF}}(x) \wedge \nu_{B_{IF}}(x)\rangle : x \in X\}$*;*

(v) $A_{IF} \cap B_{IF} = \{\langle x, \mu_{A_{IF}}(x) \wedge \mu_{B_{IF}}(x), \nu_{A_{IF}}(x) \vee \nu_{B_{IF}}(x)\rangle : x \in X\}$*;*

(vi) $A_{IF} \oplus B_{IF} = \{\langle x, \mu_{A_{IF}}(x) + \mu_{B_{IF}}(x) - \mu_{A_{IF}}(x)\mu_{B_{IF}}(x), \nu_{A_{IF}}(x)\nu_{B_{IF}}(x)\rangle : x \in X\}$*;*

(vii) $A_{IF} \otimes B_{IF} = \{\langle x, \mu_{A_{IF}}(x)\mu_{B_{IF}}(x), \nu_{A_{IF}}(x) + \nu_{B_{IF}}(x) - \nu_{A_{IF}}(x)\nu_{B_{IF}}(x)\rangle : x \in X\}$*;*

(viii) $\lambda A_{IF} = \{\langle x, 1 - (1 - \mu_{A_{IF}}(x))^\lambda, (\nu_{A_{IF}}(x))^\lambda\rangle : x \in X\}$*;*

(ix) $A_{IF}^\lambda = \{\langle x, (\mu_{A_{IF}}(x))^\lambda, 1 - (1 - \nu_{A_{IF}}(x))^\lambda\rangle : x \in X\}$.

Here, "$\wedge$" and "$\vee$" represent, respectively, the minimum and the maximum operator; "$\oplus$" and "$\otimes$" denote the set sum and multiplication operators, respectively.

Property 6.3.2 *[77] The distance function $d : F(X) \times F(X) \rightarrow [0, 1]$, with $F(X)$ as a subset of the field of real numbers, if applied to A_{IF}, B_{IF} and C_{IF}, assures the following properties:*

(i) $0 \leq d(A_{IF}, B_{IF}) \leq 1$*;*

(ii) $d(A_{IF}, B_{IF}) = 0$ *if and only if* $A_{IF} = B_{IF}$*;*

(iii) $d(A_{IF}, B_{IF}) = d(B_{IF}, A_{IF})$*;*

(iv) $d(A_{IF}, B_{IF}) \leq d(A_{IF}, C_{IF}) + d(C_{IF}, B_{IF})$*.*

This function d is termed as ***normalized distance*** *between* A_{IF} *and* B_{IF}*.*

Now, we remember the definitions and properties of HFS from **Chapter 5 (Definition 5.3.4, Property 5.3.1)** with the definitions and properties from **Chapter 4 (Definition 4.3.9, Property 4.3.1, Property 4.3.2)**.
Human judgements and perception always flow in hesitant environment and practically, these environments are nurtured with linguistic characters of responses, understood in fuzziness sense. Subsequently we demonstrate the hesitant fuzzy set in linguistic term, first defined by Rodriguez [124].

Definition 6.3.1 ***Hesitant Fuzzy-Linguistic Term Set (HFLTS)****: Let* $S = \{s_i : i = 1, 2, \ldots, t\}$ *be a LTS. An HFLTS* H_S *is defined as an ordered finite subset of successive linguistic terms of the set* S*. Null HFLTS and full HFLTS are defined on the linguistic variable* z*, respectively, as: Null HFLTS,* $H_{S_{null}}(z) = \{\}$*; and Full HFLTS,* $H_{S_{full}}(z) = S$*.*

Example 6.3.1 *Let* $S_2 = \{s_0 =$ *very poor,* $s_1 =$ *poor,* $s_2 =$ *slightly poor,* $s_3 =$ *fair,* $s_4 =$ *slightly good,* $s_5 =$ *good,* $s_6 =$ *very good*$\}$ *be a LTS. Different HFLTSs can be depicted as:* $H^1_{S_2}(z) = \{s_0, s_1, s_2, s_3\}$*,* $H^2_{S_2}(z) = \{s_3, s_4\}$*,* $H^3_{S_2}(z) = \{s_4, s_5, s_6\}$*.*

Property 6.3.3 *Let* $S = \{s_i : i = 1, 2, \ldots, t\}$ *be a LTS;* H^1_S *and* H^2_S *be two HFLTS; a distance measure between* H^1_S *and* H^2_S*, denoted by* $d(H^1_S, H^2_S)$*, satisfies the following properties:*

(i) $0 \leq d(H^1_S, H^2_S) \leq 1$*;*

(ii) $d(H^1_S, H^2_S) = 0$ *if and only if* $H^1_S = H^2_S$*;*

(iii) $d(H^1_S, H^2_S) = d(H^2_S, H^1_S)$*.*

HFLTS possesses only membership degrees to the elements. When intuitionistic behaviour is included, this HFLTS is described as Hesitant intuitionistic fuzzy-linguistic term set (HIFLTS).

Definition 6.3.2 ***Hesitant Intuitionistic Fuzzy-Linguistic Term Set (HIFLTS)*** *[11]: An HIFLTS,* A_{HILFTS}*, on a set of values X, is defined as:* $A_{HIFLTS} = \{x, h_{A_{HIFLTS_\mu}}(x), h_{A_{HIFLTS_\nu}}(x)\}$*. Here,* $h_{A_{HIFLTS_\mu}}(x), h_{A_{HIFLTS_\nu}}(x)$ *give, respectively, the fuzzy membership and non-membership degrees of the variable* x *with hesitant characters taken the values from the linguistic term set* $S = \{s_i : i = 1, 2, \ldots, t\}$*. Furthermore,* $h_{A_{HIFLTS_\mu}}(x), h_{A_{HIFLTS_\nu}}(x) \in S$ *and* $\max(h_{A_{HIFLTS_\mu}}(x)) + \min(h_{A_{HIFLTS_\nu}}(x)) \leq s_t$ *with* $\min(h_{A_{HIFLTS_\mu}}(x)) + \max(h_{A_{HIFLTS_\nu}}(x)) \leq s_t$*.*
Here, $(h_{A_{HIFLTS_\mu}}(x), h_{A_{HIFLTS_\nu}}(x))$ *is called a hesitant intuitionistic fuzzy element of the linguistic term set.*

Example 6.3.2 *Let $S_2 = \{s_0 =$ very poor, $s_1 =$ poor, $s_2 =$ slightly poor, $s_3 =$ fair, $s_4 =$ slightly good, $s_5 =$ good, $s_6 =$ very good$\}$ be a linguistic term set. Different HIFLTSs can be depicted as, $A = \{(x_1, \langle(s_1, s_2, s_3); (s_3, s_5)\rangle), (x_2, \langle(s_4, s_5); (s_2, s_3)\rangle)\}$, and $B = \{(x_1, \langle(s_0, s_1, s_3); (s_3, s_4)\rangle), (x_2, \langle(s_2, s_4); (s_5, s_6)\rangle)\}$.*

6.3.2 HIVIFLTS

In this subsection, HIVIFLTS and the corresponding properties are discussed briefly.

Definition 6.3.3 ***Hesitant Interval Valued Intuitionistic Fuzzy Linguistic Term Set (HIVIFLTS)****: An HIVIFLTS $\breve{A}$ on X are functions $h_{\breve{A}_{HIVIFLTS_\mu}}$ and $h_{\breve{A}_{HIVIFLTS_\nu}}$ so that when applied to X, they return ordered finite subsets in interval-intuitionistic form of the consecutive linguistic terms of $S = \{s_i : i = 1, 2, \ldots, t\}$; this can be represented as $\breve{A} = \{\langle x, h_{\breve{A}_{HIVIFLTS_\mu}}(x), h_{\breve{A}_{HIVIFLTS_\nu}}(x)\rangle : x \in X\}$, where $h_{\breve{A}_{HIVIFLTS_\mu}}(x) = \cup_i[h^l{}_{\breve{A}_{HIVIFLTS_\mu}}(x), h^u{}_{\breve{A}_{HIVIFLTS_\mu}}(x)] \subseteq [s_1, s_t]$ represents the membership characters and consequently, the non-membership characters are described by $h_{\breve{A}_{HIVIFLTS_\nu}}(x) = \cup_i[h^l{}_{\breve{A}_{HIVIFLTS_\nu}}(x), h^u{}_{\breve{A}_{HIVIFLTS_\nu}}(x)] \subseteq [s_1, s_t]$, for $i = 1, 2, \ldots, n$ and we have $h^u{}_{\breve{A}_{HIVIFLTS_\mu}}(x) + h^u{}_{\breve{A}_{HIVIFLTS_\nu}}(x) \leq s_t$.*
For convenience, $\langle h_{\breve{A}_{HIVIFLTS_\mu}}(x), h_{\breve{A}_{HIVIFLTS_\nu}}(x)\rangle$ is termed as hesitant interval-valued intuitionistic fuzzy-linguistic term element.

Example 6.3.3 *Let $S_2 = \{s_0 =$ very poor, $s_1 =$ poor, $s_2 =$ slightly poor, $s_3 =$ fair, $s_4 =$ slightly good, $s_5 =$ good, $s_6 =$ very good$\}$ be a linguistic term set. Different HIVIFLTSs can be depicted as:*

$$\begin{aligned}
\breve{A} &= \Big\{(x_1, \{\langle[s_2, s_3], [s_1, s_2]\rangle, \langle[s_1, s_5], [s_0, s_1]\rangle, \langle[s_2, s_3], [s_0, s_2]\rangle\}), \\
&\quad (x_2, \{\langle[s_0, s_2], [s_2, s_4]\rangle, \langle[s_2, s_5], [s_0, s_1]\rangle, \langle[s_2, s_3], [s_0, s_3]\rangle\})\Big\}, \\
\breve{B} &= \Big\{(x_1, \{\langle[s_2, s_4], [s_0, s_2]\rangle, \langle[s_0, s_2], [s_3, s_4]\rangle, \langle[s_2, s_3], [s_0, s_1]\rangle\}), \\
&\quad (x_2, \{\langle[s_0, s_1], [s_2, s_4]\rangle, \langle[s_3, s_5], [s_0, s_1]\rangle, \langle[s_0, s_3], [s_0, s_3]\rangle\})\Big\}.
\end{aligned}$$

Motivated by the ideas of **Property 5.3.1 (Chapter 5)** and **Property 6.3.1**, we define the following properties on HIVIFLTS:

Property 6.3.4 *Assume $\breve{A} = \{\langle x, h_{\breve{A}_\mu}(x), h_{\breve{A}_\nu}(x)\rangle : x \in X\}$ having membership and non membership functions $h_{\breve{A}_\mu}(x) = \cup[h^l_{\breve{A}_\mu}(x), h^u_{\breve{A}_\mu}(x)]$ and $h_{\breve{A}_\nu}(x) = \cup[h^l_{\breve{A}_\nu}(x), h^u_{\breve{A}_\nu}(x)]$, respectively and $\breve{B} = \{\langle x, h_{\breve{B}_\mu}(x), h_{\breve{B}_\nu}(x)\rangle : x \in X\}$ with membership and non-membership functions $h_{\breve{B}_\mu}(x) = \cup[h^l_{\breve{B}_\mu}(x), h^u_{\breve{B}_\mu}(x)]$ and $h_{\breve{B}_\nu}(x) = \cup[h^l_{\breve{B}_\nu}(x), h^u_{\breve{B}_\nu}(x)]$ respectively, be two HIVIFLTS on X; i.e., the membership functions and the non-membership functions when applied to X return ordered finite subsets in interval form with intuitionistic nature of the consecutive linguistic terms of $S = \{s_i : i = 1, 2, \ldots, t\}$. Then,*

(i) Complement of $\breve{A}$, $\breve{A}^c = \{\langle x, h_{\breve{A}_\nu}(x), h_{\breve{A}_\mu}(x)\rangle : x \in X\}$;

(ii) $\breve{A} \cup \breve{B} = \{\langle x, (h_{\breve{A}_\mu}(x) \cup h_{\breve{B}_\mu}(x)), (h_{\breve{A}_\nu}(x) \cap h_{\breve{B}_\nu}(x))\rangle : x \in X\}$;

(iii) $\breve{A} \cap \breve{B} = \{\langle x, (h_{\breve{A}_\mu}(x) \cap h_{\breve{B}_\mu}(x)), (h_{\breve{A}_\nu}(x) \cup h_{\breve{B}_\nu}(x))\rangle : x \in X\}$.

6.4 Mathematical Model

In this section, mathematical models of two-person non-zero-sum game is discussed. This section provides mathematical calculations on games in HIVIFLTS based environment through TOPSIS.

6.4.1 Two-person non-zero-sum game in HIVIFLTS based environment

In a non-zero-sum game, each player is possessed with his/her own payoff matrix, and describes his/her corresponding payoffs. Let us assume that the players I and II hold the set of pure strategies S_1 and S_2 respectively and also are allowed to adopt for mixed strategies Y and Z if they are constrained to choose of their own. Here, S_1, S_2, Y, Z are defined in **Chapter 2 (Section 2.4.2)**. Then payoff matrices for players I and II are described, respectively, as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1q} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2q} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & a_{p3} & \dots & a_{pq} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1q} \\ b_{21} & b_{22} & b_{23} & \dots & b_{2q} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{p1} & b_{p2} & b_{p3} & \dots & b_{pq} \end{bmatrix}.$$

Thus, a finite two-person non-zero-sum game in matrix form, $(Y, Z; A, B)$, is sometimes called a *bi-matrix game*.
Nash equilibrium, in a non-cooperative game played by two or more players, provides a solution concept which explains that every player is believed to know the equilibrium strategies of others, and changing of strategy/strategies only will not ensure anybody's gain. In such cases, when one chooses a strategy and nobody is benefited by changing strategies, others keep their strategies unchanged. The current choices of strategy-set with the corresponding payoffs frame a *Nash equilibrium*.

Definition 6.4.1 ***Nash-equilibrium solution***: *A bi-matrix game (Y, Z, A, B) possesses a Nash equilibrium solution (y^*, z^*) when*

$$y^{*T} A z^* \geq y^T A z^*, \forall y \in Y, \text{ for player I,} \tag{6.1}$$

$$y^{*T} B z^* \geq y^{*T} B z, \forall z \in Z, \text{ for player II,} \tag{6.2}$$

where y is a $p \times 1$-matrix of mixed strategies $(y \geq 0)$, z is a $q \times 1$-matrix of mixed strategies $(z \geq 0)$, T is used for transposition of a vector of matrix and without loss of generality, both player I and player II wish to maximize their own payoffs.

Definition 6.4.2 ***Expected Pay-off of players***: *Considering the mixed strategies by players I and II, the expected payoff of players I and II are, respectively, $y^T A z^*$ and $y^{*T} B z$. Therefore, two person bi-matrix game with mixed strategies are defined by the following quadratic programming* *problems:*

$$\begin{aligned} &\textit{maximize} && y^T A z^* \\ &\textit{subject to} && y \in Y, \qquad (6.3) \\ \textit{and}\ &\textit{maximize} && {y^*}^T B z \\ &\textit{subject to} && z \in Z. \qquad (6.4) \end{aligned}$$

Eq.(6.3) can be regarded as maximization of expected payoff of player I and Eq.(6.4) can be treated as maximizing the expected payoff of player II. The optimal (y^*, z^*) can be attained by simultaneous solution of Eqs.(6.3-6.4).

6.4.2 TOPSIS in classical environment

TOPSIS method is a simple and effective method to address multi-criteria decision making problems consisting of m alternatives $A = \{A_1, A_2, \ldots, A_m\}$, n criteria $C = \{C_1, C_2, \ldots, C_n\}$. The weighting vector of criteria is denoted by $w = (w_1, w_2, \ldots, w_n)^T$ where w_j is the weight of the criterion C_j, satisfying $\sum_{j=1}^{n} w_j = 1$ and $w_j \geq 0$ $(j \in \{1, 2, \ldots, n\})$. Here, x_{ij} be the criteria value of alternative A_i with respect to criterion C_j and all x_{ij} are presented in a matrix form, called the decision matrix, and comprised by $(x_{ij})_{m\times n}$. TOPSIS is defined by the following steps [37; 57]:

Step 1. Normalization of the decision matrix by $\bar{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{\xi=1}^{m}(x_{\xi j})^2}}$ where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n)$.

Step 2. Determination of the PIS A^+ and the NIS A^- as:
$A^+ = (\bar{x}_1^+, \bar{x}_2^+, \ldots, \bar{x}_n^+) = \big((\max_i \bar{x}_{ij} : C_j \in J_1)\ \textit{or}\ (\min_i \bar{x}_{ij} : C_j \in J_2)\big)$
and $A^- = (\bar{x}_1^-, \bar{x}_2^-, \ldots, \bar{x}_n^-) = \big((\min_i \bar{x}_{ij} : C_j \in J_1)\ \textit{or}\ (\max_i \bar{x}_{ij} : C_j \in J_2)\big)$. Here, J_1 denotes a subset of benefit criteria, J_2 is that of cost criteria, and $J_1 \cup J_2 = C$, $J_1 \cap J_2 = \phi$, the empty set.

Step 3. Calculation of the distances between the potential alternative and the PIS as well as the NIS, respectively,

$$d(A_i, A^+) = \sqrt{\textstyle\sum_{j=1}^{n} \omega_j (\bar{x}_{ij} - \bar{x}_j^+)^2} \text{ and } d(A_i, A^-) = \sqrt{\textstyle\sum_{j=1}^{n} \omega_j (\bar{x}_{ij} - \bar{x}_j^-)^2}.$$

Step 4. Computation of the relative Closeness Index (CI) of each alternative to the PIS:
$CI(A_i) = \frac{d(A_i, A^-)}{d(A_i, A^+) + d(A_i, A^-)}$. It is easily observed that $CI(A_i) \in [0, 1]$.

Step 5. Ranking of the alternatives according to the CI of alternatives: the bigger $CI(A_i)$, the better the alternative A_i.

6.4.3 TOPSIS in HIVIFLTS environment

In this section, we introduce first a hesitant interval-valued intuitionistic fuzzy-linguistic term oriented decision matrix, as $\mathfrak{R}$, defined as: $\mathfrak{R} = \Big(\bigcup_{i=1}^{m} (\bigcup_{j=1}^{n} \langle [{}^{\mu}h^l_{s_{ij}}, {}^{\mu}h^u_{s_{ij}}], [{}^{\nu}h^l_{s_{ij}}, {}^{\nu}h^u_{s_{ij}}] \rangle)\Big)_{m\times n}$.

Here, $i\ (=1,2,\ldots,m)$ denotes the number of alternatives, $j\ (=1,2,\ldots,n)$ chooses the number of criteria; $Q=\{Q_1,Q_2,\ldots,Q_n\}$ indicates the set of criteria, $P=\{P_1,P_2,\ldots,P_m\}$ is the set of alternatives; $w=(w_1,w_2,\ldots,w_n)$ is the set of weight vectors where, each w_j is related with each Q_j. Here, $w_j\in[0,1],\ j=1,2,\ldots,n$, and $\sum_{j=1}^{n}w_j=1$. TOPSIS is defined by the following steps:

Step 1. Construction of HIVIFLTS decision matrix

In this step, HIVIFLTS decision matrix, $\mathfrak{R}$ is constructed as, $\mathfrak{R}=\left(\bigcup_{i=1}^{m}(\bigcup_{j=1}^{n}\langle[{}^{\mu}h_{s_{ij}}^{l},{}^{\mu}h_{s_{ij}}^{u}],[{}^{\nu}h_{s_{ij}}^{l},{}^{\nu}h_{s_{ij}}^{u}]\rangle)\right)_{m\times n}$.
Here, $\langle[{}^{\mu}h_{s_{ij}}^{l},{}^{\mu}h_{s_{ij}}^{u}],[{}^{\nu}h_{s_{ij}}^{l},{}^{\nu}h_{s_{ij}}^{u}]\rangle$ represents the hesitant interval-valued intuitionistic fuzzy-linguistic term set based numbers, $[{}^{\mu}h_{s_{ij}}^{l},{}^{\mu}h_{s_{ij}}^{u}]$ represents the membership interval and $[{}^{\nu}h_{s_{ij}}^{l},{}^{\nu}h_{s_{ij}}^{u}]$ presents the non-membership interval of the alternative P_m satisfying the criterion Q_n. This step includes the definition of the universe of discourse for every alternative against each criterion by considering their linguistic values with hesitance character towards the choice of intervals and construction of the appropriate linguistic fuzzy sets for each criterion. HIVIFLTS decision matrix can be presented as follows:

$$\mathfrak{R}=\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n}\\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,n}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ a_{m,1} & a_{m,2} & a_{m,3} & \cdots & a_{m,n}\end{bmatrix},$$

where $a_{m,n}=\bigcup_{i=1}^{m}(\bigcup_{j=1}^{n}(\langle[{}^{\mu}h_{s_{ij}}^{l},{}^{\mu}h_{s_{ij}}^{u}],[{}^{\nu}h_{s_{ij}}^{l},{}^{\nu}h_{s_{ij}}^{u}]\rangle))/P_z$ indicates the position of the element in the HIVIFLTS-based matrix with P_z as z different alternatives. For example, we assume $\mathfrak{R}_1$ as

$$\begin{bmatrix} (a_{1,1}/P_1,a_{1,1}/P_2,a_{1,1}/P_3) & (a_{1,2}/P_1,a_{1,2}/P_2,a_{1,2}/P_3) & \cdots & (a_{1,n}/P_1,a_{1,n}/P_2,a_{1,n}/P_3)\\ (a_{2,1}/P_1,a_{2,1}/P_2,a_{2,1}/P_3) & (a_{2,2}/P_1,a_{2,2}/P_2,a_{2,2}/P_3) & \cdots & (a_{2,n}/P_1,a_{2,n}/P_2,a_{2,n}/P_3)\\ \vdots & \vdots & \ddots & \vdots\\ (a_{m,1}/P_1,a_{m,1}/P_2,a_{m,1}/P_3) & (a_{m,2}/P_1,a_{m,2}/P_2,a_{m,2}/P_3) & \cdots & (a_{m,n}/P_1,a_{m,n}/P_2,a_{m,n}/P_3)\end{bmatrix}.$$

Step 2. Construction of modified decision matrix

Here, using the enveloping of the intervals over the criteria, $j\ (=1,2,\ldots,n)$, we get the modified decision matrix as

$$\begin{aligned}(a_{p,q}) &= (\bigcup_{i=1}^{m}(\langle[\min_{j=1}^{n}({}^{\mu}h_{s_{ij}}^{l}),\max_{j=1}^{n}({}^{\mu}h_{s_{ij}}^{u})],[\min_{j=1}^{n}({}^{\nu}h_{s_{ij}}^{l}),\\ &\quad \max_{j=1}^{n}({}^{\nu}h_{s_{ij}}^{u})]\rangle))/P_z)_{m\times n}\\ &= (\bigcup_{i=1}^{m}(\bigcup_{j=1}^{n}(\langle[{}^{\mu}\hat{h}_{s_{ij}}^{l},{}^{\mu}\hat{h}_{s_{ij}}^{u}],[{}^{\nu}\hat{h}_{s_{ij}}^{l},{}^{\nu}\hat{h}_{s_{ij}}^{u}]\rangle))/P_z)_{m\times n}.\end{aligned}$$

Step 3. Construction of weighted HIVIFLTS decision matrix

Here, we consider the decision matrix with the weights related to criteria by w_j as $\bar{\mathfrak{R}}$, which is defined as

$$\begin{aligned}\bar{\mathfrak{R}} &= \Big(\bigcup_{i=1}^{m}\Big(\bigcup_{j=1}^{n} w_j(\langle[{}^{\mu}\hat{h}^l_{s_{ij}}, {}^{\mu}\hat{h}^u_{s_{ij}}], [{}^{\nu}\hat{h}^l_{s_{ij}}, {}^{\nu}\hat{h}^u_{s_{ij}}]\rangle)\Big)/P_z\Big)_{m\times n} \\ &= \Big(\bigcup_{i=1}^{m}\Big(\bigcup_{j=1}^{n}(\langle[h_{s_t} - (h_{s_t} -{}^{\mu}\hat{h}^l_{s_{ij}})^{w_j}, h_{s_t} - (h_{s_t} -{}^{\mu}\hat{h}^u_{s_{ij}})^{w_j}], \\ &\quad [({}^{\nu}\hat{h}^l_{s_{ij}})^{w_j}, ({}^{\nu}\hat{h}^u_{s_{ij}})^{w_j}]\rangle)\Big)/P_z\Big)_{m\times n}.\end{aligned}$$

Here, $h_{s_t} = 1$, if hesitant interval-valued intuitionistic fuzzy number is considered, and $h_{s_t} = s_t$, if hesitant interval-valued intuitionistic fuzzy-linguistic semantics are used. If criteria are equally weighted, then we can eliminate weights from the weight vector.

Step 4. Aggregated-weighted HIVIFLTS decision matrix

We aggregate the weighted decision matrix $\bar{\mathfrak{R}}$ as $\bar{\bar{\mathfrak{R}}}$ considering the decision makers' opinion, and then the aggregated weighted decision matrix is: $\bar{\bar{\mathfrak{R}}} = (\langle[s^l_{\mu_{ij}}, s^u_{\mu_{ij}}], [s^l_{\nu_{ij}}, s^u_{\nu_{ij}}]\rangle)_{m\times n}$ where, $s^l_{\mu_{ij}} = \min_{i=1}^{m}\{h_{s_t} - (h_{s_t} -{}^{\mu}\hat{h}^l_{s_{ij}})^{w_j}\}$, $s^u_{\mu_{ij}} = \max_{i=1}^{m}\{h_{s_t} - (h_{s_t} -{}^{\mu}\hat{h}^u_{s_{ij}})^{w_j}\}$, $s^l_{\nu_{ij}} = \min_{i=1}^{m}\{({}^{\nu}\hat{h}^l_{s_{ij}})^{w_j}\}$, and $s^u_{\nu_{ij}} = \max_{i=1}^{m}\{({}^{\nu}\hat{h}^u_{s_{ij}})^{w_j}\}$. Here, $s^l_{\mu_{ij}} + s^u_{\nu_{ij}} \leq s_t,\ s^u_{\mu_{ij}} + s^l_{\nu_{ij}} \leq s_t,\ ({}^{\mu}\hat{h}^l_{s_{ij}}) + ({}^{\nu}\hat{h}^u_{s_{ij}}) \leq s_t,\ ({}^{\mu}\hat{h}^u_{s_{ij}}) + ({}^{\nu}\hat{h}^l_{s_{ij}}) \leq s_t$, respectively.

Step 5. Construction of HIVIFLT positive ideal solution (HIVIFLT-PIS) and negative ideal solution (HIVIFLT-NIS)

In TOPSIS method, the evaluation criteria may be chosen along two categories, namely, benefit criteria and cost criteria. Let J_1 be a collection of benefit criteria and J_2 be a collection of cost criteria. According to the principle of TOPSIS method, HIVIFLT-PIS and HIVIFLT-NIS, denoted by $\mathfrak{R}^+$ and $\mathfrak{R}^-$, are defined as:

$$\begin{aligned}\bar{\bar{\mathfrak{R}}}^+ = &\Big\langle[((\max(s^l_{\mu_{ij}})|C_j \in J_1), or, (\min(s^l_{\mu_{ij}})|C_j \in J_2)), \\ &((\max(s^u_{\mu_{ij}})|C_j \in J_1), or, (\min(s^u_{\mu_{ij}})|C_j \in J_2))], \\ &[((\min(s^l_{\nu_{ij}})|C_j \in J_1), or, (\max(s^l_{\nu_{ij}})|C_j \in J_2)), \\ &((\min(s^u_{\nu_{ij}})|C_j \in J_1), or, (\max(s^u_{\nu_{ij}})|C_j \in J_2))]\Big\rangle \\ = &\Big((\langle[s^{\overset{+}{l}}_{\mu_{ij}}, s^{\overset{+}{u}}_{\mu_{ij}}], [s^{\overset{+}{l}}_{\nu_{ij}}, s^{\overset{+}{u}}_{\nu_{ij}}]\rangle)_m\Big)^T, \\ \bar{\bar{\mathfrak{R}}}^- = &\Big\langle[((\min(s^l_{\mu_{ij}})|C_j \in J_1), or, (\max(s^l_{\mu_{ij}})|C_j \in J_2)), \\ &((\min(s^u_{\mu_{ij}})|C_j \in J_1), or, (\max(s^u_{\mu_{ij}})|C_j \in J_2))], \\ &[((\max(s^l_{\nu_{ij}})|C_j \in J_1), or, (\min(s^l_{\nu_{ij}})|C_j \in J_2)), \\ &((\max(s^u_{\nu_{ij}})|C_j \in J_1), or, (\min(s^u_{\nu_{ij}})|C_j \in J_2))]\Big\rangle \\ = &\Big((\langle[s^{\overset{-}{l}}_{\mu_{ij}}, s^{\overset{-}{u}}_{\mu_{ij}}], [s^{\overset{-}{l}}_{\nu_{ij}}, s^{\overset{-}{u}}_{\nu_{ij}}]\rangle)_m\Big)^T.\end{aligned}$$

Here, J_1 denotes a subset of benefit criteria, J_2 is that of cost criteria, $J_1 \cup J_2 = C$, $J_1 \cap J_2 = \phi$, the empty set.

Step 6. Calculation of distance measures from HIVIFLT-PIS and HIVIFLT-NIS.

Motivating by the Euclidean 3-dimensional distance between intuitionistic fuzzy sets $\hat{A}$

and $\hat{B}$, we have,

$$d(\hat{A},\hat{B}) = \left\{\frac{1}{2}\sum_i[(\mu_{\hat{A}}(x_i)-\mu_{\hat{B}}(x_i))^2+(\nu_{\hat{A}}(x_i)-\nu_{\hat{B}}(x_i))^2+(\tau_{\hat{A}}(x_i))-\tau_{\hat{B}}(x_i))^2]\right\}^{1/2},$$

where $\mu_{\hat{A}}(x_i)$, $\nu_{\hat{A}}(x_i)$ and $\tau_{\hat{A}}(x_i)$ denote membership, non-membership and hesitance degrees of x_i $(\in \hat{A})$, according to the definition of IFS, to measure distance of each alternative of $\bar{\bar{\mathfrak{R}}}$ (given in Step 4 of Section 6.4.3) from HIVIFLT-PIS and HIVIFLT-NIS, the distances are expressed as:

$$\begin{aligned} d(\bar{\bar{\mathfrak{R}}},\bar{\mathfrak{R}}^+) &= \left\{\frac{1}{2}\sum_i[(s^l_{\mu_{ij}}-\overset{+}{s^l_{\mu_{ij}}})^2+(s^u_{\mu_{ij}}-\overset{+}{s^u_{\mu_{ij}}})^2+(s^l_{\nu_{ij}}-\overset{+}{s^l_{\nu_{ij}}})^2+(s^u_{\nu_{ij}}-\overset{+}{s^u_{\nu_{ij}}})^2\right.\\ &\qquad \left.+(s^l_{\tau_{ij}}-\overset{+}{s^l_{\tau_{ij}}})^2+(s^u_{\tau_{ij}}-\overset{+}{s^u_{\tau_{ij}}})^2]\right\}^{1/2},\\ d(\bar{\bar{\mathfrak{R}}},\bar{\mathfrak{R}}^-) &= \left\{\frac{1}{2}\sum_i[(s^l_{\mu_{ij}}-\overset{-}{s^l_{\mu_{ij}}})^2+(s^u_{\mu_{ij}}-\overset{-}{s^u_{\mu_{ij}}})^2+(s^l_{\nu_{ij}}-\overset{-}{s^l_{\nu_{ij}}})^2+(s^u_{\nu_{ij}}-\overset{-}{s^u_{\nu_{ij}}})^2\right.\\ &\qquad \left.+(s^l_{\tau_{ij}}-\overset{-}{s^l_{\tau_{ij}}})^2+(s^u_{\tau_{ij}}-\overset{-}{s^u_{\tau_{ij}}})^2]\right\}^{1/2}. \end{aligned}$$

Step 7. Calculation of relative closeness coefficient (CC)

Finally, CC of each alternative with respect to intuitionistic fuzzy ideal solutions is computed by using the following expression and then ranking of the preference order of all alternatives is done:

$$CC = \frac{d(\bar{\bar{\mathfrak{R}}},\bar{\mathfrak{R}}^-)}{d(\bar{\bar{\mathfrak{R}}},\bar{\mathfrak{R}}^+)+d(\bar{\bar{\mathfrak{R}}},\bar{\mathfrak{R}}^-)}.$$

A larger value of relative closeness coefficient indicates that an alternative is closer to HIVIFLT-PIS and farther from HIVIFLT-NIS simultaneously. Therefore, the ranking order of all the alternatives can be determined according to the descending order of relative coefficient values. The most preferred alternative is the one with the highest value.

6.5 Prisoners' Dilemma

The Prisoners' Dilemma (PD) (in 1950, by Merrill Flood and Melvin Dresher at RAND) [44], by its name expresses the very difficult and conflicting characters of the prisoners when they are exposed to interrogation according to laws and orders. Suppose that two persons are taken to custody being caught in same guilt. Due to non-availability of strong evidences always, the law personnels are to take some tricks. Individual interrogation is made by keeping them separate, and each of them is asked to either confess (C) or not confess (NC) one's guilt. It is observed that the interrogation depends on the prisoners dilemmatically. Each one may choose whether to confess/cooperate with (C) or defect/not confess (NC). If both do not confess (NC, NC), the sentence of jail or custody is mitigated to $(\alpha - p)$ years (each one gets p years of freedom), where α denotes the maximum years of imprisonment, and p being a certain number of years. Obviously, $\alpha > p$. If both confess, i.e., (C, C), suspects are set free after p years because of absence of proof (each one gets $(\alpha - p)$ years of freedom). When only one cooperates, (C, NC) or (NC, C), the one who federates is released instantly (α years of freedom from custody), while the other is condemned to the maximal discipline of α years (i.e., 0 years of freedom). The game is illustrated in bi-matrix form, as displayed in the matrix $PD_{classical}$. In a purely noncooperative situation, every player will choose to non-confess (NC) considering that he has no assurance on the choices of the other and, therefore, the solution is (NC, NC). In an unexpected way, in a cooperative game where the two players can connive and arrange joint activities, almost certainly, both will wind up in confessing (C, C). Prisoners' Dilemma bi-matrix game in classical format can be pictured as:

$$PD_{classical} = \begin{matrix} C_{PrisonerI} \\ NC_{PrisonerI} \end{matrix} \overset{\begin{matrix} C_{PrisonerII} & NC_{PrisonerII} \end{matrix}}{\begin{pmatrix} (p,p) & (0,\alpha) \\ (\alpha,0) & (\alpha-p,\alpha-p) \end{pmatrix}}.$$

Here, dominant strategies are the best strategies, unconditionally. Therefore, the optimal strategy of prisoner I and prisoner II is to confess (C) but the punishment is worse. Though non-confess (NC) of both gives the best result according to their punishment, but there is no guarantee whether one confesses by the interrogators to get less punishment. Paradoxically, since each prisoner has a dominant strategy, it remains true that each prisoner is individually better off using it and confessing. This unique equilibrium outcome in our game is termed as non-Pareto-optimal or Pareto-deficient. Here, both prisoners prefer $(\alpha - p, \alpha - p)$ to (p, p). Eventually, the remaining three non-equilibria are all Pareto-optimal, i.e., each is preferred to any other outcome by at least one player. For instance, $(\alpha - p, \alpha - p)$ is preferred over $(0, \alpha)$ by prisoner II, over $(\alpha, 0)$ by prisoner I, and over (p, p) by both prisoners.

6.6 Numerical Simulation

We consider the Prisoners' Dilemma game problem in HIVIFLTS environment in the first subsection and solve the problem in the second subsection via TOPSIS method and dominance approach of game theory.

6.6.1 A case study on Human-trafficking

Human trafficking poses to be a serious problem all over the World. India is no exception in it [40; 46; 47; 116; 127]. It appears in different forms of exploitation such as recruitment, transfer, transportation, harbouring of persons, etc., and which come into play by means of threat, fraudness, force, coercion, transaction of money, abduction, forced labour and services, evacuation of organs, subjection and comparable practices and such other illegal and immoral activities.

We are concerned with the punishment to be given to the offenders whose number exceeds two and who are caught in suspect of trafficking. As most of the cases suffer from lack of strong evidences, this type of problem may be treated as Prisoner's Dilemma where suspects are interrogated separately by different agencies in terms of language. Convicts use their own strategies and respond in a hesitant manner, sometimes in some intervals to consider their own profit, i.e., to minimize their punishment. That's why this Prisoners' Dilemma is considered in hesitant interval-valued intuitionistic fuzzy-linguistic term set-based environment. In this section, provision of punishment for human-trafficking is considered in accordance with Indian Penal Code proposed to cover every substantive part of criminal law [115], where the provisions for punishment in different stages have been prescribed clearly under Section 370 and Section 370A.

In this context, two suspects, assumed as player I (PI) and player II (PII), caught for trafficking activities, are interrogated by different agencies and different weights are assigned to the punishment years. The punishment years vary from person to person and that from agency to agency. As for example, due to sufficient knowledge/information about Indian Penal Code and Constitution, one suspect may undergo detainments for minimum eight years which may be extended to life imprisonment of fourteen years along with a fine imposed by the concerned agency. Whereas for the same offence, another suspect may be given a punishment of minimum ten years' imprisonment and extended upto forever detainment along with fine imposed by another concerned agency. So different weights are assigned to different agencies for the same offence. In our case, four agencies interrogating the suspects on the same offence carry different weights- Panchayat member (P) with weight 0.2, Inspector of Police (I) carrying weight 0.3, Superintendent of Police with weight 0.4 and Director General of Police with weight 0.1, have been considered. The punishments are categorised linguistically, i.e., 'no punishment', 'very low punishment', 'low punishment', 'medium punishment', 'high punishment', 'very high punishment', and 'extremely high punishment'. These are defined in terms of number of years under custody. Extremely high punishment means 21 years' custody, low punishment is ascribed to 7 years' custody, and very low and low punishments are assigned to custody from 3 years 6 months to seven years, and thus numbers of years under custody are defined as an interval with linguistic terms, the concept of which is depicted in Fig 6.1.

Here, we consider the seven-element set of semantics of linguistic terms as s_0 = none, s_1 =

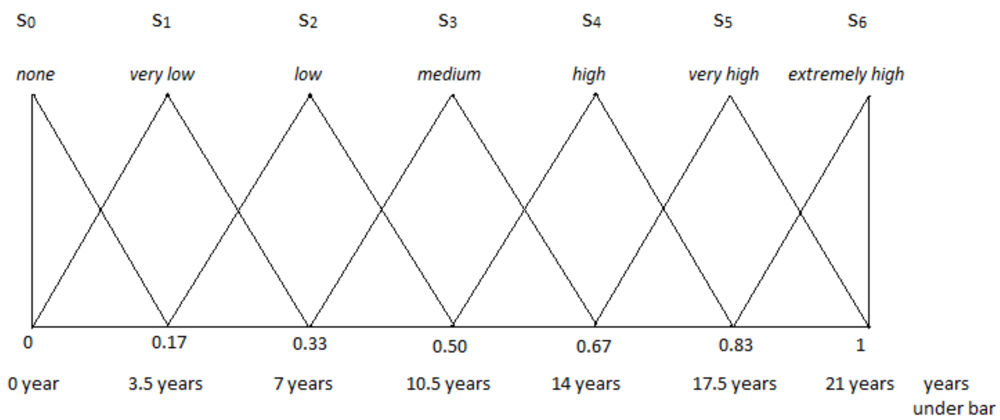

Figure 6.1: Linguistic term sets with semantics and the years of imprisonment.

very low, s_2 = low, s_3 = medium, s_4 = high, s_5 = very high and s_6 = extremely high in case of punishment and then $[3.5, 7]$, i.e., [very low, low] can be expressed as $[s_1, s_2]$.
The problem, described as a bi-matrix game, has the following representation:

$$\breve{A}_{HIVIFL} = \begin{matrix} \\ C_{PI} \\ NC_{PI} \end{matrix} \begin{matrix} C_{PII} & NC_{PII} \\ \left(\begin{matrix} ((h_1/P, h_2/I, h_3/S), (h_1/P, h_2/I, h_3/S)) \\ ((h_1/P, h_2/I, h_3/S, h_4/D), (h_1/P, h_2/I)) \end{matrix} \right. & \left. \begin{matrix} ((h_1/P, h_2/I), (h_1/P, h_2/I, h_3/S, h_4/D)) \\ ((h_1/P), (h_1/P)) \end{matrix} \right) \end{matrix},$$

where
$h_1/P = \{\langle [s_0, s_2], [s_2, s_4] \rangle, \langle [s_0, s_2], [s_3, s_4] \rangle, \langle [s_1, s_2], [s_3, s_4] \rangle\}/P,$

$h_2/I = \{\langle[s_1, s_2], [s_0, s_2]\rangle, \langle[s_1, s_4], [s_0, s_2]\rangle, \langle[s_1, s_3], [s_2, s_2]\rangle\}/I$,
$h_3/S = \{\langle[s_3, s_4], [s_0, s_1]\rangle, \langle[s_3, s_5], [s_0, s_1]\rangle, \langle[s_3, s_5], [s_0, s_0]\rangle\}/S$, and
$h_4/D = \{\langle[s_4, s_5], [s_0, s_0]\rangle, \langle[s_3, s_6], [s_0, s_0]\rangle, \langle[s_3, s_5], [s_0, s_0]\rangle\}/D$.
Here, the $(1, 1)$-positional element of the matrix $\breve{A}_{HIVIFL}$ is $((h_1/P, h_2/I, h_3/S),$ $(h_1/P, h_2/I, h_3/S))$, which is the outcome for player PI (when he or she confesses) and player PII (when he or she confesses). Similarly, other payoff elements are explained.

6.6.2 PD via TOPSIS and the solution

In this section, we define the algorithmic steps to relate the problem of Prisoners' Dilemma in HIVIFLTS environment through TOPSIS first, and then solve the aforementioned PD problem using the algorithm.

Step 1. The original decision matrix related to the fuzzy bi-matrix game problem in hesitant interval-valued intuitionistic fuzzy-linguistic environment is identified. Here, the choices of the interval are the criteria when the selection depends on human mind and intuition, which are imprecise and more basic in linguistic variables, depending upon the responses of the suspects in the course of interrogation.

Step 2. Using Steps 2 and 3 of algorithm of Section 6.4.3, the weighted HIVIFLTS decision matrix becomes

$$\bar{\mathfrak{R}} = \begin{matrix} & C_{PII} & NC_{PII} \\ C_{PI} & ((h_1/P, h_2/I, h_3/S), (h_1/P, h_2/I, h_3/S)) & ((h_1/P, h_2/I), (h_1/P, h_2/I, h_3/S, h_4/D)) \\ NC_{PI} & ((h_1/P, h_2/I, h_3/S, h_4/D), (h_1/P, h_2/I)) & ((h_1/P), (h_1/P)) \end{matrix},$$

here $h_1/P = \langle[s_{4.57}, s_{4.68}], [s_{1.15}, s_{1.32}]\rangle/P,\ h_2/I = \langle[s_{4.38}, s_{4.77}], [s_0, s_{1.23}]\rangle/I$,
$h_3/S = \langle[s_{4.45}, s_5], [s_0, s_1]\rangle/S$ and $h_4/D = \langle[s_{4.83}, s_6], [s_0, s_0]\rangle/D$.

Step 3. Utilizing Step 4 of the algorithm of Section 6.4.3, the aggregated weighted HIVIFLTS decision matrix, $\bar{\bar{\mathfrak{R}}}$ is obtained as:

$$\bar{\bar{\mathfrak{R}}} = \begin{matrix} & C & NC \\ C & M_1 & M_2 \\ NC & M_3 & M_4 \end{matrix}.$$

where $M_1 = ((\langle[s_{4.38}, s_5], [s_0, s_{1.32}]\rangle), (\langle[s_{4.38}, s_5], [s_0, s_{1.32}]\rangle))$,
$M_2 = (\langle[s_{4.38}, s_{4.77}], [s_0, s_{1.32}]\rangle, (\langle[s_{4.38}, s_6], [s_0, s_{1.32}]\rangle))$,
$M_3 = ((\langle[s_{4.38}, s_6], [s_0, s_{1.32}]\rangle), (\langle[s_{4.38}, s_{4.77}], [s_0, s_{1.32}]\rangle))$, and,
$M_4 = ((\langle[s_{4.57}, s_{4.68}], [s_{1.15}, s_{1.32}]\rangle), (\langle[s_{4.57}, s_{4.68}], [s_{1.15}, s_{1.32}]\rangle))$.
This is the payoff of the bi-matrix game. If we separate the pay-off matrix corresponding to each suspects PI and PII, the game matrix for PI and PII becomes as,

$$\bar{\bar{\mathfrak{R}}}_{PI} = \begin{matrix} & C & NC \\ C & \{\langle[s_{4.38}, s_5], [s_0, s_{1.32}]\rangle\} & \{\langle[s_{4.38}, s_{4.77}], [s_0, s_{1.32}]\rangle\} \\ NC & \{\langle[s_{4.38}, s_6], [s_0, s_{1.32}]\rangle\} & \{\langle[s_{4.57}, s_{4.68}], [s_{1.15}, s_{1.32}]\rangle\} \end{matrix}$$

$$\text{and } \bar{\bar{\mathfrak{R}}}_{PII} = \begin{array}{c} \\ C \\ NC \end{array} \begin{array}{c} \begin{array}{cc} C & NC \end{array} \\ \begin{pmatrix} (\langle [s_{4.38}, s_5], [s_0, s_{1.32}] \rangle) & (\langle [s_{4.38}, s_{4.6}], [s_0, s_{1.32}] \rangle) \\ (\langle [s_{4.38}, s_{4.77}], [s_0, s_{1.32}] \rangle) & (\langle [s_{4.57}, s_{4.68}], [s_{1.15}, s_{1.32}] \rangle) \end{pmatrix} \end{array}.$$

Step 4. Using Step 5 of the algorithm in Section 6.4.3, HIVIFLT-PIS and HIVIFLT-NIS of each $\bar{\bar{\mathfrak{R}}}_{PI}$ and $\bar{\bar{\mathfrak{R}}}_{PII}$ are obtained as:
$\bar{\mathfrak{R}}^{+}_{PI} = (\langle [s_{4.38}, s_{4.77}], [s_0, s_{1.32}] \rangle, \langle [s_{4.38}, s_{4.68}], [s_{1.15}, s_{1.32}] \rangle)^T$,
$\bar{\mathfrak{R}}^{-}_{PI} = (\langle [s_{4.38}, s_5], [s_0, s_{1.32}] \rangle, \langle [s_{4.57}, s_6], [s_0, s_{1.32}] \rangle)^T$,
$\bar{\mathfrak{R}}^{+}_{PII} = (\langle [s_{4.38}, s_5], [s_0, s_{1.32}] \rangle, \langle [s_{4.38}, s_{4.68}], [s_{1.15}, s_{1.32}] \rangle)^T$,
$\bar{\mathfrak{R}}^{-}_{PII} = (\langle [s_{4.38}, s_6], [s_0, s_{1.32}] \rangle, \langle [s_{4.57}, s_{4.77}], [s_0, s_{1.32}] \rangle)^T$.

Step 5. Since the distance calculation depends upon α of s_α [158], the semantics of linguistic term sets S and $s_\alpha, s_\beta \in S$, we use the values of α at the time of calculation of distance measure utilizing Step 6 of algorithm in Section 6.4.3. For the sake of calculation, we consider here the payoff matrix for suspect II, $\bar{\bar{\mathfrak{R}}}_{PII}$, and its HIVIFLT-PIS and HIVIFLT-NIS, for different alternatives, say x_1 and x_2:

$$\begin{aligned}
d({}^{x_1}\bar{\bar{\mathfrak{R}}}_{PII}, \bar{\mathfrak{R}}^{+}_{PII}) &= \left\{\frac{1}{2}\{(-0.32+1.32)^2+(4.77-4.68)^2+(0-1.15)^2+(-0.09-0)^2+(1.62-0.47)^2\}\right\}^{1/2} \\
&= 1.35299, \\
d({}^{x_2}\bar{\bar{\mathfrak{R}}}_{PII}, \bar{\mathfrak{R}}^{+}_{PII}) &= \left\{\frac{1}{2}\{(6-5)^2+(4.57-4.38)^2+(0.28-0.47)^2\}\right\}^{1/2} \\
&= 0.73218, \\
d({}^{x_1}\bar{\bar{\mathfrak{R}}}_{PII}, \bar{\mathfrak{R}}^{-}_{PII}) &= \left\{\frac{1}{2}\{(5-6)^2+(-0.32+1.32)^2+(4.38-4.57)^2+(1.62-1.43)^2\}\right\}^{1/2} \\
&= 1.01788, \\
d({}^{x_2}\bar{\bar{\mathfrak{R}}}_{PII}, \bar{\mathfrak{R}}^{-}_{PII}) &= \left\{\frac{1}{2}\{(4.68-4.77)^2+(1.15-0)^2+(0-(-0.09))^2+(0.28-1.43)^2\}\right\}^{1/2} \\
&= 1.15351.
\end{aligned}$$

Step 6. Using Step 7 of the algorithm in Section 6.4.3, the closeness coefficients for the alternatives are derived as:
${}^{x_1}CC_{PII} = \frac{1.01788}{1.35299+1.01788} = 0.42932$,
${}^{x_2}CC_{PII} = \frac{1.15351}{0.73218+1.15351} = 0.61171$.
So, in case of ranking the alternatives: $x_2 \succ x_1$. Similarly, calculating from payoff matrix $\bar{\bar{\mathfrak{R}}}_{PI}$, we see: ${}^{x_1}CC_{PI} = 0.09715$ and ${}^{x_2}CC_{PI} = 0.90284$. Here, $x_2 \succ x_1$.

Now, if we draw a matrix based on the ranks of the alternatives, we get the arrangement of the issue as:

$$\begin{array}{c} \\ C_{PI} \\ NC_{PI} \end{array} \begin{array}{c} \begin{array}{cc} C_{PII} & NC_{PII} \end{array} \\ \begin{pmatrix} 2,2 & 2,1 \\ 1,2 & 1,1 \end{pmatrix} \end{array}.$$

We may opine that when none of suspects cooperates, both of them might be awarded with maximum benefits, i.e., they will get minimum terms of detainment. But in reality this does come to happen. As there is every chance of betrayal, none is expected to get very low imprisonment terms. In case when one confesses and another does not confess, one may enjoy benefit which is also not an optimal solution. As the ranking alternative is low, when both cooperate, solution may reach to optimality, and such situation is defined as *Nash equilibrium*. Now, while explaining the

choices of payoffs, we see that among the set of choices of first player PI, having strategy confess, the optimal solution is $(h_1/P, h_2/I)$. However, it can be possible only if the second player PII chooses the strategy of non-confess. Here, the optimal response will be $(h_1/P, h_2/I, h_3/S)$, as it will be equilibrium choice. This is considered as the best known response in terms of choices of PII, still there is a possibility of no one to very low imprisonment year-terms when the other confesses. Same cases arise when strategies of PII are discussed. Therefore, the optimal value occurs at rank-position $(2, 2)$, and the value of the game is $((h_1/P, h_2/I, h_3/S), (h_1/P, h_2/I, h_3/S))$, where:
$h_1/P = \{\langle[s_0, s_2], [s_2, s_4]\rangle, \langle[s_0, s_2], [s_3, s_4]\rangle, \langle[s_1, s_2], [s_3, s_4]\rangle\}/P$,
$h_2/I = \{\langle[s_1, s_2], [s_0, s_2]\rangle, \langle[s_1, s_4], [s_0, s_2]\rangle, \langle[s_1, s_3], [s_2, s_2]\rangle\}/I$, and
$h_3/S = \{\langle[s_3, s_4], [s_0, s_1]\rangle, \langle[s_3, s_5], [s_0, s_1]\rangle, \langle[s_3, s_5], [s_0, s_0]\rangle\}/S$.

6.7 Conclusion

Practically, everything in nature is encountered with some kinds of vagueness and vulnerability. Human beings think in linguistic mode but count and measure by numbers giving birth the linguistic form of fuzzy set extended in different models which are more acceptable than the crisp data to meet the real-life situations.

We have attempted in our work to solve the Prisoners' Dilemma game in human trafficking depending on the ergonomics in information process and organisation management in OR. Human trafficking must be treated as a serious illegal activity as it has transnational implications in the wrong doings, and consequently every country should come forward to battle this trafficking by sanctioning stringent laws and affording reserves and prosecutorial assets.

This work starts with the introduction of preliminaries, relevant definitions and properties of hesitant interval-valued intuitionistic fuzzy-linguistic term set considered with two-person non-zero-sum games with HIVIFLTS-based payoff values. We have attempted to develop and present categorically the concept of two-person non-zero-sum game coincided with Prisoners' Dilemma in this problem and achieved realistic results. In our work we observe that when both convicts do not cooperate with interrogating agencies they are punished with maximum terms of detainment, but there is a chance of getting low or very low punishment when any one of them betrays with another and cooperates with the interrogators. It is also seen that when both of the suspects confess their guilt the get lesser punishment in terms of years in custody. So, we may conclude from our achieved results that to obtain more information about trafficking and other related matters a situation of betrayal between traffickers should be made. This is the focused insights of our considered problems. The operations, strategic assumptions and governance strategies we have used in our work may also be applied to encounter various likewise real-life problems in future.

Advantages: Our main advantage is that we have been able to present uncertainty into linguistic terms representation. Here we have considered HIVIFLTS payoffs as human judgements are facilitated by preference information that is concerned with memberships among a set of possible hesitant fuzzy-linguistic values in interval.

Disdvantages: In our problem we have used linguistic variables, but the calculations have been made on non-linguistic terms and the final results automatically do not match with the linguistic terms assumed initially. So we are to approximate the results admitting a loss of information, and these are the drawbacks of our proposed study which can be replaced by improving the measure-

ment of uncertainty with its different degrees.
Our cited problem has been considered with its merits and demerits. The collaboration of hesitant interval-valued intuitionistic fuzzy-linguistic term set (HIVIFLTS) with TOPSIS has made our proposed method different from all other methods in every aspect of game theory. This method gives vent to a new approach towards the solution of problems like human-trafficking using the fruits of Prisoners' Dilemma.