Chapter 4

Two-person zero-sum game in linguistic neutrosophic environment *

Neutrosophic concept in set and logic are now-a-days considered as another type of uncertainty. Neutrosophic numbers liberally assume the indeterminacy in choice of elements based upon decision makers' intuition, assumption, judgement, behaviour, evaluation and decision. Here, we introduce the concept of linguistic neutrosophic numbers as payoff elements of two-person zero-sum game through the fundamental concept of game theory. Finally, we achieve a real-life problematic-example from medical point of view and solve it according to generated concept.

4.1 Motivation

Mental health illness is growing rapidly throughout the World and consequently diagnoses of depression are required. This chapter is motivated from the real-world problems of diagnoses and analyses of mental health issues which are solved here through two-person zero-sum game in another type of uncertainty, say as, linguistic neutrosophic environment.

4.2 Introduction

Uncertainties and ambiguities are exhibited through several environments. Linguistic term set in neutrosophic environment through game theory can be considered as new concept of two-person zero-sum game theory, when treated in uncertain, ambiguous environment. Here, we develop a two-person zero-sum matrix game model with linguistic neutrosophic numbers (both single-valued and interval-valued) as payoff elements. The game problem is solved using matrix method of game theory and the solution is verified through a real-life problem having medical background.

* A part of this chapter has been communicated to an international journal.

4.3 Basic Concepts

In this section, neutrosophic set, single-valued neutrosophic set, interval neutrosophic set, linguistic term set and their properties, operational laws are discussed.

Definition 4.3.1 *[130] Let X be a universe of discourse with a generic element $x \in X$. A neutrosophic set (NS) A in X is defined as, $A = \{\langle x,\ T_A(x),\ I_A(x),\ F_A(x)\rangle : x \in X\}$, where, $T_A : X \to]0^-, 1^+[, I_A : X \to]0^-, 1^+[$ and $F_A : X \to]0^-, 1^+[$, with restriction on $T_A(x) + I_A(x) + F_A(x)$, i.e., $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+, \forall\ x \in X$. The numbers $T_A(x)$, $I_A(x)$ and $F_A(x)$ are respectively the truth membership, indeterminacy membership and falsity membership degrees of the element x to the set A.*

The representation of neutrosophic set was presented philosophically, earlier. But, to express reality, standardization of non-standard subsets of $]0^-, 1^+[$ are improved to $[0, 1]$ by Wang [144] through engineering or scientific point of view.

Definition 4.3.2 *[144] $\acute{A}$, a single-valued neutrosophic set (SVNS), in a universe of discourse X is given by $\acute{A} = \{\langle x,\ T_{\acute{A}}(x),\ I_{\acute{A}}(x),\ F_{\acute{A}}(x)\rangle : x \in X\}$, where $T_{\acute{A}} : X \to [0, 1], I_{\acute{A}} : X \to [0, 1]$ and $F_{\acute{A}} : X \to [0, 1]$, with the condition $0 \leq T_{\acute{A}}(x) + I_{\acute{A}}(x) + F_{\acute{A}}(x) \leq 3, \forall\ x \in X$. The numbers $T_{\acute{A}}(x)$, $I_{\acute{A}}(x)$ and $F_{\acute{A}}(x)$ are respectively the truth membership, indeterminacy membership and falsity membership degrees of the element x to the set $\acute{A}$.*

Definition 4.3.3 *A SVNS $\acute{A} = \{\langle x, T_{\acute{A}}(x), I_{\acute{A}}(x), F_{\acute{A}}(x)\rangle : \forall\ x \in X\}$ is said to be neutrosophic-normal, if there exist at least three points $a, b, c \in X$ such that $T_{\acute{A}}(a) = I_{\acute{A}}(b) = F_{\acute{A}}(c) = 1$.*

Definition 4.3.4 *A SVNS $\acute{A} = \{\langle x, T_{\acute{A}}(x), I_{\acute{A}}(x), F_{\acute{A}}(x)\rangle : \forall\ x \in X\}$ is called neutrosophic-convex if for all $a, b \in X$ and $\lambda \in [0, 1]$, the following conditions are satisfied.*

(i) $T_{\acute{A}}(\lambda a + (1 - \lambda)b) \geq \min(T_{\acute{A}}(a), T_{\acute{A}}(b))$,

(ii) $I_{\acute{A}}(\lambda a + (1 - \lambda)b) \leq \max(I_{\acute{A}}(a), I_{\acute{A}}(b))$,

(iii) $F_{\acute{A}}(\lambda a + (1 - \lambda)b) \leq \max(F_{\acute{A}}(a), F_{\acute{A}}(b))$,

i.e., truth membership function is fuzzy convex and indeterminacy, falsity membership functions are fuzzy concave.

Definition 4.3.5 *A single-valued neutrosophic number (SVNN) is $\acute{A} = \{\langle x, T_{\acute{A}}(x), I_{\acute{A}}(x), F_{\acute{A}}(x)\rangle : x \in X\}$, when*

(i) *$\acute{A}$ is neutrosophic-normal and neutrosophic-convex;*

(ii) *$T_{\acute{A}}$ is upper semi-continuous, $I_{\acute{A}}$ and $F_{\acute{A}}$ are lower semi-continuous;*

(iii) *Support $S(\acute{A})$ of $\acute{A}$ is bounded, i.e., $S(\acute{A}) = \{T_{\acute{A}}(x) > 0, I_{\acute{A}}(x) < 1, F_{\acute{A}}(x) < 1, \forall\ x \in X\}$.*

Definition 4.3.6 *[165] Let* $x = \Big\langle T_{\acute{A}}(x), I_{\acute{A}}(x), F_{\acute{A}}(x) \Big\rangle, y = \Big\langle T_{\acute{A}}(y), I_{\acute{A}}(y), F_{\acute{A}}(y) \Big\rangle$ *be two SVNNs such that* $x,\ y \in \acute{A}$ *and* $\lambda > 0$. *Then operations can be defined as follows:*

(i) $x^c = \Big\langle F_{\acute{A}}(x), 1 - I_{\acute{A}}(x), T_{\acute{A}}(x) \Big\rangle$;

(ii) $x \cup y = \Big\langle \max(T_{\acute{A}}(x), T_{\acute{A}}(y)), \min(I_{\acute{A}}(x), I_{\acute{A}}(y)), \min(F_{\acute{A}}(x), F_{\acute{A}}(y)) \Big\rangle$;

(iii) $x \cap y = \Big\langle \min(T_{\acute{A}}(x), T_{\acute{A}}(y)), \max(I_{\acute{A}}(x), I_{\acute{A}}(y)), \max(F_{\acute{A}}(x), F_{\acute{A}}(y)) \Big\rangle$;

(iv) $x \oplus y = \Big\langle (T_{\acute{A}}(x) + T_{\acute{A}}(y) - T_{\acute{A}}(x)T_{\acute{A}}(y)), (I_{\acute{A}}(x)I_{\acute{A}}(y)), (F_{\acute{A}}(x)F_{\acute{A}}(y)) \Big\rangle$;

(v) $x \otimes y = \Big\langle (T_{\acute{A}}(x)T_{\acute{A}}(y)), (I_{\acute{A}}(x) + I_{\acute{A}}(y) - I_{\acute{A}}(x)I_{\acute{A}}(y)), (F_{\acute{A}}(x) + F_{\acute{A}}(y) - F_{\acute{A}}(x)F_{\acute{A}}(y)) \Big\rangle$;

(vi) $x \ominus y = \Big\langle \frac{T_{\acute{A}}(x) - T_{\acute{A}}(y)}{1 - T_{\acute{A}}(y)}, \frac{I_{\acute{A}}(x)}{I_{\acute{A}}(y)}, \frac{F_{\acute{A}}(x)}{F_{\acute{A}}(y)} \Big\rangle$ *provided,* $T_{\acute{A}}(y) \neq 1, I_{\acute{A}}(y) \neq 0, F_{\acute{A}}(y) \neq 0$;

(vii) $x \oslash y = \Big\langle \frac{T_{\acute{A}}(x)}{T_{\acute{A}}(y)}, \frac{I_{\acute{A}}(x) - I_{\acute{A}}(y)}{1 - I_{\acute{A}}(y)}, \frac{F_{\acute{A}}(x) - F_{\acute{A}}(y)}{1 - F_{\acute{A}}(y)} \Big\rangle$ *provided,* $T_{\acute{A}}(y) \neq 0, I_{\acute{A}}(y) \neq 1, F_{\acute{A}}(y) \neq 1$;

(viii) $\lambda x = \Big\langle 1 - (1 - T_{\acute{A}}(x))^{\lambda}, I_{\acute{A}}(x)^{\lambda}, F_{\acute{A}}(x)^{\lambda} \Big\rangle$;

(ix) $x^{\lambda} = \Big\langle T_{\acute{A}}(x)^{\lambda}, 1 - (1 - I_{\acute{A}}(x))^{\lambda}, 1 - (1 - F_{\acute{A}}(x))^{\lambda} \Big\rangle$.

Here, $\oplus$, $\otimes$, $\ominus$ and $\oslash$ are used for neutrosophic addition, multiplication, subtraction and division, respectively. Also, in subtraction and division operations (given in (vi) and (vii) of **Definition 4.3.6**), components of neutrosophic numbers are assumed in the interval $[0, 1]$ as classical case. But, for the general case, when dealing with neutrosophic overset, underset and offset [165], or the neutrosophic number components are in the interval $[f, g]$, where f is called underlimit and g is called overlimit, with $f \leq 0 < 1 \leq g$, i.e., one has neutrosophic overnumbers, undernumbers and offnumbers, respectively, then the components of neutrosophic numbers due to subtraction and division lie within $[f, g]$.

Definition 4.3.7 *[145] Let X be a space of points (objects) with generic element x in X. An interval neutrosophic set* $\acute{A}$ *in X is characterized by truth-membership function* $T_{\acute{A}}(x)$*, indeterminacy membership function* $I_{\acute{A}}(x)$ *and falsity-membership function* $F_{\acute{A}}(x)$*. For each x in X,* $T_{\acute{A}}(x),\ I_{\acute{A}}(x),\ F_{\acute{A}}(x) \subseteq [0, 1]$. *Therefore, an interval neutrosophic set can be shown as follows:* $\acute{A} = \Big\{ \Big\langle x, [T^l_{\acute{A}}(x), T^u_{\acute{A}}(x)], [I^l_{\acute{A}}(x), I^u_{\acute{A}}(x)], [F^l_{\acute{A}}(x), F^u_{\acute{A}}(x)] \Big\rangle : x \in X \Big\}$. *Simply, it can be written as:* $\acute{A} = \Big\{ \Big\langle [T^l_{\acute{A}}, T^u_{\acute{A}}], [I^l_{\acute{A}}, I^u_{\acute{A}}], [F^l_{\acute{A}}, F^u_{\acute{A}}] \Big\rangle \Big\}$*, where,* $T^u_{\acute{A}} + I^u_{\acute{A}} + F^u_{\acute{A}} \leq 3$. *Only the subunitary interval of* $[0, 1]$ *is considered, and it is a subclass of a neutrosophic set. Therefore, an interval neutrosophic set can be viewed as a collection of interval neutrosophic numbers.*

Definition 4.3.8 *[145] Let* $\grave{x} = \Big\langle [T^l_{\acute{A}}(x), T^u_{\acute{A}}(x)], [I^l_{\acute{A}}(x), I^u_{\acute{A}}(x)], [F^l_{\acute{A}}(x), F^u_{\acute{A}}(x)] \Big\rangle, \grave{y} = \Big\langle [T^l_{\acute{A}}(y), T^u_{\acute{A}}(y)], [I^l_{\acute{A}}(y), I^u_{\acute{A}}(y)], [F^l_{\acute{A}}(y), F^u_{\acute{A}}(y)] \Big\rangle$ *be two interval neutrosophic numbers such that* $x,\ y \in$

$\acute{A}$ and $\lambda > 0$. These can be simply written as, $\grave{x} = \left\langle [T_1^l, T_1^u], [I_1^l, I_1^u], [F_1^l, F_1^u] \right\rangle$ and $\grave{y} = \left\langle [T_2^l, T_2^u], [I_2^l, I_2^u], [F_2^l, F_2^u] \right\rangle$. Then operational laws can be defined as follows:

(i) $\grave{x}^c = \left\langle [F_1^l, F_1^u], [1 - I_1^l, 1 - I_1^u], [T_1^l, T_1^u] \right\rangle$;

(ii) $\grave{x} \oplus \grave{y} = \left\langle [T_1^l + T_2^l - T_1^l T_2^l, T_1^u + T_2^u - T_1^u T_2^u], [I_1^l I_2^l, I_1^u I_2^u], [F_1^l F_2^l, F_1^u F_2^u] \right\rangle$;

(iii) $\grave{x} \otimes \grave{y} = \left\langle [T_1^l T_2^l, T_1^u T_2^u], [I_1^l + I_2^l - I_1^l I_2^l, I_1^u + I_2^u - I_1^u I_2^u], [F_1^l + F_2^l - F_1^l F_2^l, F_1^u + F_2^u - F_1^u F_2^u] \right\rangle$;

(iv) $\grave{x} \ominus \grave{y} = \left\langle [\frac{T_1^l - T_2^l}{1 - T_2^l}, \frac{T_1^u - T_2^u}{1 - T_2^u}], [\frac{I_1^l}{I_2^l}, \frac{I_1^u}{I_2^u}], [\frac{F_1^l}{F_2^l}, \frac{F_1^u}{F_2^u}] \right\rangle$ *provided* $T_2^l \neq 1, T_2^u \neq 1, I_2^l \neq 0, I_2^u \neq 0, F_2^l \neq 0, F_2^u \neq 0$;

(v) $\grave{x} \oslash \grave{y} = \left\langle [\frac{T_1^l}{T_2^l}, \frac{T_1^u}{T_2^u}], [\frac{I_1^l - I_2^l}{1 - I_2^l}, \frac{I_1^u - I_2^u}{1 - I_2^u}], [\frac{F_1^l - F_2^l}{1 - F_2^l}, \frac{F_1^u - F_2^u}{1 - F_2^u}] \right\rangle$ *provided* $T_2^l \neq 0, T_2^u \neq 0, I_2^l \neq 1, I_2^u \neq 1, F_2^l \neq 1, F_2^u \neq 1$;

(vi) $\lambda \grave{x} = \left\langle [1 - (1 - T_1^l)^\lambda, 1 - (1 - T_1^u)^\lambda], [(I_1^l)^\lambda, (I_1^u)^\lambda], [(F_1^l)^\lambda, (F_1^u)^\lambda] \right\rangle$;

(vii) $\grave{x}^\lambda = \left\langle [(T_1^l)^\lambda, (T_1^u)^\lambda], [1 - (1 - I_1^l)^\lambda, 1 - (1 - I_1^u)^\lambda], [1 - (1 - F_1^l)^\lambda, 1 - (1 - F_1^u)^\lambda] \right\rangle$.

Here, each components of interval neutrosophic numbers in (iv) and (v) (of **Definition 4.3.8**) are assumed in the interval $[0, 1]$, as classical case. But, for the general case, when dealing with neutrosophic overset, underset and offset [165], or the neutrosophic number components are in the interval $[f, g]$, where f is called underlimit and g is called overlimit, with $f \leq 0 < 1 \leq g$, i.e., one has neutrosophic overnumbers, undernumbers and offnumbers, respectively, then the components of neutrosophic numbers due to subtraction and division lie within $[f, g]$.
We observe that in some cases of real-life, society gives preference to those variables which are expressed by words, sentences or language rather than these expressed in terms of numerical values. Zadeh [168] introduced this type of variables, called *linguistic variables* and also linguistic approach in fuzzy set theory.

Definition 4.3.9 *[168]: A linguistic variable is characterized by a quintuple $(C, T(C), U, G, M)$, where C denotes the name of the variable. Here, $T(C)$ indicates the term set of C, i.e., the set of its linguistic values; U is a universe of discourse; G is the way by which the terms of $T(C)$ are generated; and M is a semantic rule for associating each linguistic value X with its meaning; finally, $M(X)$ is a fuzzy subset of U. A linguistic variable is described logically by its semantics.*

Several ways [51; 124] are there to express the linguistic descriptors and the corresponding semantics. Among these, seven scales of linguistic term-based semantics are used frequently, given as: $S_{11} = \{s_0$ (nothing), s_1 (very low), s_2 (low), s_3 (medium), s_4 (high), s_5 (very high), s_6 (perfect)$\}$; $S_{12} = \{s_0$ (very poor), s_1 (poor), s_2 (slightly poor), s_3 (fair), s_4 (slightly good), s_5 (good), s_6 (very good)$\}$; $S_{13} = \{s_1$ (extremely poor), s_2 (very poor), s_3 (poor), s_4 (medium), s_5 (good), s_6 (very good), s_7 (extremely good)$\}$; $S_{14} = \{s_{-3}$ (none), s_{-2} (very low), s_{-1} (low), s_0 (medium), s_1 (high), s_2 (very high), s_3 (perfect)$\}$.

Property 4.3.1 *Consider a linguistic term set* $S = \{s_i : i = 1, 2, \ldots, t\}$. *Then we have:*

(i) *The set is ordered, i.e., for* $i > j$, $s_i > s_j$;

(ii) *A negation operator exists, i.e.,* $\neg(s_i) = s_j$, *for* $i + j = t + 1$;

(iii) *A maximizing operator exists, i.e.,* $\max(s_i, s_j) = s_j$, *if* $s_j \geq s_i$;

(iv) *A minimizing operator exists, i.e.,* $\min(s_i, s_j) = s_j$, *if* $s_j \leq s_i$.

This discrete term set can be converted into continuous term set as:
$\overline{S} = \{s_i : s_1 \leq s_i \leq s_q, i \in [1, q]\}$, where q is sufficiently large positive number. Here, s_i is called the *original linguistic term* if $s_i \in S$, otherwise, the *virtual linguistic term*.

Property 4.3.2 *Assume* s_γ *and* s_δ *be two linguistic variables;* $s_\gamma, s_\delta \in \overline{S}; \lambda, \kappa \in [0, 1]$. *The operational laws are elucidated as [159]:*

(i) $s_\gamma \oplus s_\delta = s_{\gamma+\delta}$;

(ii) $s_\gamma \otimes s_\delta = s_{\gamma\delta}$;

(iii) $\lambda s_\gamma = s_{\lambda\gamma}$;

(iv) $(\lambda + \kappa)s_\gamma = \lambda s_\gamma \oplus \kappa s_\gamma$;

(v) $(s_\gamma)^\lambda = s_{\gamma^\lambda}$.

Human judgements and perception always flow neutrosophically and basically these environments are nurtured with linguistic characters of responses, understood in fuzziness sense. Subsequently, we demonstrate the linguistic single-valued and interval-valued neutrosophic sets.

4.4 Linguistic Neutrosophic Set

Based on the combination of linguistic term set and neutrosophic set, this section promotes the concept of linguistic neutrosophic set.

Definition 4.4.1 *Let X be a non-empty subset of the universe and consider a linguistic term set* $S = \{s_i : i = 1, 2, \ldots, t\}$. *Then linguistic neutrosophic set is defined as:* $\breve{S} = \{\langle s_{T_{\breve{x}}}, s_{I_{\breve{x}}}, s_{F_{\breve{x}}}\rangle : \breve{x} \in X\}$. *Each component of* $\breve{S}$, *i.e.,* $s_{T_{\breve{x}}}, s_{I_{\breve{x}}}$, *and* $s_{F_{\breve{x}}}$ *are in linguistic form.*

From this definition, $\langle s_{0.4}, s_{0.5}, s_{0.2}\rangle$ is a linguistic neutrosophic number. Sometimes, it is called linguistic single-valued neutrosophic number.

Property 4.4.1 *Let* $\breve{l} = \left\langle s_{T_{\breve{l}}},\ s_{I_{\breve{l}}},\ s_{F_{\breve{l}}}\right\rangle$, $\breve{l}_1 = \left\langle s_{T_{\breve{l}_1}},\ s_{I_{\breve{l}_1}},\ s_{F_{\breve{l}_1}}\right\rangle$ *and* $\breve{l}_2 = \left\langle s_{T_{\breve{l}_2}},\ s_{I_{\breve{l}_2}},\ s_{F_{\breve{l}_2}}\right\rangle$ *be any three linguistic single-valued neutrosophic numbers and* $\lambda > 0$, *then the operational laws of linguistic neutrosophic numbers are defined as follows:*

(i) $\breve{l}_1 \oplus \breve{l}_2 = \left\langle s_{T_{\breve{l}_1}} + s_{T_{\breve{l}_2}} - s_{T_{\breve{l}_1}} s_{T_{\breve{l}_2}},\ s_{I_{\breve{l}_1}} s_{I_{\breve{l}_2}},\ s_{F_{\breve{l}_1}} s_{F_{\breve{l}_2}} \right\rangle$;

(ii) $\breve{l}_1 \ominus \breve{l}_2 = \left\langle \frac{s_{T_{\breve{l}_1}} - s_{T_{\breve{l}_2}}}{s_1 - s_{T_{\breve{l}_2}}},\ \frac{s_{I_{\breve{l}_1}}}{s_{I_{\breve{l}_2}}},\ \frac{s_{F_{\breve{l}_1}}}{s_{F_{\breve{l}_2}}} \right\rangle$ *provided* $s_{T_{\breve{l}_2}} \neq s_1, s_{I_{\breve{l}_2}} \neq s_0, s_{F_{\breve{l}_2}} \neq s_0$;

(iii) $\breve{l}_1 \otimes \breve{l}_2 = \left\langle s_{T_{\breve{l}_1}} s_{T_{\breve{l}_2}},\ s_{I_{\breve{l}_1}} + s_{I_{\breve{l}_2}} - s_{I_{\breve{l}_1}} s_{I_{\breve{l}_2}},\ s_{F_{\breve{l}_1}} + s_{F_{\breve{l}_2}} - s_{F_{\breve{l}_1}} s_{F_{\breve{l}_2}} \right\rangle$;

(iv) $\breve{l}_1 \oslash \breve{l}_2 = \left\langle \frac{s_{T_{\breve{l}_1}}}{s_{T_{\breve{l}_2}}},\ \frac{s_{I_{\breve{l}_1}} - s_{I_{\breve{l}_2}}}{s_1 - s_{I_{\breve{l}_2}}},\ \frac{s_{F_{\breve{l}_1}} - s_{F_{\breve{l}_2}}}{s_1 - s_{F_{\breve{l}_2}}} \right\rangle$ *provided* $s_{T_{\breve{l}_2}} \neq s_0, s_{I_{\breve{l}_2}} \neq s_1, s_{F_{\breve{l}_2}} \neq s_1$;

(v) $\lambda \breve{l} = \left\langle 1 - (1 - s_{T_{\breve{l}}})^{\lambda},\ (s_{I_{\breve{l}}})^{\lambda},\ (s_{F_{\breve{l}}})^{\lambda} \right\rangle$;

(vi) $\lambda \breve{l} = \left\langle (s_{T_{\breve{l}}})^{\lambda},\ 1 - (1 - s_{I_{\breve{l}}})^{\lambda},\ 1 - (1 - s_{F_{\breve{l}}})^{\lambda} \right\rangle$.

Since calculations of linguistic neutrosophic terms depend upon α of s_α [158], therefore calculation of $s_{T_{\breve{l}_1}} + s_{T_{\breve{l}_2}} - s_{T_{\breve{l}_1}} s_{T_{\breve{l}_2}}$ is nothing but $s_{(T_{\breve{l}_1} + T_{\breve{l}_2} - T_{\breve{l}_1} T_{\breve{l}_2})}$, i.e., $T_{\breve{l}_1} + T_{\breve{l}_2} - T_{\breve{l}_1} T_{\breve{l}_2}$. Similarly other terms are calculated during operational laws.

Definition 4.4.2 *Let X be a space of points (objects) with generic element x in X. A linguistic interval neutrosophic set $\grave{A}$ in X is characterized by truth-membership linguistic interval neutrosophic function $s_{T_{\grave{A}}(x)}$, indeterminacy membership linguistic interval neutrosophic function $s_{I_{\grave{A}}(x)}$ and falsity-membership linguistic interval neutrosophic function $s_{F_{\grave{A}}(x)}$. For each x in X, $T_{\grave{A}}(x),\ I_{\grave{A}}(x),\ F_{\grave{A}}(x) \subseteq [0,1]$, and $T^u_{\grave{A}} + I^u_{\grave{A}} + F^u_{\grave{A}} \leq 3$. Each of $s_{T_{\grave{A}}(x)},\ s_{T_{\grave{A}}(x)},\ s_{T_{\grave{A}}(x)} \in S$, the linguistic term set. A linguistic interval neutrosophic set can be represented, mathematically as: $\grave{A} = \left\{ \left\langle [s_{T^l_{\grave{A}}(x)}, s_{T^u_{\grave{A}}(x)}],\ [s_{I^l_{\grave{A}}(x)}, s_{I^u_{\grave{A}}(x)}],\ [s_{F^l_{\grave{A}}(x)}, s_{F^u_{\grave{A}}(x)}] \right\rangle : x \in X \right\}$. Elements of linguistic interval neutrosophic set are called as linguistic interval neutrosophic number. For example, $\left\langle [s_{0.75}, s_{0.89}], [s_{0.17}, s_{0.43}], [s_{0.10}, s_{0.30}] \right\rangle$ is a linguistic interval neutrosophic number.*

Property 4.4.2 *Several operational laws can be found on linguistic interval neutrosophic numbers. Let $p_1 = \left\langle [s_{T^l_{p_1}}, s_{T^u_{p_1}}], [s_{I^l_{p_1}}, s_{I^u_{p_1}}], [s_{F^l_{p_1}}, s_{F^u_{p_1}}] \right\rangle$, $p_2 = \left\langle [s_{T^l_{p_2}}, s_{T^u_{p_2}}], [s_{I^l_{p_2}}, s_{I^u_{p_2}}], [s_{F^l_{p_2}}, s_{F^u_{p_2}}] \right\rangle$ be two linguistic interval neutrosophic numbers and $\lambda > 0$. Then we have the following operational relations:*

(i) $p_1^c = \left\langle [s_{F^l_{p_1}}, s_{F^u_{p_1}}], [s_1 - s_{I^l_{p_1}}, s_1 - s_{I^u_{p_1}}], [s_1 - s_{T^l_{p_1}}, s_1 - s_{T^u_{p_1}}] \right\rangle$;

(ii) $p_1 \oplus p_2 = \left\langle [s_{T^l_{p_1}} + s_{T^l_{p_2}} - s_{T^l_{p_1}} s_{T^l_{p_2}}, s_{T^u_{p_1}} + s_{T^u_{p_2}} - s_{T^u_{p_1}} s_{T^u_{p_2}}], [s_{I^l_{p_1}} s_{I^l_{p_2}}, s_{I^u_{p_1}} s_{I^u_{p_2}}], [s_{F^l_{p_1}} s_{F^l_{p_2}}, s_{F^u_{p_1}} s_{F^u_{p_2}}] \right\rangle$;

(iii) $p_1 \otimes p_2 = \left\langle [s_{T^l_{p_1}} s_{T^l_{p_2}}, s_{T^u_{p_1}} s_{T^u_{p_2}}], [s_{I^l_{p_1}} + s_{I^l_{p_2}} - s_{I^l_{p_1}} s_{I^l_{p_2}}, s_{I^u_{p_1}} + s_{I^u_{p_2}} - s_{I^u_{p_1}} s_{I^u_{p_2}}], [s_{F^l_{p_1}} + s_{F^l_{p_2}} - s_{F^l_{p_1}} s_{F^l_{p_2}}, s_{F^u_{p_1}} + s_{F^u_{p_2}} - s_{F^u_{p_1}} s_{F^u_{p_2}}] \right\rangle$;

(iv) $p_1 \ominus p_2 = \left\langle [\frac{s_{T^l_{p_1}} - s_{T^l_{p_2}}}{s_1 - s_{T^l_{p_2}}}, \frac{s_{T^u_{p_1}} - s_{T^u_{p_2}}}{s_1 - s_{T^u_{p_2}}}], [\frac{s_{I^l_{p_1}}}{s_{I^l_{p_2}}}, \frac{s_{I^u_{p_1}}}{s_{I^u_{p_2}}}], [\frac{s_{F^l_{p_1}}}{s_{F^l_{p_2}}}, \frac{s_{F^u_{p_1}}}{s_{F^u_{p_2}}}] \right\rangle$ *provided* $s_{T^l_{p_2}} \neq s_1, s_{T^u_{p_2}} \neq s_1, s_{I^l_{p_2}} \neq s_0, s_{I^u_{p_2}} \neq s_0, s_{F^l_{p_2}} \neq s_0, s_{F^u_{p_2}} \neq s_0$;

(v) $p_1 \oslash p_2 = \left\langle [\frac{s_{T^l_{p1}}}{s_{T^l_{p2}}}, \frac{s_{T^u_{p1}}}{s_{T^u_{p2}}}], [\frac{s_{I^l_{p1}} - s_{I^l_{p2}}}{s_1 - s_{I^l_{p2}}}, \frac{s_{I^u_{p1}} - s_{I^u_{p2}}}{s_1 - s_{I^u_{p2}}}], [\frac{s_{F^l_{p1}} - s_{F^l_{p2}}}{s_1 - s_{F^l_{p2}}}, \frac{s_{F^u_{p1}} - s_{F^u_{p2}}}{s_1 - s_{F^u_{p2}}}] \right\rangle$ provided $s_{T^l_{p2}} \neq s_0$, $s_{T^u_{p2}} \neq s_0, s_{I^l_{p2}} \neq s_1, s_{I^u_{p2}} \neq s_1, s_{F^l_{p2}} \neq s_1, s_{F^u_{p2}} \neq s_1$;

(vi) $\lambda p_1 = \left\langle [s_1 - (s_1 - s_{T^l_{p1}})^\lambda, s_1 - (s_1 - s_{T^u_{p1}})^\lambda], [(s_{I^l_{p1}})^\lambda, (s_{I^u_{p1}})^\lambda], [(s_{F^l_{p1}})^\lambda, (s_{F^u_{p1}})^\lambda] \right\rangle$;

(vii) $p_1^\lambda = \left\langle [(s_{T^l_{p1}})^\lambda, (s_{T^u_{p1}})^\lambda], [s_1 - (s_1 - s_{I^l_{p1}})^\lambda, s_1 - (s_1 - s_{I^u_{p1}})^\lambda], [s_1 - (s_1 - s_{F^l_{p1}})^\lambda, s_1 - (s_1 - s_{F^u_{p1}})^\lambda] \right\rangle$.

4.5 Mathematical Model

In this Section, basically we introduce the concept of matrix game in crisp environment and in linguistic neutrosophic environment.

4.5.1 Matrix game in crisp environment

We consider two-person zero-sum game in crisp nature with pure strategies S_1, S_2 and mixed strategies Y, Z, as discussed in **Chapter 2** and denote the game by $G \equiv (Y, Z, A)$. Therefore, the payoff matrix for player I can be described as: $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1q} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2q} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & a_{p3} & \dots & a_{pq} \end{bmatrix}$. When we consider 2×2 matrix game, then the payoff matrix becomes: $A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

Definition 4.5.1 *[111; 113] Consider a choice of mixed strategies Y for player I and Z for player II, chosen independently, the expected payoff to player I of the game is given as:*

$$\begin{aligned} E(Y, Z) &= \sum_{i=1}^{p}\sum_{j=1}^{q} a_{ij} Prob(PI \ uses \ i \ and \ PII \ uses \ j) \\ &= \sum_{i=1}^{p}\sum_{j=1}^{q} a_{ij} Prob(PI \ uses \ i) \ Prob(PII \ uses \ j) \\ &= \sum_{i=1}^{p}\sum_{j=1}^{q} y_i a_{ij} z_j = Y^T A Z. \end{aligned}$$

Theorem 4.5.1 *Assuming that there are no pure optimal strategies, for any two-person zero-sum game, then optimal mixed strategies are (y_1^*, y_2^*) for player I and (z_1^*, z_2^*) for player II, so from the payoff matrix $A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, we get, $y_1^* = \frac{a_{22} - a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$, $y_2^* = 1 - y_1^*$; $z_1^* = \frac{a_{22} - a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$, $z_2^* = 1 - z_1^*$. The value of the game becomes $v^* = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$.*

Proof: The proof of the theorem is straightforward. □

4.5.2 Matrix game in linguistic neutrosophic environment

In this Section, we describe two-person matrix game in linguistic neutrosophic environment, and the game is represented as $\breve{G} \equiv (Y, Z; \breve{A}_1)$ shortly, $\breve{G}_{(\breve{A}_1)}$, where

(1) the set for strategies of player I is Y, a non-empty set,

(2) the set for strategies of player II is Z, a non-empty set, and

(3) $\breve{A}_1$ is linguistic neutrosophic set over $Y \times Z$, defined as:
$\breve{A}_1 = \{\langle (y, z), (s_{T_{\breve{A}_1}(y,z)}, s_{I_{\breve{A}_1}(y,z)}, s_{F_{\breve{A}_1}(y,z)}) \rangle : (y, z) \in Y \times Z\}$.

Here, for the sake of simplicity, we consider 2×2 matrix game, i.e., when player I uses y_1 and player II uses z_2, the payoff for player I is described as: $\langle s_{T_{\breve{A}_1}(y_1,z_2)}, s_{I_{\breve{A}_1}(y_1,z_2)}, s_{F_{\breve{A}_1}(y_1,z_2)} \rangle$.

Definition 4.5.2 *Let the linguistic neutrosophic game is* $\breve{G} \equiv (Y, Z; \breve{A}_1)$ *where,* $\breve{A}_1 = \{\langle (y, z), (s_{T_{\breve{A}_1}(y,z)}, s_{I_{\breve{A}_1}(y,z)}, s_{F_{\breve{A}_1}(y,z)}) \rangle : (y, z) \in Y \times Z\}$. *If* $\max_{y_i}\{\langle s_{T_{\breve{A}_1}(y_i,z_j)}, s_{I_{\breve{A}_1}(y_i,z_j)}, s_{F_{\breve{A}_1}(y_i,z_j)} \rangle\} = \min_{z_j}\{\langle s_{T_{\breve{A}_1}(y_i,z_j)}, s_{I_{\breve{A}_1}(y_i,z_j)}, s_{F_{\breve{A}_1}(y_i,z_j)} \rangle\}$, *then* $s_{(y_i,z_j)}$ *is called linguistic neutrosophic saddle point of matrix game.*

Similar cases arise for linguistic interval neutrosophic numbers.

Definition 4.5.3 *Let player I chooses any mixed strategy* $y \in Y$, *and player II selects any mixed strategy* $z \in Z$, *then expected payoffs for player I is* $E(\breve{A}_1) = y^T \breve{A}_1 z$ *and that for player II is the corresponding negation of* $E(\breve{A}_1)$.

Theorem 4.5.2 *Assuming that there are no pure optimal strategies, for any two-person zero-sum game, in linguistic neutrosophic environment (both single-valued and interval-valued), then optimal mixed strategies are* (y_1^*, y_2^*) *for player I and* (z_1^*, z_2^*) *for player II, then from the payoff matrix* $A_1 = \begin{bmatrix} \breve{a}_{11} & \breve{a}_{12} \\ \breve{a}_{21} & \breve{a}_{22} \end{bmatrix}$, *we get,* $y_1^* = \Big(\breve{a}_{22} \ominus \breve{a}_{21}\Big) \oslash \Big((\breve{a}_{11} \oplus \breve{a}_{22}) \ominus (\breve{a}_{12} \oplus \breve{a}_{21})\Big)$, $y_2^* = 1 \ominus y_1^*$; $z_1^* = \Big(\breve{a}_{22} \ominus \breve{a}_{12}\Big) \oslash \Big((\breve{a}_{11} \oplus \breve{a}_{22}) \ominus (\breve{a}_{12} \oplus \breve{a}_{21})\Big)$, $z_2^* = 1 \ominus z_1^*$. *The value of the game becomes* $v^* = \Big(\breve{a}_{11} \otimes \breve{a}_{22} \ominus \breve{a}_{12} \otimes \breve{a}_{21}\Big) \oslash \Big((\breve{a}_{11} \oplus \breve{a}_{22}) \ominus (\breve{a}_{12} \oplus \breve{a}_{21})\Big)$.

Proof: The proof of the theorem is simple and straightforward. □

4.6 Computative Example

We, in this Section, discuss on real-life problems from medical sectors through linguistic single-valued neutrosophic environment and linguistic interval-valued neutrosophic environment.

4.6.1 Problem description: From medical diagnoses phenomena

Mental health problem [41; 49; 74] is growing rapidly throughout the World. The analyses described in this part are focused on the factors affecting treatment (and no treatment, also) of mental health affected people by a mental health specialist or by a general physician. Diagnosis, aggressive behaviour, age, pressures on work-place, family structure, etc. are significant factors for treatment by health professionals. These are explicitly tested by professionals using linguistic variables taken directly from patient or from relatives, friends and others known to patient. Among several factors, mainly two factors [89; 143] are more important, namely,

(A) Beliefs and attitudes toward treatment behaviours, within which

  (1) Biological changes in the brain cause depression,

  (2) Medications are effective,

  (3) Treatment preferences,

  (4) Medications are addictive, and

(B) Subjective social norms, within which

  (1) Embarrassed if friends knew,

  (2) Employer should not know,

  (3) Family would be disappointed

exist. All these factors are under questionnaire [48; 133] during interaction within doctors, practitioners and patients, guardian, etc. Interestingly, all outputs are in linguistic format, for example, when doctor asks guardian about mental health disorder due to patient's biological changes in the brain causing depression, output is any one of strongly disagree, disagree, slightly disagree, neither, slightly agree, agree and strongly agree. Thus linguistic term sets are important to describe the situation. Here, these seven linguistic terms can be put as: $S = \{s_0$ (strongly disagree), s_1 (disagree), s_2 (slightly disagree), s_3 (neither), s_4 (slightly agree), s_5 (agree), s_6 (strongly agree)$\}$ in numerical scales.

We convert the numerical scales of semantics within the interval $[0,1]$, as: $S = \{s_{0.00}$ (strongly disagree), $s_{0.17}$ (disagree), $s_{0.34}$ (slightly disagree), $s_{0.50}$ (neither), $s_{0.67}$ (slightly agree), $s_{0.84}$ (agree), $s_{1.00}$ (strongly agree)$\}$ in numerical scales. This scale change is due to the introduction of neutrosophic concept in problem phenomenon. For example, a single-valued neutrosophic number may be $\langle$almost agree, neither agree, neither agree$\rangle$, and this can be mathematically expressed as, $\langle s_{0.75}, s_{0.50}, s_{0.50}\rangle$ or $\langle s_{0.89}, s_{0.50}, s_{0.50}\rangle$ or $\langle s_{0.83}, s_{0.50}, s_{0.50}\rangle$, etc. Similarly, linguistic interval neutrosophic number may be expressed in linguistic form as, $\Big\langle [near\ about\ disagree, near\ about\ slightly\ agree], [near\ about\ disagree, near\ about\ slightly\ disagree], [near\ about\ disagree, within\ disagree\ and\ slightly\ disagree]\Big\rangle$ i.e., semantically, $\Big\langle [s_{0.1}, s_{0.6}], [s_{0.1}, s_{0.2}], [s_{0.1}, s_{0.3}]\Big\rangle$. In this scenario, we consider a game, where two players, namely, mental health specialist (player I) and general physician (player II) are involved in a hospital. When one treats a mental-ill patient, other have nothing to do. So this situation can be seen as two-person zero-sum matrix game. Here, player I, i.e., mental health specialist, in time of conversation with guardian or friends of

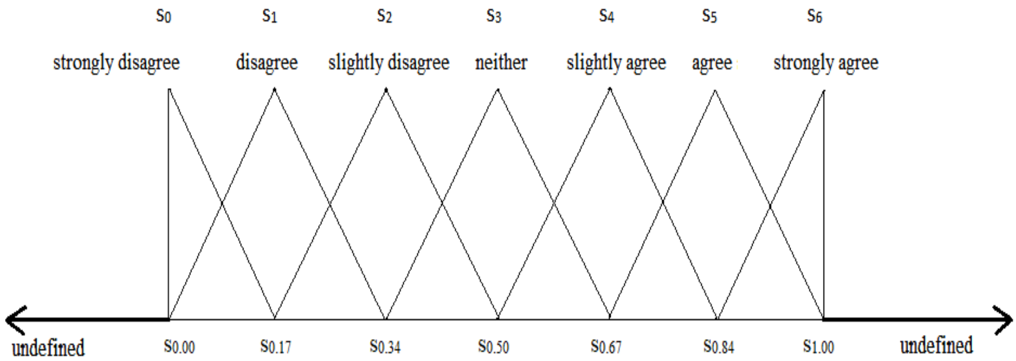

Figure 4.1: Linguistic term sets with semantics in course of patient's diagnosis.

patient, wants to know the causes of illness asking questions, considered as two strategies, α_1 : Beliefs and attitudes toward treatment behaviours due to biological changes in the brain causing depression and α_2 : Subjective social norms due to family-disappointment, whereas player II, i.e., general physician performs the diagnosis with the strategies β_1 : Beliefs and attitudes toward treatment behaviours due to biological changes in the brain causing depression and β_2 : Subjective social norms due to friends' embarrassment. In course of diagnosis, all outputs are in neutrosophic linguistic form. Here we consider the game problem in two separate linguistic environment, one with single-valued neutrosophic numbers and other for interval-valued neutrosophic numbers.

4.6.2 Problem environment 1: linguistic single-valued neutrosophic

The game problem in linguistic single-valued neutrosophic environment can be put in matrix form, as a matrix game $\breve{G}_1$:

$$\breve{G}_1 = \begin{array}{c c} & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{pmatrix} \breve{a}_{11} & \breve{a}_{12} \\ \breve{a}_{21} & \breve{a}_{22} \end{pmatrix} \end{array} = \begin{array}{c c} & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{pmatrix} \langle s_{0.4}, s_{0.2}, s_{0.3} \rangle & \langle s_{0.6}, s_{0.4}, s_{0.3} \rangle \\ \langle s_{0.4}, s_{0.4}, s_{0.3} \rangle & \langle s_{0.7}, s_{0.3}, s_{0.2} \rangle \end{pmatrix} \end{array}.$$

Here, $\breve{a}_{11} = \langle s_{0.4}, s_{0.2}, s_{0.3} \rangle, \breve{a}_{12} = \langle s_{0.6}, s_{0.4}, s_{0.3} \rangle, \breve{a}_{21} = \langle s_{0.4}, s_{0.4}, s_{0.3} \rangle$ and $\breve{a}_{22} = \langle s_{0.7}, s_{0.3}, s_{0.2} \rangle$. From $\breve{a}_{22}$, i.e., from $\langle s_{0.7}, s_{0.3}, s_{0.2} \rangle$, it can be easily described that when player I adopts strategy α_2 and player II uses strategy β_2, then due to factors or predictors of interrogation in course of diagnoses time, output is $\langle$near about agree, slightly disagree, near about disagree$\rangle$. This, in semantic, is put as $\langle s_{0.7}, s_{0.3}, s_{0.2} \rangle$. In elaborated form, we may say that, when mental health specialist assumes the health problem due to family-disappointment and general physician predicts the health problem of patient due to friends' embarrassment as a subjective social norm, the relatives or guardian of patient near about agree with doctor's diagnosis truly, slightly disagree for indeterminacy and near about disagree with a consideration that diagnosis as falsity with respect to predictors. Other payoff elements are considered similarly. Thus, we want to compute the linguistic neutrosophic strategy and optimal game value from the game-matrix with its zero-sum no saddle point concept.

4.6.3 Solution through algorithmic steps with results

Algorithm 3: Algorithm of solving game in linguistic single-valued neutrosophic environment

Input: Payoff matrix
Output: Optimal solutions
1 Firstly, payoff matrix is considered. (Here, as $\breve{G}_1$)
2 Secondly, consider the operational laws (as given in **Property 4.4.1**).
3 Finally, optimal strategies are obtained (using **Theorem 4.5.2**).

Solving the considered problem and using the proposed algorithm, we get,
$y_1^* = \frac{\langle 0.50,0.75,0.66\rangle}{\langle 0.25,0.37,0.66\rangle} = \langle \text{undefined}, 0.60, 0.00\rangle$, $y_2^* = \langle 1.00, 0.00, \text{undefined}\rangle$,
$z_1^* = \frac{\langle 0.25,0.75,0.66\rangle}{\langle 0.25,0.37,0.66\rangle} = \langle 1.00, 0.60, 0.00\rangle$, $z_2^* = \langle 1.00, 0.00,\text{undefined}\rangle$,
and $v^* = \frac{\langle 0.28,0.44,0.44\rangle - \langle 0.24,0.64,0.51\rangle}{\langle 0.25,0.37,0.66\rangle} = \langle 0.20, 0.49, 0.58\rangle$.
Here we achieve the optimal solutions of two-person 2×2 matrix game in linguistic term set based expression as: $y_1^* = \langle \text{undefined}, s_{0.60}, s_{0.00}\rangle$, $y_2^* = \langle s_{1.00}, s_{0.00}, \text{undefined}\rangle$, $z_1^* = \langle s_{1.00}, s_{0.60}, s_{0.00}\rangle$, $z_2^* = \langle s_{1.00}, s_{0.00}, \text{undefined}\rangle$, and $v^* = \langle s_{0.20}, s_{0.49}, s_{0.58}\rangle$. Thus the optimal outcome, figured from semantic scales to linguistic scale is: $y_1^* = \langle$undefined, near about slightly disagree, slightly disagree$\rangle$, $y_2^* = \langle$strongly agree, strongly disagree, undefined$\rangle$, $z_1^* = \langle$strongly agree, near about slightly agree, strongly disagree$\rangle$, $z_2^* = \langle$strongly agree, strongly disagree, undefined$\rangle$, and $v^* = \langle$near about disagree, neither, near about slightly agree$\rangle$. From this linguistic game value, it can be easily commented that, under diagnosis, truly, the imposed strategies face near about disagree situation; no indeterminacy exists about diagnosis-predictors; and shows a falsity towards strategies having some slightly agreeableness.

4.6.4 Problem environment 2: linguistic interval-valued neutrosophic

When the game problem is considered in linguistic interval-valued neutrosophic environment, then the matrix game can be described as, $\breve{G}_2 = \begin{array}{c c} & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{pmatrix} \breve{a}_{11} & \breve{a}_{12} \\ \breve{a}_{21} & \breve{a}_{22} \end{pmatrix} \end{array}$, i.e.,

$$\breve{G}_2 = \begin{array}{c c} & \begin{array}{cc} \beta_1 & \beta_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} & \begin{pmatrix} \langle [s_{0.1}, s_{0.6}], [s_{0.1}, s_{0.2}], [s_{0.1}, s_{0.3}]\rangle & \langle [s_{0.3}, s_{0.6}], [s_{0.2}, s_{0.3}], [s_{0.3}, s_{0.4}]\rangle \\ \langle [s_{0.4}, s_{0.5}], [s_{0.2}, s_{0.3}], [s_{0.3}, s_{0.4}]\rangle & \langle [s_{0.6}, s_{0.7}], [s_{0.3}, s_{0.4}], [s_{0.8}, s_{0.9}]\rangle \end{pmatrix} \end{array}.$$

Here, $\breve{a}_{11} = \langle [s_{0.1}, s_{0.6}], [s_{0.1}, s_{0.2}], [s_{0.1}, s_{0.3}]\rangle$ and similarly for others. From $\breve{a}_{22}$, i.e., from $\langle [s_{0.6}, s_{0.7}], [s_{0.3}, s_{0.4}], [s_{0.8}, s_{0.9}]\rangle$, it can be easily described that when player I adopts strategy α_2 and player II uses strategy β_2, then due to factors or predictors of interrogation in course of diagnosis time, output is $\langle$[near about slightly agree, slightly agree], [near about slightly disagree, slightly disagree], [near about agree, strongly agree]$\rangle$. This, in semantic, is put as

$\langle [s_{0.6}, s_{0.7}], [s_{0.3}, s_{0.4}], [s_{0.8}, s_{0.9}] \rangle$. In elaborated form, we may say that, when mental health specialist assumes the health problem due to family-disappointment and general physician predicts the health problem of patient due to friends' embarrassment as a subjective social norm, then the relatives or guardian of patient feel near about slightly agreeableness to slightly agreeableness with doctor's diagnosis truly, from near about almost slightly disagree to slightly disagree for indeterminacy and lie within almost agree to strongly agree about the consideration that diagnosis as falsity with respect to predictors. Others payoff elements are considered similarly.

4.6.5 Solution through algorithmic steps with results

Algorithm 4: Algorithm of solving game in linguistic interval-valued neutrosophic environment

---

Input: Payoff matrix
Output: Optimal solutions
1 Firstly, payoff matrix is considered. (Here, as $\breve{G}_2$)
2 Remembering the operational laws (as given in **Definition 4.3.7** with **Property 4.4.2**) perform corresponding operation.
3 Optimal strategies are achieved (using **Theorem 4.5.2**).

Solving the considered problem and using the proposed algorithm, we get the optimal solutions of two-person 2×2 matrix game in linguistic term set based interval neutrosophic expression as: $y_1^* = \langle$[undefined, $s_{1.00}$], [undefined, undefined], [undefined, undefined] $\rangle$, $y_2^* = \langle$[undefined, $s_{0.00}$], [undefined, undefined], [undefined, undefined]$\rangle$, $z_1^* = \langle$[undefined, $s_{0.43}$], [undefined, undefined], [undefined, undefined]$\rangle$, $z_2^* = \langle$[undefined, $s_{0.00}$], [undefined, undefined], [undefined, undefined]$\rangle$, and $v^* = \langle$[undefined, $s_{0.00}$], [undefined, undefined], [undefined, $s_{0.34}$]$\rangle$. The obtained results significantly show that in course of diagnosis of patient, interaction may truly affect the optimality with its highest value but indeterminacy and falsity cannot be attained, which is represented by undefined characteristics.

4.7 Conclusion

In this work, we have proposed linguistic neutrosophic environment, both single-valued and interval-valued, to solve two-person matrix game. For this purpose, we have considered neutrosophic characteristics, i.e., degree of acceptance, degree of rejection and degree of indeterminacy to judge the object's behaviour. In matrix game model, we have used the traditional approach as procedure of game solution. The development of such mathematical models used to stimulate diagnosis in medical sciences. Here, the problem and the corresponding solutions depict that when medical interactions are done in linguistic mode, the outputs have some true values, indeterminacy values and falsity values; and this is the main findings of this chapter.
We have reviewed a series of studies related to mental health problems. Characteristics of the proposed approach of our work have been summarized in Table 4.1. This study significantly shows that discussion on matrix game theory under neutrosophic environment has a significant effect in real-life problems like medical diagnosis, where treatment mostly depends upon linguistic behaviour.

Table 4.1: Comparative study with other literatures.

Literature	Application area	Domain representation	Computational methods	Domain of aggregated results	Exploitation
[58]	Mental illness	Genetic structure	Specific therapies	N/A	N/A
[74]	Stigma of mental illness	N/A	N/A	N/A	N/A
[89]	Mental disorder	Real number	Meta-analytic study	N/A	N/A
[143]	Diagnosis of depression	N/A	N/A	N/A	N/A
Our proposed work	Diagnosis and analysis of mental health problem	Neutrosophic, linguistic, interval numbers	Matrix method of Game theory	Neutrosophic, linguistic, interval numbers	Computing analysis