Chapter 3

Two-person zero-sum game in triangular type-2 intuitionistic fuzzy environment*

In this chapter, we consider two-person zero-sum game with payoffs as triangular type-2 intuitionistic fuzzy number (TT2IFN), i.e., we originate Triangular Type-2 Intuitionistic Fuzzy Matrix Game (TT2IFMG) as a new concept. A new ranking function is used to get relevant solutions of TT2IFMG. A real-life problematic situations are also taken for validating the proposed methodology.

3.1 Motivation

Water preservation plays the most vital role in maintenance of daily lives of all living creatures, in ensuring public health and acts as a pre-requisite for socio-economic growth and development of a country. It must be preserved as an integral part of the habitat of flora and fauna and the natural environment. Realizing this real-life situation, we extend the concept of triangular intuitionistic fuzzy number (from **Chapter 2**) into triangular type-2 intuitionistic fuzzy number with a new ranking function approach and then apply in game theory.

3.2 Introduction

Water is regarded as the greatest gift of nature. But it is unfortunate that we the humans conciously or unconciously are squandering this gift. Pollution of water, decrease in ground-water level, human waste, ill management of water-use, increase in infrastructural growth etc. have aggravated the situation. At present 2 million tons of sewage along with agricultural and industrial wastes are being discharged everyday to World's water and sixty percent of stream-flows of World's 227 biggest rivers are being disrupted by the constructions of dams and infrastructures causing radical decrease in sediment and nutrient transport to downstream stretches. All these ultimately reduce the water quality and damage the eco-system health. The waste-water produced annually, estimated by The United Nations World Water Assessment Programme 2003

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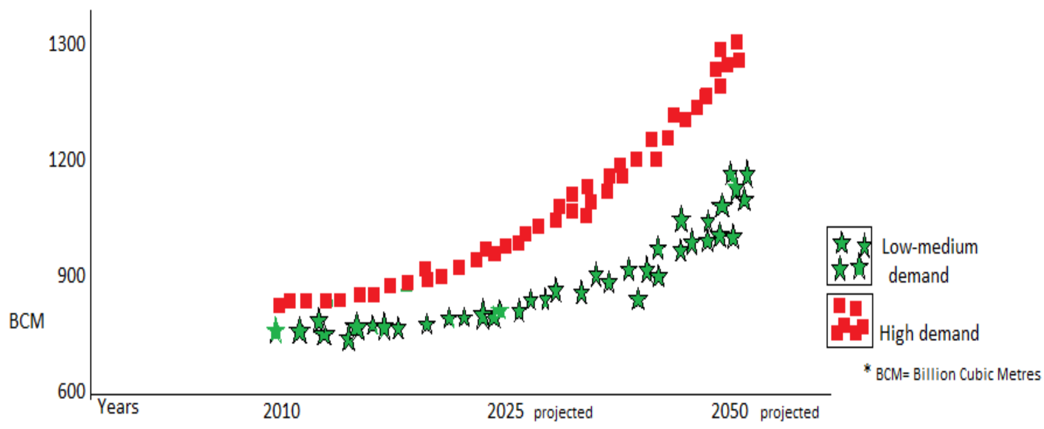

Figure 3.1: India's water demand (projected years 2025 and 2050).

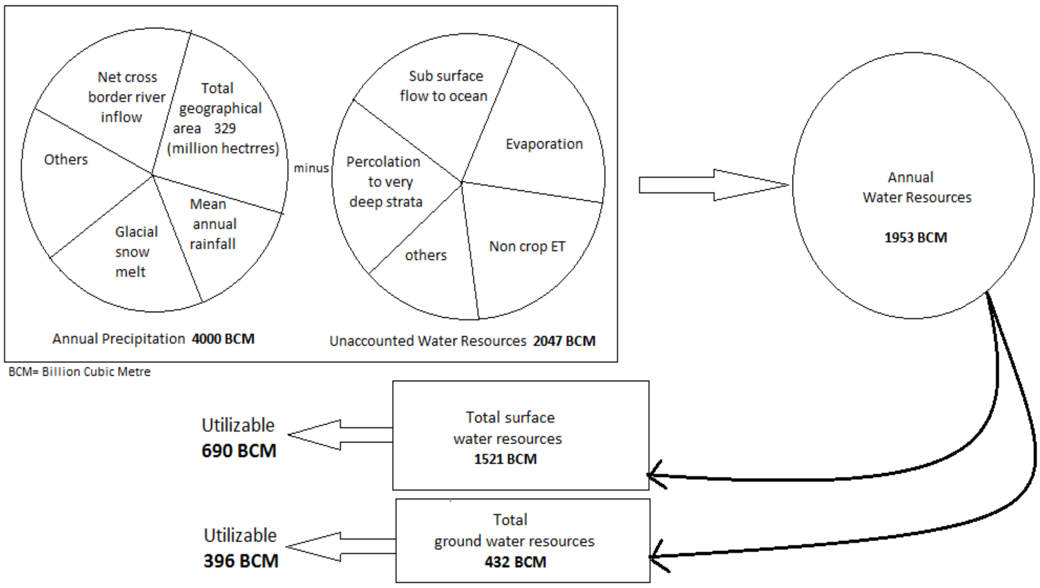

Figure 3.2: India's water resources (projected year 2025).

[UN-WWAP 2003] [172], amounts to 1500 cubic kilometres. Nearabout 70 million people of Bangladesh are exposed to ground-water arsenic and more than 140 million people from 70 countries are badly affected by ground-water arsenic pollution [UN-WWAP 2009] [172]. Excess or over-extraction of ground water has made the situation worse in the populated coastal areas all over the World by making the ground-water reserve saline below the sea-level upto to a few kilometres. In Chennai, in India, this extraction has made the ground-water saline nearly 10 kilometres inland from sea-level. So precautionary measures must be taken to prevent hazardous substances discharged by industries, pollution from surface and ground water and misuse of water. A proper skillfull eco-friendly intelligent and sustainable management of water only can serve the interest of the general public as well as that of the individuals. In India, water demand and supply are shown in Fig. 3.1 and Fig. 3.2. That the World rapidly approaches to a crisis in water is reflected in the management of its supply and access. At local level also the people are engaged in quarrel and sometimes resulting in loss of lives over the sharing of water, which means a chaotic situation arises. To get rid of such chaotic phenomenon we should strive for a stable management, mathematically which may be programmed or sketched as fuzzy game theory. Due to problem complexities, we consider two-fold uncertainties as type-2 fuzzy sense in triangular intuitionistic fuzzy environment and apply it in two-person zero-sum game phenomenon.

3.3 Basic Concepts

Fuzzy set of type-1 or simply the fuzzy set was due to Zadeh [167] which is the very beginning of fuzzy logic. Zadeh also logged the concept of type-2 fuzzy set.

Definition 3.3.1 ***Fuzzy set of type-2:*** *[96] A Type-2 Fuzzy Set $\tilde{\tilde{A}}$, denoted as T2FS, is characterized by a membership function as stated below:*

$$\mu_{\tilde{\tilde{A}}}(x) : X \times [0,1] \to [0,1], \text{ i.e., } \mu_{\tilde{\tilde{A}}}(x) : X \times U \to V.$$

Here, X is referred to the primary domain, U as the secondary domain of the T2FS and V is considered as the secondary membership of x, $x \in X$, i.e., we can write

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{A}}(x,u)) : \forall x \in X, \forall\, u \in J_x \subseteq [0,1]\}, \tag{3.1}$$

in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$. From Eq. (3.1), $\tilde{\tilde{A}}$ can also be expressed as

$$\begin{aligned} \tilde{\tilde{A}} &= \{\int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u), J_x \subseteq [0,1]\} \\ &= \{\int_{x \in X} \{\int_{u \in J_x} \mu_{\tilde{A}}(x,u)/u\}/x, J_x \subseteq [0,1]\}. \end{aligned} \tag{3.2}$$

where, x is the primary variable, J_x is its primary membership function, $J_x \subseteq [0,1], u$ is the secondary variable and $\int_{u \in J_x} \mu_{\tilde{A}}(x,u)/u$ is the secondary membership function. $\int \int$ denotes union over all admissible x and u. In case of universe of discrete discourses, $\int$ is replaced by $\sum$.

Example 3.3.1 *Assume*

$$\tilde{\tilde{A}} = \{(x, \mu_{\tilde{\tilde{A}}}(x)) : \forall\, x \in X\},\ X = \{10, 11, 12\} \tag{3.3}$$

and the primary memberships of X are respectively

$$\begin{cases} J_{10} = \{0.1,\ 0.2,\ 0.3\}, \\ J_{11} = \{0.4,\ 0.5,\ 0.6,\ 0.7\}, \\ J_{12} = \{0.8,\ 0.9,\ 0.1\}. \end{cases} \tag{3.4}$$

The secondary membership function of 10 is

$$\begin{cases} \mu_{\tilde{\tilde{A}}}(10) = \mu_{\tilde{A}}(10,\ u) = (0.6/0.1) + (1.0/0.2) + (0.7/0.3). \\ \mu_{\tilde{\tilde{A}}}(11) = \mu_{\tilde{A}}(11,\ u) = (0.7/0.4) + (0.8/0.5) + (0.6/0.6) + (0.3/0.7) \\ \mu_{\tilde{\tilde{A}}}(12) = \mu_{\tilde{A}}(12,\ u) = (0.3/0.8) + (0.4/0.9) + (0.7/0.1). \end{cases} \tag{3.5}$$

So, type-2 discrete fuzzy number $\tilde{\tilde{A}}$ is given by

$$\begin{aligned} \tilde{\tilde{A}} = (0.6/0.1)/10 + (1.0/0.2)/10 + (0.7/0.3)/10 + (0.7/0.4)/11 \\ +(0.8/0.5)/11 + (0.6/0.6)/11 + (0.3/0.7)/11 \\ +(0.3/0.8)/12 + (0.4/0.9)/12 + (0.7/0.1)/12. \end{aligned} \tag{3.6}$$

$\tilde{\tilde{A}}$ *can be written as*

$$\tilde{\tilde{A}} = \begin{cases} 10, & \textit{with membership } \mu_{\tilde{\tilde{A}}}(10), \\ 11, & \textit{with membership } \mu_{\tilde{\tilde{A}}}(11), \\ 12, & \textit{with membership } \mu_{\tilde{\tilde{A}}}(12). \end{cases} \tag{3.7}$$

This type-2 fuzzy set in discrete universe is shown in Fig. 3.3 (using Eqs. (3.6) and (3.7)).

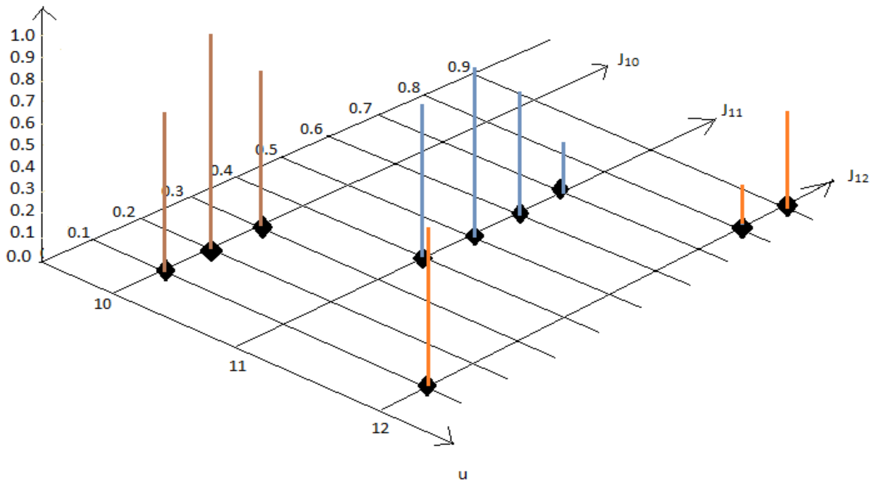

Figure 3.3: Type-2 fuzzy set (in discrete universe).

Here, we consider another form of TIFN (sometimes called Triangular Type-1 IFN) apart from **Definition 2.3.3 (Chapter 2)**.

Definition 3.3.2 ***Triangular IFN*** *[77]: An IF number $\hat{A} = \langle(\underline{\varphi}, \varphi, \overline{\varphi});\ \phi_{\hat{\varphi}},\ \Phi_{\hat{\varphi}}\rangle$ having membership and non-membership degrees of an element $x = \varphi$ in $\hat{A}$, being $\epsilon_{\hat{\varphi}}$ and $\rho_{\hat{\varphi}}$, is said to be triangular IFN if its membership and non-membership functions respectively are defined as pursues:*

$$\phi_{\hat{A}}(x) = \begin{cases} \epsilon_{\hat{\varphi}}\left(\frac{x-\underline{\varphi}}{\varphi-\underline{\varphi}}\right), & \textit{if } \underline{\varphi} \leq x < \varphi, \\ \epsilon_{\hat{\varphi}}, & \textit{if } x = \varphi, \\ \epsilon_{\hat{\varphi}}\left(\frac{\overline{\varphi}-x}{\overline{\varphi}-\varphi}\right), & \textit{if } \varphi < x \leq \overline{\varphi}, \\ 0, & \textit{if } x < \underline{\varphi} \textit{ or } x > \overline{\varphi}, \end{cases} \tag{3.8}$$

and

$$\Phi_{\hat{A}}(x) = \begin{cases} \frac{\varphi-x+\rho_{\hat{\varphi}}(x-\underline{\varphi})}{\varphi-\underline{\varphi}}, & \textit{if } \underline{\varphi} \leq x < \varphi, \\ \rho_{\hat{\varphi}}, & \textit{if } x = \varphi, \\ \frac{x-\varphi+\rho_{\hat{\varphi}}(\overline{\varphi}-x)}{\overline{\varphi}-\varphi}, & \textit{if } \varphi < x \leq \overline{\varphi}, \\ 1, & \textit{if } x < \underline{\varphi} \textit{ or } x > \overline{\varphi}. \end{cases} \tag{3.9}$$

Here, $\epsilon_{\hat{\varphi}}$ and $\rho_{\hat{\varphi}}$, in Eqs. (3.8) and (3.9), satisfy the conditions: $0 \leq \epsilon_{\hat{\varphi}} \leq 1$, $0 \leq \rho_{\hat{\varphi}} \leq 1$, $0 \leq \epsilon_{\hat{\varphi}} + \rho_{\hat{\varphi}} \leq 1$. Also, $\pi_{\hat{A}}(x) = 1 - \phi_{\hat{A}}(x) - \Phi_{\hat{A}}(x)$ is defined as intuitionistic fuzzy index of an element $x \in \hat{A}$.

Arithmetic Operations on Triangular IFNs:
Let $\hat{\varphi} = \langle(\underline{\varphi}, \varphi, \overline{\varphi}); \epsilon_{\hat{\varphi}}, \rho_{\hat{\varphi}}\rangle$ and $\hat{\vartheta} = \langle(\underline{\vartheta}, \vartheta, \overline{\vartheta}); \epsilon_{\hat{\vartheta}}, \rho_{\hat{\vartheta}}\rangle$ appear for two triangular IFNs, then the addition, subtraction, multiplication, division and scalar multiplication of the numbers are conveyed as below:

$$\text{Addition: } \hat{\varphi} \oplus \hat{\vartheta} = \langle(\underline{\varphi} + \underline{\vartheta}, \varphi + \vartheta, \overline{\varphi} + \overline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle. \tag{3.10}$$

$$\text{Substraction: } \hat{\varphi} \ominus \hat{\vartheta} = \langle(\underline{\varphi} - \overline{\vartheta}, \varphi - \vartheta, \overline{\varphi} - \underline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle. \tag{3.11}$$

$$\text{Multiplication: } \hat{\varphi} \otimes \hat{\vartheta} = \begin{cases} \langle(\underline{\varphi}\underline{\vartheta}, \varphi\vartheta, \overline{\varphi}\overline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle, & if\ \hat{\varphi} > 0,\ \hat{\vartheta} > 0, \\ \langle(\underline{\varphi}\overline{\vartheta}, \varphi\vartheta, \overline{\varphi}\underline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle, & if\ \hat{\varphi} < 0,\ \hat{\vartheta} > 0, \\ \langle(\overline{\varphi}\overline{\vartheta}, \varphi\vartheta, \underline{\varphi}\underline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle, & if\ \hat{\varphi} < 0,\ \hat{\vartheta} < 0. \end{cases} \tag{3.12}$$

$$\text{Division: } \hat{\varphi} \oslash \hat{\vartheta} = \begin{cases} \langle(\underline{\varphi}/\overline{\vartheta}, \varphi/\vartheta, \overline{\varphi}/\underline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle, & if\ \hat{\varphi} > 0,\ \hat{\vartheta} > 0, \\ \langle(\overline{\varphi}/\overline{\vartheta}, \varphi/\vartheta, \underline{\varphi}/\underline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle, & if\ \hat{\varphi} < 0,\ \hat{\vartheta} > 0, \\ \langle(\overline{\varphi}/\underline{\vartheta}, \varphi/\vartheta, \underline{\varphi}/\overline{\vartheta}); \epsilon_{\hat{\varphi}} \wedge \epsilon_{\hat{\vartheta}}, \rho_{\hat{\varphi}} \vee \rho_{\hat{\vartheta}}\rangle, & if\ \hat{\varphi} < 0,\ \hat{\vartheta} < 0. \end{cases} \tag{3.13}$$

where " $\wedge$ " and " $\vee$ " individually denote min and max operators in fuzzy sense.

Scalar Multiplication: For any real number k,

$$k\hat{\varphi} = \begin{cases} \langle(k\underline{\varphi}, k\varphi, k\overline{\varphi}); \epsilon_{\hat{\varphi}}, \rho_{\hat{\varphi}}\rangle, & \text{if } k \geq 0, \\ \langle(k\overline{\varphi}, k\varphi, k\underline{\varphi}); \epsilon_{\hat{\varphi}}, \rho_{\hat{\varphi}}\rangle, & \text{if } k < 0. \end{cases} \tag{3.14}$$

Here the assumption of $\hat{\varphi}$ (> 0 or < 0) are decided by the extension principle of fuzzy set [36].

3.4 Triangular Type-2 Intuitionistic Fuzzy Number (TT2IFN)

In this section of our study, the Triangular type-1 IFNs (Eq.(3.10) to Eq.(3.14)) have been extended to Triangular type-2 IFNs along with their definitions and arithmetic operations. In context of our problem emphasis has been laid on TT2IFN due to the characteristics of triangular type fuzzy numbers and also as because this type of fuzzy numbers can be used both in symmetric and asymmetric cases of fuzziness which is more aligned with real-life situations.

Definition 3.4.1 ***TT2IFN****: A triangular type-2 intuitionistic fuzzy number $\breve{A}$ on X is given by*

$$\breve{A} = \langle(\hat{A}_1, \hat{A}_2, \hat{A}_3); \mu_{\breve{A}}, \gamma_{\breve{A}}\rangle, \tag{3.15}$$

where $\hat{A}_1$, $\hat{A}_2$ and $\hat{A}_3$ are again triangular type-1 intuitionistic fuzzy numbers and $\mu_{\breve{A}}$ and $\gamma_{\breve{A}}$ represent the membership degree and the non-membership degree of $\breve{A}$ respectively. Therefore, we can write from Eq.(3.15),

$$\begin{aligned} \breve{A} &= (\langle(\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}); \omega_1, \omega_2\rangle) \\ &= \langle(\langle(\grave{a}_{ij}, a_{ij}, \acute{a}_{ij}); \mu_{\hat{a}_{ij}}, \gamma_{\hat{a}_{ij}}\rangle, \langle(\grave{b}_{ij}, b_{ij}, \acute{b}_{ij}); \mu_{\hat{b}_{ij}}, \gamma_{\hat{b}_{ij}}\rangle, \\ &\quad \langle(\grave{c}_{ij}, c_{ij}, \acute{c}_{ij}); \mu_{\hat{c}_{ij}}, \gamma_{\hat{c}_{ij}}\rangle); \omega_1, \omega_2\rangle \end{aligned} \tag{3.16}$$

where, $\omega_1 = \min\{\mu_{\hat{a}_{ij}}, \mu_{\hat{b}_{ij}}, \mu_{\hat{c}_{ij}}\}$, $\omega_2 = \max\{\gamma_{\hat{a}_{ij}}, \gamma_{\hat{b}_{ij}}, \gamma_{\hat{c}_{ij}}\}$ and ω_1, ω_2 respectively denote the degree of membership and degree of non-membership of $\breve{A}$ respectively.

Arithmetic Operations on TT2IFNs:
Assume that $\breve{A}$ and $\breve{B}$ are two TT2IFNs such that

$$\begin{aligned}\breve{A} &= \langle(\langle(\grave{a}_{11}, a_{11}, \acute{a}_{11}); \mu_{a_{11}}, \gamma_{a_{11}}\rangle, \langle(\grave{b}_{11}, b_{11}, \acute{b}_{11}); \mu_{b_{11}}, \gamma_{b_{11}}\rangle, \\ &\quad \langle(\grave{c}_{11}, c_{11}, \acute{c}_{11}); \mu_{c_{11}}, \gamma_{c_{11}}\rangle); \theta_1, \theta_2\rangle, \qquad (3.17)\\ \breve{B} &= \langle(\langle(\grave{a}_{22}, a_{22}, \acute{a}_{22}); \mu_{a_{22}}, \gamma_{a_{22}}\rangle, \langle(\grave{b}_{22}, b_{22}, \acute{b}_{22}); \mu_{b_{22}}, \gamma_{b_{22}}\rangle, \\ &\quad \langle(\grave{c}_{22}, c_{22}, \acute{c}_{22}); \mu_{c_{22}}, \gamma_{c_{22}}\rangle); \theta_3, \theta_4\rangle, \qquad (3.18)\end{aligned}$$

where $\theta_1 = \min\{\mu_{a_{11}}, \mu_{b_{11}}, \mu_{c_{11}}\}$, $\theta_2 = \max\{\gamma_{a_{11}}, \gamma_{b_{11}}, \gamma_{c_{11}}\}$
and $\theta_3 = \min\{\mu_{a_{22}}, \mu_{b_{22}}, \mu_{c_{22}}\}$, $\theta_4 = \max\{\gamma_{a_{22}}, \gamma_{b_{22}}, \gamma_{c_{22}}\}$,
then the addition, substraction and scalar multiplication of the numbers (taken from Eqs.(3.17) and (3.18)) are stated as below:

$$\begin{aligned}\text{Addition: } \breve{A} \oplus \breve{B} &= \langle(\langle(\grave{a}_{11} + \grave{a}_{22}, a_{11} + a_{22}, \acute{a}_{11} + \acute{a}_{22}); \mu_{a_{11}} \wedge \mu_{a_{22}}, \gamma_{a_{11}} \vee \gamma_{a_{22}}\rangle, \\ &\quad \langle(\grave{b}_{11} + \grave{b}_{22}, b_{11} + b_{22}, \acute{b}_{11} + \acute{b}_{22}); \mu_{b_{11}} \wedge \mu_{b_{22}}, \gamma_{b_{11}} \vee \gamma_{b_{22}}\rangle, \\ &\quad \langle(\grave{c}_{11} + \grave{c}_{22}, c_{11} + c_{22}, \acute{c}_{11} + \acute{c}_{22}); \mu_{c_{11}} \wedge \mu_{c_{22}}, \gamma_{c_{11}} \vee \gamma_{c_{22}}\rangle); \delta_1, \delta_2\rangle, \qquad (3.19)\end{aligned}$$

where $\delta_1 = \min\{\mu_{a_{11}} \wedge \mu_{a_{22}}, \mu_{b_{11}} \wedge \mu_{b_{22}}, \mu_{c_{11}} \wedge \mu_{c_{22}}\}$,
$\delta_2 = \max\{\gamma_{a_{11}} \vee \gamma_{a_{22}}, \gamma_{b_{11}} \vee \gamma_{b_{22}}, \gamma_{c_{11}} \vee \gamma_{c_{22}}\}$.

$$\begin{aligned}\text{Subtraction: } \breve{A} \ominus \breve{B} &= \langle(\langle(\grave{a}_{11} - \acute{c}_{22}, a_{11} - c_{22}, \acute{a}_{11} - \grave{c}_{22}); \mu_{a_{11}} \wedge \mu_{c_{22}}, \gamma_{a_{11}} \vee \gamma_{c_{22}}\rangle, \\ &\quad \langle(\grave{b}_{11} - \acute{b}_{22}, b_{11} - b_{22}, \acute{b}_{11} - \grave{b}_{22}); \mu_{b_{11}} \wedge \mu_{b_{22}}, \gamma_{b_{11}} \vee \gamma_{b_{22}}\rangle, \\ &\quad \langle(\grave{c}_{11} - \acute{a}_{22}, c_{11} - a_{22}, \acute{c}_{11} - \grave{a}_{22}); \mu_{c_{11}} \wedge \mu_{a_{22}}, \gamma_{c_{11}} \vee \gamma_{a_{22}}\rangle); \delta_3, \delta_4\rangle, \qquad (3.20)\end{aligned}$$

where $\delta_3 = \min\{\mu_{a_{11}} \wedge \mu_{c_{22}}, \mu_{b_{11}} \wedge \mu_{b_{22}}, \mu_{c_{11}} \wedge \mu_{a_{22}}\}$,
$\delta_4 = \max\{\gamma_{a_{11}} \vee \gamma_{c_{22}}, \gamma_{b_{11}} \vee \gamma_{b_{22}}, \gamma_{c_{11}} \vee \gamma_{a_{22}}\}$.

Scalar Multiplication: For any real number k,

$$k\breve{A} = \begin{cases} \langle(\langle(k\grave{a}_{11}, ka_{11}, k\acute{a}_{11}); \mu_{a_{11}}, \gamma_{a_{11}}\rangle, \langle(k\grave{b}_{11}, kb_{11}, k\acute{b}_{11}); \mu_{b_{11}}, \gamma_{b_{11}}\rangle, \\ \langle(k\grave{c}_{11}, kc_{11}, k\acute{c}_{11}); \mu_{c_{11}}, \gamma_{c_{11}}\rangle); \theta_1, \theta_2\rangle, \quad \text{if } k \geq 0, \\ \langle(\langle(k\acute{c}_{11}, kc_{11}, k\grave{c}_{11}); \mu_{c_{11}}, \gamma_{c_{11}}\rangle, \langle(k\acute{b}_{11}, kb_{11}, k\grave{b}_{11}); \mu_{b_{11}}, \gamma_{b_{11}}\rangle, \\ \langle(k\acute{a}_{11}, ka_{11}, k\grave{a}_{11}); \mu_{a_{11}}, \gamma_{a_{11}}\rangle); \theta_1, \theta_2\rangle, \quad \text{if } k < 0. \end{cases}$$

3.5 Ranking Function

In the real-life problems and their solutions, the comparison or ranking of fuzzy numbers plays a significant role. For constructing mathematical models and methods, ranking/comparison among the fuzzy variables and parameters are required to be manipulated, though it is not an easy task as the fuzzy numbers are not ordered, and overlapping comes to happen. Various fuzzy ranking methods such as methods by integration, linguistic approach method, methods using α-cut and probability concept are being applied to address such complex real-life situations.
Here, an efficient technique for ordering, i.e., comparing the triangular type-2 intuitionistic fuzzy numbers is followed by the implementation of a new ranking function, as $\mathfrak{R}(\breve{A})$, which maps triangular type-2 intuitionistic fuzzy numbers to real line, i.e., $\mathfrak{R} : \mathbb{F}(\breve{A}) \rightarrow \mathbb{R}$ where $\mathbb{F}(\breve{A})$ is the

collection of all TT2IFNs and $\mathbb{R}$ is the set of real numbers, is defined as follows:

$$\mathfrak{R}(\breve{A}) = \left(\frac{\omega_1 + \omega_2}{2}\right)\left(\frac{1}{3}\right)\left(\frac{\grave{a}_{ij} + \grave{b}_{ij} + \grave{c}_{ij}}{3} + \frac{a_{ij} + b_{ij} + c_{ij}}{3} + \frac{\acute{a}_{ij} + \acute{b}_{ij} + \acute{c}_{ij}}{3}\right), \qquad (3.21)$$

where, $\breve{A}$ is expressed by the Eq. (3.16).

This satisfies linearity and additive properties and provides results associated with practical human intuition.

Let $\breve{A}$ and $\breve{B}$ be two triangular type-2 intuitionistic fuzzy numbers. Then the following comparison are taken into consideration.

Case (i): $\mathfrak{R}(\breve{A}) > \mathfrak{R}(\breve{B}) \Rightarrow \breve{A} >_{\mathfrak{R}} \breve{B}$, i.e., min$\{\breve{A}, \breve{B}\} = \breve{B}$,

Case (ii): $\mathfrak{R}(\breve{A}) < \mathfrak{R}(\breve{B}) \Rightarrow \breve{A} <_{\mathfrak{R}} \breve{B}$, i.e., min$\{\breve{A}, \breve{B}\} = \breve{A}$,

Case (iii): $\mathfrak{R}(\breve{A}) = \mathfrak{R}(\breve{B}) \Rightarrow \breve{A} =_{\mathfrak{R}} \breve{B}$, i.e., min$\{\breve{A}, \breve{B}\} = \breve{A} = \breve{B}$.

Here simple type linear ranking functions have been used to compare TT2IFNs with its converted crisp numbers and we have been able to overcome the time-complexity and the complexities arose out of getting results directly from the real-world problems which are common to type-2 fuzzy numbers owing to its two-fold characteristics.

3.6 Mathematical Model

Matrix game with TT2IFNs and its solutions are discussed in the following subsections.

3.6.1 Triangular Type-2 Intuitionistic Fuzzy Matrix Game (TT2IFMG)

When the game is concerned with real-world situation, payoff elements of the matrix game are not obtained in precise form due to the inaccuracy of the available information occurred in the system. As a more general case of fuzzy sets, triangular type-2 intuitionistic fuzzy sets can be used in matrix games to deal with the fuzziness of information; and we call them triangular type-2 intuitionistic fuzzy payoff matrix, exhibited as $\breve{A} = (\breve{a}_{ij})$ where $(\breve{a}_{ij}) = (\langle(\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}); \omega_1, \omega_2\rangle)$ $(i = 1, 2, \ldots, p;\ j = 1, 2, \ldots, q)$ where $\hat{a}_{ij} = \langle(\grave{a}_{ij}, a_{ij}, \acute{a}_{ij}); \mu_{\hat{a}_{ij}}, \gamma_{\hat{a}_{ij}}\rangle$, and

$$\omega_1 = \min_{ij}\{\mu_{\hat{a}_{ij}}, \mu_{\hat{b}_{ij}}, \mu_{\hat{c}_{ij}}\},\ \forall\, i,\ j, \qquad (3.22)$$

$$\omega_2 = \max_{ij}\{\gamma_{\hat{a}_{ij}}, \gamma_{\hat{b}_{ij}}, \gamma_{\hat{c}_{ij}}\},\ \forall\, i,\ j, \qquad (3.23)$$

and the game is considered as $G \equiv (Y, Z, \breve{A})$.

The value of the game is assigned to the maximum guaranteed gain to the maximizing player I or the minimum possible loss to the minimizing player II, which is governed by the maximin or minimax principle as discussed earlier.

If (m, n)-th position of the payoff matrix $\breve{A} = (\breve{a}_{ij})_{p\times q} = (\langle(\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}); \omega_1, \omega_2\rangle)_{p\times q}$ is a saddle point then,

$$\langle(\hat{a}_{mn}, \hat{b}_{mn}, \hat{c}_{mn}); \omega_1, \omega_2\rangle = \max_i\{\min_j\langle(\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}); \omega_1, \omega_2\rangle\} \qquad (3.24)$$

$$= \min_j\{\max_i\langle(\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}); \omega_1, \omega_2\rangle\}, \qquad (3.25)$$

and this suggests that the saddle point entry is, (m, n)-th, value of the game, where ω_1 and ω_2 respectively denote the degree of acceptance and degree of non-acceptance of $(\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij})$.

Definition 3.6.1 ***Solution of TT2IFMG:*** *If there exist $\breve{v}^* \in V$ and $\breve{w}^* \in W$ and if there exist no* *other $\breve{v}$ and $\breve{w}$ such that $\breve{v}^* \breve{\leq} \breve{v}$ and $\breve{w}^* \breve{\geq} \breve{w}$, where V and W finger the sets of all reasonable game values $\breve{v}$ and $\breve{w}$ for players I and II respectively, then $(y^*, z^*, \breve{v}^*, \breve{w}^*)$ is called a solution of Triangular Type-2 Intuitionistic Fuzzy Matrix Game (TT2IFMG).*

Definition 3.6.2 ***Fuzzy value of TT2IFMG:*** *y^* and z^* are called the maximin and minimax strategies for players I and II respectively and $\breve{v}^*$ and $\breve{w}^*$ are called respectively player I's gain-floor and player II's loss-ceiling. Let $\breve{\nu}^* = \breve{v}^* \wedge \breve{w}^*$ with the membership function $\mu_{\breve{\nu}^*}(x) = \min\{\mu_{\breve{v}^*}(x), \mu_{\breve{w}^*}(x)\}$. Then $\breve{\nu}^*$ is called a fuzzy value of TT2IFMG.*

Now y^* and z^*, maximin and minimax strategies for players I and II respectively are obtained by solving the following fuzzy mathematical programming problems:

$$\begin{aligned} &\text{maximize} \quad \breve{v}, \\ &\text{subject to} \quad y^T \breve{A} z \breve{\geq} \breve{v}, \ \forall z \in Z, \\ &\qquad y \in Y, \ \ \breve{v} \in TT2IFN(\mathbb{R}), \end{aligned} \tag{3.26}$$

$$\begin{aligned} \text{and} \quad &\text{minimize} \quad \breve{w}, \\ &\text{subject to} \quad y^T \breve{A} z \breve{\leq} \breve{w}, \ \forall y \in Y, \\ &\qquad z \in Z, \ \ \breve{w} \in TT2IFN(\mathbb{R}). \end{aligned} \tag{3.27}$$

where, '$\breve{\geq}$' and '$\breve{\leq}$' denote intuitionistic fuzzy inequalities and 'TT2IFN($\mathbb{R}$)' is the abridged form of Triangular Type-2 Intuitionistic Fuzzy Number whose entries are real numbers. Then the expected payoff $E(\breve{A})$ to the player I of the game is given by

$$\begin{aligned} E(\breve{A}) &= y^T \breve{A} z \\ &= \sum_{i=1}^{p} \sum_{j=1}^{q} \breve{\breve{a}}_{ij} y_i z_j \\ &= \sum_{i=1}^{p} \sum_{j=1}^{q} \langle (\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}); \omega_1, \omega_2 \rangle y_i z_j \\ &= \langle (\sum_{i=1}^{p} \sum_{j=1}^{q} \hat{a}_{ij} y_i z_j, \ \sum_{i=1}^{p} \sum_{j=1}^{q} \hat{b}_{ij} y_i z_j, \ \sum_{i=1}^{p} \sum_{j=1}^{q} \hat{c}_{ij} y_i z_j); \ \omega_1, \ \omega_2 \rangle. \end{aligned}$$

Similarly,

$$\begin{aligned} E(-\breve{A}) &= y^T (-\breve{A}) z \\ &= \sum_{i=1}^{p} \sum_{j=1}^{q} (-\breve{\breve{a}}_{ij}) y_i z_j \\ &= \sum_{i=1}^{p} \sum_{j=1}^{q} \langle -(\hat{a}_{ij}, \hat{b}_{ij}, \hat{c}_{ij}); \omega_1, \omega_2 \rangle y_i z_j \\ &= \langle (-\sum_{i=1}^{p} \sum_{j=1}^{q} \hat{c}_{ij} y_i z_j, \ -\sum_{i=1}^{p} \sum_{j=1}^{q} \hat{b}_{ij} y_i z_j, \ -\sum_{i=1}^{p} \sum_{j=1}^{q} \hat{a}_{ij} y_i z_j); \\ &\quad \omega_1, \ \omega_2 \rangle. \end{aligned}$$

Expressions of $E(\breve{A})$ and $E(-\breve{A})$ are due to TT2IFNs.

3.6.2 Pragmatic solutions and strategies

Assume $\breve{v} = \langle(\underline{\hat{v}}, \hat{v}, \hat{\overline{v}}); \breve{v}_1, \breve{v}_2\rangle \in TT2IFN(\mathbb{R})$ and $\breve{w} = \langle(\underline{\hat{w}}, \hat{w}, \hat{\overline{w}}); \breve{w}_3, \breve{w}_4\rangle \in TT2IFN(\mathbb{R})$ be two triangular type-2 intuitionistic fuzzy numbers. Suppose that there exist $y^* \in Y$, $z^* \in Z$, then $(y^*, z^*, \breve{v}, \breve{w})$ is called a pragmatic or reasonable (rationally, highly acceptable than others in the correspondence situation) solution of the matrix game and for any $y^* \in Y$ and $z^* \in Z$, $y^{*T}\breve{A}z \gtrsim \breve{v}$ and $y^T\breve{A}z^* \lesssim \breve{w}$. If $(y^*, z^*, \breve{v}, \breve{w})$ is a pragmatic or reasonable solution of the TT2IFMG, then $\breve{v}$ and $\breve{w}$ are called pragmatic or reasonable values for players I and II respectively. Similarly y^* and z^* are called pragmatic or reasonable strategies for players I and II respectively.

Theorem 3.6.1 *If a payoff matrix with TT2IFN as payoff has at (m, n)-th position the value of the game as $\breve{a}_{mn}$, then after defuzzification with the help of ranking function $\mathfrak{R}$, the value of the game is $\mathfrak{R}(\breve{a}_{mn})$ at (m, n)-th position.*

Proof: The proof of the theorem can be obtained easily as the proof of the **Theorem 2.4.1**. □

Theorem 3.6.2 *If (y^*, z^*) be the solution of the payoff matrix with mixed strategies then (y^*, z^*) is also the solution of the payoff matrix after defuzzification by the ranking $\mathfrak{R}(\breve{a}_{ij})$.*

Proof: This theorem can be easily proved as an extension of **Theorem 2.4.2** in TT2IFN based environment. □

3.6.3 Algorithm for solving TT2IFMG

We consider the fuzzy matrix game with payoff as triangular type-2 intuitionistic fuzzy number. Next, we use our proposed ranking function, and get the corresponding defuzzified crisp matrix of game problem. Thus, we solve the game problem using the following algorithm.

Algorithm 2: Algorithm for solving TT2IFMG.

Input: Fuzzy matrix games with payoffs as triangular type-2 intuitionistic form
Output: Optimal solutions

1. Defuzzified form (crisp equivalence) of TT2IFMG is evaluated.
2. Checking the saddle point using maximin-minimax criterion based principle [113] is done:
   (I) If the saddle point exists then solution of the game from the defuzzified matrix is obtained.
   (II) Otherwise, using mixed strategies method [113], solution of the game is achieved.
3. The defuzzified solutions of the game arise.
4. Using problem's environment, required solutions are attained.

3.7 Numerical Illustration

We consider here a computational simulation from the problem of water management, a real-life problem.

3.7.1 Water Management Problem

Safe water and clean air are considered as the birth rights of humans. In the one hand, World's population increases exponentially and on the other, much of its population can not access to safe water. Two-third of Earth's surface is covered by water, but mostly salt-water. Only three percent of Earth's surface are covered by fresh-water but mostly found in Antarctic and polar regions in frozen form. Underground water, water available in lakes and coming from rivers support to human consumable water, but that in total amounts to only one percent of all water on the Earth. So an intelligent management for using, preserving and recycling/re-using the water is a crying need of the day.

Why water shortage:

The following three factors may be marked as the major reasons for shortage of fresh water in the World.

- Withdrawals of ground water have doubled in the last 40 to 50 years, due to the unscientific and inefficient irrigation practices promoted by modernisation. These practices have accelerated the rapid degradation of soil quality and reduced farm productivity.
- Uncontrolled rise in the population of World, mainly in the developing countries, has sharply decreased the amount of per person per year allotment of water from about 8000 cubic metres to about 5000 cubic metres [173; 174].
- The unequal accessing of water is another important cause of fresh-water shortage. City and town dwellers use more water than their village counterparts. Even the developed countries have wider access to water in households and industry rather than agriculture in comparison to the developing and underdeveloped countries.

Moreover, global warming, change of climate, lack of proper management for preservation and use of fresh water etc. are also responsible for water shortage.

Water requirement by sector:

Though irrigation is estimated to be the major consumer of water, water required in energy, industry and domestic sectors is no less. Industrial requirement in water is calculated to be increased by more than five times during the next 40 years, from 12 Billion Cubic Meters (BCM) in 2010 to 63 BCM in 2050. Domestic requirement by this time is estimated to nearly double (56 BCM in 2010 to 102 BCM in 2050) [173; 174].

What we must-to-do:

99% of World water reserve lies in the oceans, ice, mostly saline and atmospheric water, which is unworthy of human-uses. Much of the remaining 1% is stored in rivers, lakes and wells. Surface water sources such as rivers and lakes constitutes only 0.0067% of total water reserve. The investment for rural drinking water only has increased from Rs. 2425 crores in the VI Plan (1980-1985) to Rs. 88490 crores in the XI Plan [177]. Similar escalation of costs is noticed in irrigation, industrial and domestic sectors. Water resource cost is calculated by the electricity-use [174]. Recovery costs of water during cleaning of ground water mainly river water are extremely high. The Ganga river basin, in India, a homeland of nearly 400 million people of five states, and with more than dozens of thickly populated towns on its bank and open sewage systems towards the river, is classified as high alarming to the environment. The World Bank has invested Rs.

6700 crores in loans and grants for purification of its water. In urban areas, water recovery cost from polluted resources added with transportation cost amount to crores and crores of rupees. The World population is expected to increase from 6.2 billion in 2000 to 10 billion by 2050, the developing countries seem to be worst sufferers of water stress. So, to meet the increasing demand of water, water conservation and recycling of this vital source must be increased. We may adopt the following practices which will minimize ecological and environmental hazards and fulfill the increasing demand of safe and fresh water for a better living and better civilization on this water-friendly planet. These may be mentioned as,

1. Search for new/alternative sources of water and recycling/re-use of water is needed mostly. For this, energy-efficient methods, de-salinization, reverse osmosis process are to be nurtured. Recycling might be applied to terraced farming by using water flowing from higher terrain to the irrigation in lower terrain.

2. To ensure distribution and access to safe water to every human for fulfilling his/her minimum requirement in minimum cost, plugging of leaks from tanks, pipelines and taps must be done with constant vigilance.

3. As both the developing and developed countries use water more on agriculture and industry, ways and techniques to cultivate crops which require less water should be generated and thereby reduce the demand of water. For this food habits should be changed.

Government and non-government organizations (GOs and NGOs) should come forward towards the implementation of these must-to-do practices to prevent wastage of water. Awareness building measures should be taken for the common people working in agriculture and industry for sustainable implementation of these must-to-do practices.
To overcome these problems GOs and NGOs may adopt the following strategies as,

A_1**.** Encouragement of political commitment through effective policy formulation, support for the implementation of plans, and improved budgetary allocations (considering 1, 2 and 3 of the above must-to-do).

A_2**.** Promotion of intersector coordination and cooperation to forge a policy consensus for the promotion of safe water, facilitation of access to appropriate technologies and development of legal and regulatory frameworks for private industry and non-governmental participation (remembering 1, 2 and 3 of the above must-to-do).

A_3**.** Involvement of women in water-supply activities by identifying women's groups and movements at all levels and incorporating their views into community-level water-harvesting, recycling, saving, purifying, supplying, etc (taking into account 1, 2 and 3 of the above must-to-do).

Whereas, common people's strategies are:

B_1**.** To cultivate for lives (in the light of 1, 2 and 3 of the above must-to-do).

B_2**.** To work in industry for lives (bearing in mind 1, 2 and 3 of the above must-to-do).

B_3**.** To access water wilfully (mindful of 1, 2 and 3 of the above must-to-do).

So, it is obvious from the discussions made above that the ground water or fresh water management is not an easy task to perform. A real-life problem appears when the GOs and NGOs, the service providers, are considered in one side and the common people, the consumers, on the other, as both are subjected to some constraints. This real-life situation may be regarded as a real-life game problem where GOs and NGOs may be considered as player I and the common people as player II. Here player I (GOs and NGOs) tries to maximize its agenda whereas player II (the common people) wants to live with minimum efforts and minimum costs. In this conflicting situation every strategy of each player has three sub-strategies. We consider this problem as a type-2 fuzzy environment with degrees of acceptance and non-acceptance of strategies from both ends. We introduce a triangular type-2 intuitionistic fuzzy payoff matrix $P_{TT2IFPM}$ as,

$$P_{TT2IFPM} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} \check{C}_{11} & \check{C}_{12} & \check{C}_{13} \\ \check{C}_{21} & \check{C}_{22} & \check{C}_{23} \\ \check{C}_{31} & \check{C}_{32} & \check{C}_{33} \end{pmatrix} \end{array},$$

where,

$$\begin{cases} \check{C}_{11} = \langle(\langle(165,170,180);0.65,0.1\rangle,\langle(175,180,190);0.6,0.2\rangle,\langle(185,190,200);0.7,0.2\rangle);0.6,0.2\rangle \\ \check{C}_{12} = \langle(\langle(145,151,153);0.7,0.1\rangle,\langle(150,156,158);0.6,0.4\rangle,\langle(155,161,163);0.6,0.3\rangle);0.6,0.3\rangle \\ \check{C}_{13} = \langle(\langle(110,140,170);0.8,0.1\rangle,\langle(120,150,180);0.75,0.1\rangle,\langle(130,160,190);0.8,0.2\rangle);0.75,0.2\rangle \\ \check{C}_{21} = \langle(\langle(75,86,90);0.8,0.2\rangle,\langle(80,90,100);0.85,0.1\rangle,\langle(85,94,110);0.9,0.1\rangle);0.8,0.2\rangle \\ \check{C}_{22} = \langle(\langle(165,170,180);0.65,0.1\rangle,\langle(175,180,190);0.6,0.2\rangle,\langle(185,190,200);0.7,0.2\rangle);0.6,0.2\rangle \\ \check{C}_{23} = \langle(\langle(160,165,175);0.8,0.1\rangle,\langle(165,170,180);0.6,0.3\rangle,\langle(170,175,185);0.7,0.2\rangle);0.6,0.3\rangle \\ \check{C}_{31} = \langle(\langle(145,155,165);0.5,0.5\rangle,\langle(150,160,180);0.4,0.5\rangle,\langle(155,165,195);0.5,0.4\rangle);0.4,0.5\rangle \\ \check{C}_{32} = \langle(\langle(160,165,175);0.8,0.1\rangle,\langle(165,170,180);0.6,0.3\rangle,\langle(170,175,185);0.7,0.2\rangle);0.6,0.3\rangle \\ \check{C}_{33} = \langle(\langle(90,100,110);0.7,0.1\rangle,\langle(100,110,120);0.8,0.2\rangle,\langle(110,120,130);0.8,0.1\rangle);0.7,0.2\rangle. \end{cases}$$

Here, the cell position $\check{C}_{12}$ shows payoff $\langle(\langle(145,151,153);0.7,0.1\rangle,\langle(150,156,158);0.6,0.4\rangle,$ $\langle(155,161,163);0.6,0.3\rangle);0.6,0.3\rangle$ which indicates that when the players I and II, i.e., government and common people use alternatives A_1 and B_2 respectively then we say that after government's political commitment through effective policy formulation, supporting for the implementation of plans, and improving budgetary allocations, and common people's more wish to work in industry for lives, seeking new sources of water and recycling the water are 151 percent with minimum 145 percent and maximum 153 percent having acceptance 0.7 percent, non-acceptance 0.1 percent; distribution and accessing of water in waste-free and inexpensive manner is 156 percent with minimum 150 percent and maximum 158 percent having acceptance 0.6 percent, non-acceptance 0.4 percent and reduction of the demand of water is 161 percent with minimum 155 percent and maximum 163 percent having acceptance 0.6 percent, non-acceptance 0.3 percent. With an overall acceptance 0.6 percent and non-acceptance as 0.3 percent; $\check{C}_{12}$ represents the payoff when government and non-government organizations choose their strategies A_1 and common people choose their strategies B_2, both considering their must-to-do practices. Similarly, other cells represent the payoffs.

Now using the ranking function, given by Eq.(3.21), we get $\Re(\check{C}_{12})$ which is shown as below:

$$\begin{aligned}\Re(\check{C}_{12}) &= \left(\frac{0.6+0.3}{2}\right)\left(\frac{1}{3}\right)\left(\frac{145+150+155}{3}+\frac{151+156+161}{3}+\frac{153+158+163}{3}\right)\\ &= 69.6000002 \qquad (3.28)\end{aligned}$$

Similarly, we calculate other indexes from $P_{TT2IFPM}$ as shown below:
$\Re(\check{C}_{11}) = 72.6666667$, $\Re(\check{C}_{13}) = 71.2500000$, $\Re(\check{C}_{21}) = 45.0000000$, $\Re(\check{C}_{22}) = 72.6666667$, $\Re(\check{C}_{23}) = 77.2500000$, $\Re(\check{C}_{31}) = 73.5000000$, $\Re(\check{C}_{32}) = 77.2500000$, $\Re(\check{C}_{33}) = 49.5000000$, and then the defuzzified payoff matrix Q_{DPM} is described as:

$$Q_{DPM} = \begin{matrix} & B_1 & B_2 & B_3 \\ A_1 & 72.6666667 & 69.6000002 & 71.2500000 \\ A_2 & 45.0000000 & 72.6666667 & 77.2500000 \\ A_3 & 73.5000000 & 77.2500000 & 49.5000000 \end{matrix}.$$

Since maximin-minimax criterion implies the non-existence of saddle point, we use mixed strategy method by solving strategically the matrix game and we get (applying Eqs.(3.26-3.27), **Theorem 3.6.1** and **Theorem 3.6.2**), using LINGO with a 32-bit machine, from the following pairs of linear programming problems where we search for player I, the strategies $(x_1, x_2, x_3), x_i \geq 0, \sum x_i = 1, x_i = p_i.v, v = \frac{1}{\sum p_i}, i = 1, 2, 3$.

$$\begin{aligned} \text{minimize} \quad & p_1 + p_2 + p_3, \\ \text{subject to} \quad & 72.6666667p_1 + 45.0000000p_2 + 73.5000000p_3 \geq 1, \\ & 69.6000002p_1 + 72.6666667p_2 + 77.2500000p_3 \geq 1, \qquad (3.29)\\ & 71.2500000p_1 + 77.2500000p_2 + 49.5000000p_3 \geq 1, \\ & p_1, p_2, p_3 \geq 0. \end{aligned}$$

And for player II, the strategies $(y_1, y_2, y_3),\ y_j \geq 0,\ \sum y_j = 1, y_j = q_j.w, w = \frac{1}{\sum q_j}, j = 1, 2, 3.$

$$\begin{aligned} \text{maximize} \quad & q_1 + q_2 + q_3, \\ \text{subject to} \quad & 72.6666667q_1 + 69.6000002q_2 + 71.2500000q_3 \leq 1, \\ & 45.0000000q_1 + 72.6666667q_2 + 77.2500000q_3 \leq 1, \qquad (3.30)\\ & 73.5000000q_1 + 77.2500000q_2 + 49.5000000q_3 \leq 1, \\ & q_1, q_2, q_3 \geq 0. \end{aligned}$$

Consequently, we get the optimal strategies as indicated below:

$$(x_1^*, x_2^*, x_3^*) = (0.8499083, 0.0854442, 0.0646475), \qquad (3.31)$$

$$(y_1^*, y_2^*, y_3^*) = (0.1219201, 0.6461442, 0.2319357). \qquad (3.32)$$

The defuzzified value of the game is $v = 70.3565883 = w$ and the value of the game as TT2IFN is given by $(0.8499083) \times \check{C}_{11} + (0.0854442) \times \check{C}_{21} + (0.0646475) \times \check{C}_{31}$
$= \langle(\langle(156.0170720,\ 161.8529747,\ 171.3403095); 0.5,\ 0.5\rangle, \langle(165.2666135,\ 171.0170730,\ 181.6635470); 0.4,\ 0.5\rangle, \langle(174.5161550,\ 180.1811693,\ 191.9867845); 0.5,\ 0.4\rangle); 0.4, 0.5\rangle$,
and this indicates that the seeking of new sources of water and recycling the water are 161.8529747 percent with minimum 156.0170720 percent and maximum 171.3403095 percent having acceptance 0.5 percent, non-acceptance 0.5 percent; distribution and accessing of water in waste-free and inexpensive manner is 171.0170730 percent with minimum 165.2666135 percent and maximum 181.6635470 percent having acceptance 0.4 percent, non-acceptance 0.5 percent and reduction of the demand of water is 180.1811693 percent with minimum 174.5161550 percent and

maximum 191.9867845 percent having acceptance 0.5 percent, non-acceptance 0.4 percent. And overall percentage of acceptance and non-acceptance of must-to-do practices are 0.4 and 0.5 respectively.

3.8 Discussion

The per capita storage availability in India (207 cubic meters) is very low compared to that of other countries. During the last 50 years, renewable fresh water available per person per year has reduced from 5177 cubic meters (CM) in 1951 to 1816 CM in 2001 to 1545 CM in 2011, and which is further estimated to be reduced to 1341 CM in 2025 and to 1140 CM in 2050. This continual sharp decline points to a water scarce condition by which water availability per person per year becomes less than 1000 CM [173; 174].

The results of this chapter help to reach to a persuasive conclusion that overall percentage of acceptance and non-acceptance of must-to-do practices/things are respectively 0.4 and 0.5, which indicates that humans are sincerely searching for new sources of Earth water, wish to recycle the water from the earlier methods (heating, evaporation, condensation and collection etc.) to the new methods such as desalinization based on reverse osmosis process to get fresh water from sea, wish to re-use water in terrain cultivation and at the same time want to live on water dependent crops, to work in industries and to enhance infrastructural development with water etc.

Advantages: The slight difference between 0.4 and 0.5 arises due to the arbitrary collection of data to construct TT2IFMG which can be easily manipulated by the change in weather/climate or by humans or by both. The main advantage of this method is that it alarms us with an immediate management of fresh or waste water by recycling or re-using, urges upon various GOs and NGOs to search for effective sustainable solutions.

Use of ranking function in the proposed method of this chapter points to another important advantage as it significantly simplifies the two-fold characteristics of the TT2IFNs to crisp numbers. However, a general method for solving any situation cannot be described or defined as the proposed problem is incurred with complexity. But in this chapter, ranking of fuzzy numbers has been formalised by using an improved fuzzy ranking method. Calculation-hazards and time-complexity towards getting a solution are basically reduced by this ranking function technique.

Disadvantages: The foremost drawback is that we can not explore the fuzzy characteristics of real-life data as we fail to systematize or schematize the fuzzy numbers without using their crisp equivalent. So to illustrate the results of real-life conflicts, we are to take a reverse mapping from crisp solutions to corresponding fuzzy equivalent.

3.9 Conclusion

In April 2016, Latur, a district of Maharashtra, India came under the flashlight of broadcasting and printing media due to its drought-hit worst situation. Local and central administration strived to manage the worst threat of nature. Government of India carried water about 70 lacs litre from Krishna river by a 50-wagon train and made nine trips of a 10-wagon water train. The water was stored in a huge well located nearly Latur railway station and distributed among the people after purification. During the last two decades, more water-efficient process for the agricultural, industrial and infrastructural sectors have been developed, as the water required for producing 1

ton of steel has been reduced from 80 tons in 1950 to 6 tons today. Industry's need for water has declined with the replacement of steel by different alloys and eco-friendly plastics. Crops using less water-guzzling practices are being harvested now.
To cope with such type of severe real-life problems as we face today and expect to face in near future, our mathematical methodologies with the proposed ranking function discussed in this chapter, may provide a new direction to the decision-makers of water resource management. We should conserve as well as preserve water to live and let live the next generation and all the inhabitants. Using different ranking functions and type-2 intuitionistic fuzzy environment which may be of the form triangular or trapezoidal or hexagonal or interval, etc., we can derive reliable solutions from many ecological-environmental problems.