Chapter 1

Introduction, Literature Review and Organization

This chapter briefly describes some uncertain environments, like fuzzy, type-2 fuzzy, hesitant fuzzy, linguistic fuzzy, interval fuzzy and neutrosophic concept in two-person zero-sum and two-person non-zero-sum game environment. In order to solve different types of game problems, various methodologies are introduced. A summary of related literature survey of the proposed research study, from its motivational concept to its objective and conclusively the organization style are presented as the parts of this chapter.

1.1 Introduction

Pre and post scenario of 2^{nd} World War promoted the terminology "Operations Research" (OR). Scientists, at that time used to study complex and difficult strategic and tactical problems involved in several military operations. The focus on OR was in realization of the most effective utilization in limited military resources, activities, techniques. Post war situation rapidly changed the World environment towards improvements in society and OR was spread rapidly into the other sectors, say, government sector, industry sector, education sector, even in health sector. Contemporarily, in 1944, John von Neumann and Oskar Morgenstern in their seminal book explained the game theory and some speculation about its future. Meanwhile, Williams (1954) composed *The Compleat Strategyst* as a book intended to make the theory more widely accessible to non-mathematicians. von Neumann and Morgenstern's book *Theory of Games and Economic Behavior* provides an indepth presentation of game theory and highlights how the theory from its very inception has attracted the academics from different fields so seriously. 'Fortune' in its May issue of 1949 published an article of 20-pages describing its accomplishments in World War II and further successful implements in industry. The Princeton University, the RAND Corporation and The American Mathematical Society which published a series of four volumes in the subject, have devoted themselves a lot towards the development of the theory during nineteen fifties and sixties. Not only economics but also biology, information science, statistical physics, and other social sciences are influenced by game theory structures.
Similar to other decision making problems, game theory, as an optimization method, also engaged itself to get optimal results to the problems, involved. A mathematical problem consisting

of objective/s and constraints is said to be an optimization problem. Optimization problems can be classified into two sections in terms of solution procedures- Linear and Nonlinear. Both these types can be solved through programming to get optimal (maximal or minimal) results with respect to objective/s. Based on number of objectives, optimization problems are classified as single-objective and multi-objective. Game problems can be defined within these structures.

1.1.1 Game problems and its different types

Game theory emerged as a strong and promising discipline of mathematics. It was mostly applied and developed by the economists. Nash, Selten and Harsanyi were awarded jointly the Nobel prize in Economics in 1944 for their brilliant work in game theory. Different models of game theory represent in abstract form the real-life situations arising out of cooperation and competition between several parties. A game formally describes the situation in strategic form and depends on the basic assumption that some well defined exogenous objectives be pursued by the decision-makers having a preliminary knowledge about the expectations/behaviour of other decision-makers. Game theory studies the active or passive interactions among several agents which may be individuals, groups, companies etc. or an admissible combination of these. The underlying concept lies on the formulation of a language, structuring, analysing and understanding the strategic scenario. In each play of the game, the desirability of an outcome to a player is quantitatively measured in terms of the amount each player wins or loses and is expressed by a number, called the Payoff or utility. Payoffs are valued with uncertainty when the outcomes appear as uncertain variables. The player's attitude to risk is incorporated in the expected payoff. In the strategic or normal form, the compact representation of the game, the players are able to choose simultaneously their strategies. Strategies for each player in most of the cases become very complicated as these are the plans, designed at the very starting of the game, which will guide the players in every situation for betterment. The resulting payoffs presented in tabular form provide a cell for each strategy combination. When a strategy constitutes a complete plan of choices, one for each decision point/node of the player, the game is called an extensive game. In a two person game, sometimes, one player wins and the other loses. This is far too restricted for many games, especially games in economics or politics, where both players can win something or both can lose something. In bi-matrix game, each player is possessed with his/her own individual payoff matrix and aims at maximizing his/her own payoff. Bi-matrix game is no longer assumed zero sum, or even constant sum. In a two-person non-zero sum game, we simply assume that each player has her or his own payoff matrix. In literatures, several types of games such as Cooperative and Non-cooperative games, Normal form and Extensive form games, Constant sum, Zero-sum and Non-zero-sum games, Symmetric and Asymmetric games, Discrete and Continuous games, Many-player and Population games, Metagames and some others have been defined. Among these various types of games, in this thesis, only two-person zero-sum game (matrix game) and two-person non-zero-sum game (bi-matrix game) have been discussed with.
Two-person zero-sum game: Two-person zero-sum game is considered with a situation where number of players or decision makers are two; and one player's gain is quantitatively same to other's loss, so the net value is zero. A zero-sum game may have more than two players or finitely large number of players. Two-person zero-sum games are widely used in game theory. For a simple example, distributing a chocolate, where to take a larger piece means to reduce the amount of chocolate available for others, is a zero-sum game if all participants value each unit of

chocolate equally. Poker and gambling are popular examples of zero-sum games as the combined losses of some players is equalled by the sum of amounts won by others. Games like chess and tennis, where one wins and other loses, are then termed as zero-sum games. Zero-sum games are usually solved with the maximin-minimax theorem, closely related to the duality theory of linear programming or with Nash equilibrium [105].
Two-person non-zero-sum game: Contrastingly, non-zero-sum game is dealt with a situation where the aggregate of gains and losses of the interacting parties is less than or greater than zero. In bi-matrix game, each player may opt for a finite number of actions. The game is so called (bi-matrix game) as the normal form of such game is expressed by two matrices. The payoffs of player I is described by one matrix, say matrix A, and that of player II is expressed by another matrix, say matrix B. Zero-sum game is sometimes called strictly competitive game, whereas non-zero-sum game may be either competitive or non-competitive. Every bi-matrix game possibly in mixed strategies is possessed with a Nash equilibrium which may be treated as a special case of Linear Complementary Problem (LCP) and may be found out by applying Lemke-Howson algorithm [73].

1.1.2 Several uncertain environments

Game theory, when discussed theoretically, expresses conflict or cooperative situation mathematically. But, when it is applied in real-life problematic cases, uncertainty plays an important role. Suppose, a company wishes to increase its market-share. So, a set of strategies is decided. But, in course of business-time, company has to change strategy or strategies, slightly. And these changes play the role for uncertainty. Thus, due to complexity of situations, data is imprecise, sometimes vague, sometimes hazy. Causes of uncertainties in game can be considered from:

- decision makers'/players' intuition,
- uncertain payoff elements,
- ambiguity in game structure,
- hesitance characters of game,
- suddenly introduction of new players with unknown information,
- varieties of real-life data as game example,
- uncertain data in game-constraints.

Dealing with such types of uncertainties as well as hesitancy information in game, this thesis is modelled with various kinds of uncertain environments such as stochastic environment, interval environment, fuzzy environment, intuitionistic fuzzy environment in game to get more realistic data in decision making process. Several uncertain environments are classified as follows:
Fuzzy environment: To formulate game problems under stochastic environment, either a priori predictable periodicity or posteriori frequency distribution is required, in advance. But, in fuzzy set theory, such types of predictable periodicity/frequency distribution are not needed. Fuzzy set theory provides an excellent mathematical tool to deal with uncertain information and can be

treated as superior one to probability theory. Fuzzy set is expressed through membership function. When game problem is treated using the concept of fuzzy set, it is said as game under fuzzy environment. Let X denotes a universe of discourse. A fuzzy set A_F in X is distinguished by a membership function $\mu_{A_F} : X \to [0,1]$. A fuzzy set A_F in X can be demonstrated as: $A_F = \{(x, \mu_{A_F}(x)) : \mu_{A_F}(x) \in [0,1], \ x \in X\}$. Membership degrees $\mu_{A_F}(x)$ of A_F are crisp numbers.

Interval environment: Beside fuzzy environment, interval number, interval computation and interval analysis play important roles in the representation of uncertainty. In order to fully and effectively utilize uncertainties in problems of natural phenomena, single point is not sufficient to express a physical quantity. In game theory and decision making problems, when all coefficients or some coefficients in the objective functions and constraints of games are within an interval then corresponding environment is said to be an interval environment. An interval number is the simplest way to represent imprecise, uncertain phenomena between its lower and upper bounds.

Intuitionistic fuzzy environment: In fuzzy concept, members are characterized by their membership values. But non-membership characteristics are not included there. Atanassov [4] incorporated the concept of intuitionistic fuzzy set concerning non-membership values of members including membership values. Intuitionistic fuzzy concept plays an important role in depicting real-world situations through decision making problems and game problems.

Type-2 fuzzy environment: Type-2 fuzzy sets and systems generalize fuzzy sets (sometimes say, type-1 fuzzy set) and systems so that uncertainty can be handled, vastly and broadly. Since the earliest days of fuzzy concept, somebody believe that the membership function of type-1 fuzzy set bears no uncertainty. But it seems to be contradictory as the very word “fuzzy" connotates a lot of uncertainties. This was responded in terms of type-2 fuzzy set by Zadeh. In type-2 fuzzy set, the uncertainty about the membership function is incorporated into fuzzy set theory. Generally in type-2 fuzzy set, the membership function becomes three dimensional and its value at each point on its two-dimensional domain, called the footprint of uncertainty, is determined by the third dimension. Zadeh expanded type-2 fuzzy set into its type-n version.

Hesitant fuzzy environment: When individuals settle on a choice, they generally hesitate to or falter in one thing or another, and find it difficult to reach to a final settlement. For instance, at the time of consideration of membership degree of an element to a set, decision maker may allocate 0.6 or 0.65 or 0.655 or 0.68 and the allotment of a common membership degree becomes difficult. This difficulty arises not because of any margin of error or some possibility distribution values that we have, but because of a set of potential esteems we are possessed with. Torra and Narukawa [141] have managed these difficulties by introducing the concept of hesitant fuzzy set (HFS) which is considered as one of the extensions of Zadeh's fuzzy set. This extension allows the possible membership degrees of an element to a set, and hesitant information is expressed more comprehensively than other extensions. Hesitant fuzzy element (HFE) serves as the fundamental unit of a HFS.

Linguistic term set based environment: Humanistic systems developing fields such as, artificial intelligence, decision making processes, linguistics, pattern recognition, medical diagnosis, law, psychology, economics, etc., are mostly nurtured by languages. By a linguistic variable we describe a variable with values as words or sentences, may be in a natural language or in artificial language. For example, while we describe age, we prefer to use the terms like young, old, very young, too old, not so old, quite young, etc., rather than the numerals 18, 22, 16, 30, 24, etc. The terms - very, quite, extremely, etc. also can be treated as linguistic terms. A linguistic variable promoted by such type of descriptive thoughts helps in getting some indications while we wish to approximate on the characterization of very complex phenomena. Here a mode of reasoning, neither exact nor inexact, has been developed to form a basis for approximate reasoning which is sustained by a more realistic framework in comparison to the traditional two-valued logic. Thus linguistic term set based environment arises and works.

Neutrosophic environment: Neutrosophic set (NS) and neutrosophic logical thoughts were introduced by Smarandache [130] to deal with imprecise, uncertain data. In NS concept, truth-membership (T), indeterminacy-membership (I) and falsity-membership (F) are engaged independently in standard or non-standard subsets of $[^{-}0, 1^{+}]$. Atanassov's intuitionistic fuzzy did not assume the concept of indeterminacy. But indeterminacy is assumed in NS theory. Here the main idea is - an entity, *A* in relation to its opposite *non-A*, and to that which is neither *A* nor *non-A*, denoted by *neut-A*. Thus, NS theoretical concept is applied vastly and successfully.

1.1.3 Solution methodologies

In this thesis, various methods and approaches are applied to solve games in different uncertain environments. Some introductions about the related methods and techniques are discussed, briefly.

Linear and non-linear programming: Linear programming was shown as a viable method for solving mixed strategy zero-sum games from very beginning of game theory. Formally speaking, Linear Programming is a device or technique by which linear objective function is optimized under some linear equality and inequality constraints. The feasible region of Linear programming Problem (LPP) exhibits a convex polytope, which constitutes a set defined by the intersection of finitely many half spaces, and a linear inequality defines each such space. The objective function of LPP defined on this polyhedron, is a real-valued linear function. An algorithm of a LPP helps finding a point in the polytope (if such a point exists) where the objective function attains its largest (or smallest) value. In standard form, a LPP can be written as, Maximize $c^T x$, provided $Ax \leq b$ and $x \geq 0$; x represents the vector of variables to be determined, A is the coefficient matrix, c and b are coefficient vectors, T denotes transpose. Here the expression $c^T x$ that is to be maximized or minimized is called objective function. The constraints specifying the convex polytope over which the objective function is optimized are determined by the inequalities $Ax \leq b$ and $x \geq 0$. Two vectors having the same dimensions are comparable in this context. The first vector is said to be less-than or equal-to the second vector if every entry in the first is less-than or equal-to the corresponding entry in the second. Dantzig [31] developed general LPP formulation and invented the simplex method to tackle LPPs. On the other hand, non-linear programming (NLP) may be defined as the process solving an optimization problem in which either the objective function or a few of the constraints become non-linear. In an optimization problem, the extrema (maxima/minima/stationary point) of the objective function over a set of unknown variables is calculated subject to satisfying the constraints defined by a system of equalities or inequalities. It forms a sub-field of mathematical optimization which is dealt with non-linear problems. Let $n, m,$ and p be positive integers. Let X be a subset of $\mathbb{R}^n$, let f, g_i, h_j be real-valued functions on X for each $i \in \{1, \ldots, m\}$ and each $j \in \{1, \ldots, p\}$, with at least one of $f, g_i,$ and h_j being non-linear. Then, a non-linear minimization problem is an optimization problem of the form Minimize $f(x)$ with subject to $g_i(x) \leq 0,$ for each $i \in \{1, \ldots, m\}$ & $h_j(x) =$

0 for each $j \in \{1, \ldots, p\}$, with $x \in X$.

Matrix method: Matrix game can be solved directly from the corresponding payoff matrix. In many cases, this way of getting solutions of matrix game is fastest and seems to be the best among others solution methodologies. Optimal strategies and game value are obtained from this method by some explicit formulae. This method is sometimes termed as matrix method.

Maximin-minimax method: In a two-person zero-sum finite game the strategy sets of both the players are finite. John von Neumann [109] described the fundamental theorem of game theory as the situation encountered in the game of odd or even holds for all finite two-person zero-sum games. In maximin strategy, player I wishes to maximize the minimal (in respect to the strategy of player II) his payoff and in minimax strategy player II tries to minimize the maximum (with respect to that of player I) that the player II is to pay to player I. For every finite two-person zero-sum game, there is a number V, called the value of the game. Both the players adopt mixed strategies in such a way that player I's average gain is at least V, no matter what player II does, and also player II's average loss is at most V, independent of what player I does. This method is called Maximin-minimax method, which has been stated more precisely in our later discussion and applied in depth in our cited examples.

TOPSIS: *Technique for order preference by similarity to ideal solution* (TOPSIS) is considered as one of the most applied decision making tools to handle classical multi-criteria decision making problems. The basic idea or principle behind the technique lies in the fact that to achieve to an optimal alternative shortest distance from the positive ideal solution (PIS) and the furthest distance from the negative ideal solution (NIS) should be maintained. In this thesis, TOPSIS is applied for solving game problems on uncertain environments.

Ranking approach: Fuzzy numbers are not ordered and can overlap with each other. Fuzzy numbers are represented by possibility distributions, their comparison and ordering, not akin to that of real numbers. So, to rank fuzzy quantities or a set of fuzzy numbers some specific defuzzification measures are taken and thus fuzzy numbers are converted into real numbers, where a natural order between them is defined. In fuzzy game theory, fuzzy payoff matrix is converted into crisp payoff matrix by means of ranking approach. Several ranking approaches are discussed in this thesis.

1.2 Literature Review

Zadeh [167] introduced and Zimmermann [171] later developed the fuzzy set theory. So far as human judgement, decisions, behaviour, evaluation, etc., are concerned, a sense of fuzziness comes into play. The fuzzy set theory has generalized the classical notion of set theory into this sense of fuzziness. Collection of relevant data or information for constructing standard mathematical models becomes very hard in reality in many wide applications. These data or information are termed as crisp and these are different from the present day data or information which we call fuzzy. Fuzzy numbers have been defined by Dubois and Prade [36] as fuzzy subset of the real numbers. Fuzzy set assists brilliantly in investigating the inner situations of issues identified with everyday's existence. Although it has a few constraints to deal with imprecise information, hazy and murky data when various kinds of fuzziness, murkiness and unpredictability yield up at the same time. Analysts have nurtured the fuzzy sets and stretched out the fuzzy set to intuitionistic fuzzy set, type-2 fuzzy set, hesitant fuzzy set, linguistic set, neutrosophic set, etc. Intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set have been introduced and developed by Atanassov [4; 5; 6]. After the invention of intuitionistic fuzzy set (IFS) and intuitionistic fuzzy number (IFN), a radical change is noticed in the expression of describing the fuzzy sets. Only a membership function which indicates the degree of belongingness of the member of the set under consideration is used in fuzzy set. The degree of non-belongingness is just complementary to 1. The IFS includes the degree of belongingness and that of non-belongingness and also the hesitant degree. The information described by the IFS is more close and relevant to the present day situations which are encountered with uncertainty. In many areas [35; 155], IF environment has been applied successfully.

The shape of membership function differs in the types of fuzzy sets and numbers and also depends on the problem itself. To represent the fuzzy numbers in real-life situations, triangular or trapezoidal fuzzy numbers are mostly used. Extensive review of literatures confirms that triangular fuzzy sets and numbers are widely used because of its simplicity, yet all sorts of uncertainties involved in the system can not be considered in their two-dimensional representations. Concept of type-2 fuzzy sets [168] fill this gap. Mendel and John [96] pioneered on type-2 fuzzy sets. Several articles have been published on type-2 fuzzy set, its properties, operations [29; 66; 98; 151] and type-2 fuzzy sets, type-2 fuzzy numbers, type-2 fuzzy logic have been used in various fields [65; 145; 156].

Zadeh's concept of linguistic term set [168] proposed another structure towards anew approximation of reasoning to depict real-world phenomenon. Several articles [51; 52; 64; 84; 88; 93] are in literature due to linguistic term set and the corresponding real-life applications. Mamdani with Assilian [92] experienced linguistic synthesis with a fuzzy logic controller. Xu [158; 159] applied linguistic information in decision making.

Fuzzy concept was modified from its membership characteristics to intuitionistic fuzzy concept having non-membership characteristics. But in some real cases, suppose in time of choosing candidate in voting-system, one have options to give up or to be in state of undecided besides confirm choice or no-choice. Such situations are not handled by intuitionistic characters. In such cases, neutrosophic set (NS) and neutrosophic logical concept are originated and successfully applied by Smarandache [130; 131]. In many situations, beyond the acceptance and the rejection degrees, the indeterminacy degree exists too. Several extensions of neutrosophic sets, such as single-valued neutrosophic set [146], interval neutrosophic set [147], multi-valued neutrosophic set [148], etc. have also been proposed. Ye [164] defined subtraction and division operators of simplified netrosophic sets. Ye [165] used multi-criteria decision making method and aggregation operators for simplified neutrosophic set. Neutrosophic set and logic have been applied in many areas [33; 34; 87], successfully. Among these, single valued neutrosophic set and interval neutrosophic set are highly discussed due to their easy applicability.

In reality single value becomes insufficient to afford complete information about the degree of kinship of an element belonging to the set under consideration. The very term "hesitant fuzzy set" (HFS) where every member is defined with a set of membership degrees was first coined by Torra [140]. Torra with Narukawa [141] elaborated hesitant fuzzy sets and decisions in different ways. HFS is used in many decision-making problems when the decision makers are encountered with hesitant environment and variety of vagueness such as disaccord, discrepancy situations [39; 160; 166]. Some advanced operations in HFS have been done by Xia and Xu [161]. The interval-valued hesitant fuzzy set as proposed by Chen et al. [28] posits the membership values of an element to a set with several attainable interval values and introduced some interval-valued hesitant fuzzy aggregation operators. Rodriguez [124], Beg and Rashid [11] and Özkan et al. [114] introduced hesitant fuzzy-linguistic term set (HFLTS), hesitant intuitionistic fuzzy-linguistic term set (HIFLTS) and hesitant fuzzy-linguistic multi-criteria decision making approach, respectively.

In some situations, we only can assume the lower-upper approximation of uncertainty, which can be described as intervals [77; 82; 163; 170]. Ranking method and interval computing, different from the real numbers, largely attract the academicians [56; 128] as because of the complexities of the problems. Based on Moore's pioneer work [100], several applications are studied on the fields of game and decision making [17; 85]. Several articles [27; 28] are published on interval-valued hesitant fuzzy set. Rodriguez et al. [124] introduced HFLTSs for decision making problems.

In terms of theory and applications, game theory itself stretches a vast area. Game theoretic situations can be seen not only in The Bible [16] or in The Talmud [7], but in many real-life problems from very early days (near about 2000 years back) [137] to today's evolutionary games [139]. Game theory plays the most important mathematical role in economics. And this is the reason of flourishing of game revolution through the fields of economics [30]. One of the most formal works on game theory was by Zermelo [169]. The topics *Game Theory* build from zero-sum matrix games, to non-zero-sum matrix games, to cooperative games, to population games. For this vast varieties, here some histories about zero-sum and non-zero-sum games are described, mainly. von Neumann [109] invented the mathematical theory of games with famous minimax theorem for zero-sum games and later with Morgenstern [110] co-authored the famous book *Theory of Games and Economic Behavior*. This book also laid the groundwork for the study of cooperative games. Another remarkable work was done by Luce and Raiffa [90] in their book *Games and decisions: Introduction and critical survey*, from which many examples are cited still, like the Prisoners' Dilemma and Battle of the Sexes. Seminal works were carried out by Nash: on bargaining [105], on Nash equlibrium [106], on two-person cooperative game [107]. Fuzzy sets [167] representing and explaining the payoffs of strategies of fuzzy matrix games have been utilized in many articles for measuring some uncertain elements in the games. Several monographs and articles have been published in matrix game theory [113; 122; 126; 129], having some real-life impacts. Bector and Chandra [9] have extended the fuzzy matrix games to fuzzy mathematical programming and the duality in linear programming with fuzzy payoffs and fuzzy parameters has been analyzed by Bector et al. [10]. In fuzzy arithmetic, ranking of fuzzy numbers appears to be a fundamental problem. Like the real number systems, which is ordered, fuzzy numbers represented by possibility distribution do not exhibit any ordered system as they may overlap with each other. Campos [23] used a new way of solving fuzzy matrix games through fuzzy linear programming models and Campos et al. [24] applied ranking function approach to solve fuzzy matrix games in some direct ways. Li [77], in his monograph, considered game theories and related applications in management with intuitionistic fuzzy sets, originated by Atanassov [4; 6]. Nan et al. [103; 104] made use of IF concept towards the development matrix games in triangular environment. In literatures, Atanassov's interval valued IF set comes into the fore as reality does not promote the exact values of membership as well as non-membership degrees. Xu and Zhang [153] advanced further taking into account of similarity measure and accuracy degree and introduced a ranking method. Xu [154] considered intuitionistic preference relations and their applications in group decision making. Based on the concept of [5], Xu [155] used the score function and accuracy function to present his ranking method on IF values and interval-valued IF sets. Chen and Tan [25] made use of score function in multi-attribute fuzzy decision making problems whereas Hong and Choi [55] later laid stress on accuracy function for better alternatives. A ranking function for interval-valued IF sets was first proposed by Yager [162] and later by Ye [163] and Lee [72]. Wang et al. [144] made significant contribution to this ranking function. Ranking functions are used for the defuzzification of fuzzy numbers. Bhaumik et al. [12] solved two-person zero-sum game problem using robust ranking method. Li [75; 76] solved matrix games with fuzzy payoffs. Bhaumik and Roy [14] proposed intuitionistic interval-valued hesitant fuzzy matrix games with a new aggregation operator. Jana and Roy [60] solved matrix games with generalised trapezoidal fuzzy payoffs. Jana and Roy [59] derived solutions of dual hesitant fuzzy matrix games based on new similarity measures. Fuzzy matrix games under different scenarios have been elaborated through different studies [121; 123]. Nishizaki and Sakawa [111] discussed on fuzzy and multiobjective games for conflict resolution. Roy and Mula [119] solved matrix game with rough payoffs using genetic algorithm. Li and Cheng [80] solved constrained matrix game using fuzzy multi-objective programming in fuzzy numbers. Li and Hong [81] proposed alpha-cut based linear programming method for constrained matrix games.

Two-person non-zero-sum game is another avenue of game theory. Sometimes it is termed as bi-matrix game. Bi-matrix games have been embellished through various studies by several authors in different ways in both crisp and fuzzy environments. Among these, [32; 112; 118; 120] are citable. Prisoners' Dilemma is one of the promising examples of Bi-matrix game. Based on Prisoners' Dilemma, many works are carried out in different aspects, in different fields from mathematical sciences to biological sciences. Borges et al. [15] made fuzzy approach to the Prisoner's Dilemma. Brembs [18] potentially solved Prisoner's Dilemma through chaos, cheating and cooperation. Bhaumik et al. [13] studied on human trafficking based on Prisoners' Dilemma game in hesitant interval-valued intuitionistic fuzzy-linguistic term set based environment. Maeda [91] characterized the equilibrium strategies of bi-matrix game with fuzzy payoffs. Larbani [69; 70] solved bi-matrix games in normal form, introducing nature as a player. Mula et al. [101] solved bi-matrix game through birough programming approach. Game theories and game theorists played important roles to bag the prestigious Nobel Prizes from 1970 to 2007 [179].

The best introduction to game theory is by way of examples and their representations. Some real-life problems (considered in this dissertation) are taken here with some correlated review works. Several articles [1; 19; 150] have been published concerning water and waste-water management. Several reports from non-government agencies and government organizations [172; 173; 174; 175; 176; 177; 178] have discussed on water problems. Banihabib and Shabestari [8] have considered the water management problem in agriculture under fuzzy hybrid MCDM model. On the issues of water management, various concepts and mathematical methodologies have been developed, like, the introduction of time-series analysis in water demand prediction [26]; using of multi-objective decision making concept in water resources [54] and in land, water and environmental management [38]; using of multi-criteria decision making approaches [2; 3; 53]; water consumption and waste-water network topology treatment under genetic algorithm [71]; fuzzy Analytic Hierarchy Process (AHP) assessment implementation to water management plans [134]; inter-basin water transferring under fuzzy decision making framework [157]; using fuzzy multi-criteria approach in the problems of water scarcity [142]; management of coastal aquifers using stochastic and robust multi-objective optimization [135]. Management of water resources under tremendous demand, i.e., the demand-supply of water, the reuse the waste water and to preserve the water among its users, using the fuzzy game theory approach was also discussed by Roy and Bhaumik [125].

Human trafficking has been discussed in several articles [67; 99] from mathematical point of view. In literature, human trafficking through two-person non-zero-sum game theory approach is also defined by Bhaumik et al. [13]. Here, hesitant interval-valued intuitionistic fuzzy-linguistic term set based environment is considered. Numerous works have been done on human trafficking [115; 116; 127]. Foot et al. [40] delivered a report on developments in anti-trafficking efforts from 2008 to 2011. Ghosh [47] elaborated the topics of nature, dimensions and strategies for prevention of trafficking in children and women in India.

Several articles and monographs describe medical problems related to mental healths [46; 48; 50]. Fraser et al. [41] framed mental health issues in different aspects. Spitzer et al. [133] structured a frame of clinical interview against mental health issues. Hastings and Brown [49] discussed behaviour problems of children and their health. Hymen [58] discussed on mental illness. Lersner et al. [74] focused on stigma of mental illness in Germans and Turkish immigrants through casual beliefs. Loughman and Haslam [89] neuro-scientifically explained mental disorder. Voorhees et al. [143] considered beliefs and attitudes towards diagnosis of depression as a mental health issue.

Several articles [44; 57; 63; 78; 79; 83] have been published in decision making problems using uncertain environments. Nehi and Maleki [108] applied intuitionistic fuzzy numbers in fuzzy optimization problems. Wang and Mendel [149] generated and degenerated fuzzy rules from problem-phenomena. Zhang and Wan [170] defined some models in polymorphic nonlinear programming problems and algorithms in uncertain environments. Artificial neural network in decision making problem is one of the most important topics, now-a-days. Several articles are devoted on artificial neural network, fuzzy neural systems [20; 21; 43; 61; 62; 68; 97]. Lin and Lee [86] developed neural network based fuzzy logic control and decision system. McCulloch and Pitts [94] contributed some works on logical calculus of the ideas imminent in nervous activity. Sugeno and Yasukawa [136] discussed on qualitative modeling based on fuzzy-logic based approach. Takagi and Sugeno [138] applied fuzzy concept in systems, modeling and control. Wu and Er [152] studied dynamic fuzzy neural networks on function approximation. In recent days, numerous articles are published on game theory together with neural network [42; 95; 132].

1.3 Organization of the thesis

Motivated by different thoughts and based on several objectives this thesis's chapter-wise organizations are traced in this section.

1.3.1 Motivation

As our physical World is always changing, it seems to be a common attribute of human intelligence to take decision with knowledge and solve the pertinent problems we encounter with in our real-life conflicting situations. Decision making is guided by the cognitive process which depends on individual's perception, imprecise in nature. The process attracts the researchers to model and present the incomplete/imprecise data by using some new tools and methodologies. Researchers in game theory and its allied areas for a long time have opened a door to us to deal with the cognitive process of human behaviour concerned with the imprecise premises. But two-valued or multi-valued traditional logics failed to handle such process which motivated Zadeh [167] to represent the imprecise concepts and knowledge of the cognitive process by introducing fuzzy logic and fuzzy set theory. Since much of our knowledge in real-life problems is more imprecise, it appears to be really difficult to manipulate the game problems by machines as a word might be explained in different ways by various researchers and experts. Zadeh's fuzzy sets have been applied successfully in many fields to tackle such situations. Atanassov [4] introduced intuitionistic fuzzy sets which have been proved to be highly useful to encounter with vagueness. Gau and Buehrer [45], Bustince and Burillo [22] and many other learned researchers and scientists have explored a lot in the subject and reached to its stupendous possibilities. Different optimization tools provide different optimal results, but no single technique can provide a universally accepted optimal solution as uncertainty can never be properly addressed. We are to choose those new techniques which give better results in comparison to the existing ones. Two-Person Zero-Sum Game (Matrix Game), Two-Person Non-Zero-Sum Game (Bimatrix Game), Fuzzy Matrix Game, Prisoners' Dilemma, Triangular Intuitionistic Fyzzy Number, Hesitant Intuitionistic Fuzzy Set, Single-Valued Triangular Neutrosophic Number, Linguistic Term Set, (α, β, γ)-cut set, Robust Ranking Technique, TOPSIS, Water Management, Human Trafficking, Shopping-Marketing Management Problem, etc. have come into the fore. Nash, Selten and Harsanyi [179] jointly have been awarded Nobel Prize in Economics for their excellent work in Game Theory in 1994. Water crisis along with its management and human trafficking now-a-days pose a grave concern to human civilization. All these have deeply inspired us to work in this field.

Moreover, Neumann and Morgenstern's seminal book *Theory of Games and Economic Behavior* dealt with N-person (NP) games, as well as the single-person and two-person zero-sum specializations might be considered as a great motivation to our work in the specialized area which we have tried to explore with concrete examples of real-life conflicting situations faced with uncertainty. This book puts a significant contribution to finite two-person zero-sum game. The most cited "minimax" theorem asserts the existence of rational behaviour as long as randomized strategies are allowed. Their definition of "solution" for two-person zero-sum games has stood the test of time, with subsequent years seeing the idea generalized, rather than revised. Neumann and Morgenstern descriptively considered rational behaviour in NP games. Then, the game scientists involved them on

- generalization and application of the minimax theorem for two-person zero-sum games through some constructive proofs, computationally.
- investigation of "solution" of NP games, and finding normative base of Neumann and Morgenstern's concept towards its application. In the four volumes published by the American Mathematical Society, the number of articles devoted to two-person zero-sum games in 1950, 1954, 1957, and 1964 were 52%, 47%, 30%, and 25%, respectively. Even in 1985, only about 20% of the papers published in the International Journal of Game Theory were related to two-person zero-sum games, and about 10 % in 2013. Thus, literatures established a decreasing trends.

But recently, several reasons arise for shifting the trend. Now-a-days a good number of articles have been published in reputed journals emphasising on two-person zero-sum games along with non-zero-sum games. Because, theoretical progress in two-person zero-sum game originates

quickly. The minimax theorem, its extended version and other related criteria seem to be closed in many infinite games, and serious computational and modelling issues remain to be discovered, and there are huge scopes of applying two-person zero-sum game and non-zero-sum game in several sectors.
This thesis is motivated also by the above shifting trends.

1.3.2 Objective

This thesis intentionally and practically possesses objectives. These objectives basically come from the motivation of construction of this thesis. The major intention is to apply two-person zero-sum game and two-person non-zero-sum game in real-life oriented problems and to get favouring results. Minutely, this thesis's objectives focus on:

- to develop new theories, methods, approaches related to two-person zero-sum and two-person non-zero-sum games;
- to cultivate new ranking method, distance measures, modified de-crispification method.
- to apply games in real-world problems.

1.3.3 Organization of the thesis

This thesis is organized in eight chapters. Outline of the proposed research study is explained as follows:

Chapter 1 is an overview on the history of game theory and applications of uncertain environments on game theory. A brief discussion on two-person zero-sum, two-person non-zero-sum game theory and fuzzy set, hesitant fuzzy set, linguistic term set, neutrosophic set with some methodologies and approaches in solutions of game problems are described in this chapter.

Chapter 2 analyzes *two-person zero-sum game in triangular intuitionistic fuzzy environment* with an application of robust ranking technique for comparing of fuzzy numbers. A real-life problem is discussed herewith satisfying the proposed game phenomenon. A part of this chapter has appeared in ***Journal of Intelligent & Fuzzy Systems 33 (2017) 327-336***, **SCIE, IF: 1.637**.

Chapter 3 presents *two-person zero-sum game in triangular type-2 intuitionistic fuzzy environment*. **Chapter 3** can be treated as an extension of **Chapter 2**. Here, matrix games are proposed with payoffs as triangular type-2 intuitionistic fuzzy numbers, i.e., Triangular Type-2 Intuitionistic Fuzzy Matrix Game (TT2IFMG) as a new and rare concept. A new ranking function is used to get relevant solutions of TT2IFMG. Water management problem is treated in this chapter under the scanner of TT2IFMG environment. A part of this chapter has appeared in ***Water Resources Management 32 (2018) 949-968***, **SCIE, IF: 2.987**.

Chapter 4 develops *two-person zero-sum game in linguistic neutrosophic environment* where neutrosophic and linguistic characters play important role as uncertain-characteristics. The concept of linguistic neutrosophic numbers are considered here as payoff elements of two-person 2×2 zero-sum game through the fundamental concept of game theory. Here, two-person 2×2 zero-sum game is solved using matrix method. *A part of this chapter is communicated to an international (SCI/SCIE) journal.*

Chapter 5 represents *two-person zero-sum game through artificial neural network structures* under hesitant triangular intuitionistic fuzzy environment. This hesitant triangular intuitionistic fuzzy environment can be treated as an extension of triangular intuitionistic fuzzy environment. A new way of representation of matrix game problem is assumed in terms of neural networks. Introducing neuro-fuzzy concept with logic-gate switching circuit in game problems makes a new way in decision making and expert systems. *A part of this chapter is communicated to an international (SCI/SCIE) journal.*

Chapter 6 is discussed with the environment based on the two-person non-zero-sum game in hesitant interval-valued intuitionistic fuzzy-linguistic term set. Here linguistic semantics first express the linguistic terms in interval and respective indices are used then. The corresponding problem arrives at a Nash equilibrium solution. TOPSIS and dominance property of matrix game theory have been used to get expected results which establish close contact with reality. A part of this chapter has appeared in ***Central European Journal of Operations Research 28 (2020) 797-816***, **SCIE, IF: 1.260**.

Chapter 7 attempts to get optimal solution of *two-person non-zero-sum game in single-valued triangular neutrosophic environment.* The combination of two-person non-zero-sum game with single-valued triangular neutrosophic environment through cut-set approach generates a new approach in solution of game problem. The ranking approach is based on the (α, β, γ)-cut of neutrosophic set and is applied on two-person non-zero-sum game theory by validating real-life problem. *A part of this chapter is communicated to an international (SCI/SCIE) journal.*

Finally, the overall conclusions of the research works from Chapter 2 to Chapter 7 are reported in **Chapter 8** having future scopes of the study at last.