List of Figures

1. Figure 2.1: Schematic representation of (a) WGM micro-cavity (b) OMS set up with external driving.....17
2. Figure 2.2: (a) spreading of decay line width (b) work done leads to cooling and heating effect.....23
3. Figure 2.3: Uncertainties of (a) coherent state (b) squeezed state.....27
4. Figure 2.4: Photon stream comparison of (a) antibunching (b) coherent (c) bunched light.....30
5. Figure 2.5: Representations of entangled, steerable and Bell state.....33
6. Figure 3.1: Schematic diagram of the model system with 'membrane-in-the-middle' configuration.....41
7. Figure 3.2: Variation of the field quadrature variances $(\Delta X_b)^2$ (solid line) and $(\Delta Y_b)^2$ (dashed line) with rescaled time $\omega_m t$. The parameters are $|\alpha| = 3$, $|\beta| = 2$ (a) $g/\omega_m = 2$, $g = 1.2$ MHz (b) $g/\omega_m = 2\,000$, $g = 1.2$ GHz.....44
8. Figure 3.3: Variation of squeezing factors $S_{1b}(n)$ (solid line) and $S_{2b}(n)$ (dashed line) with $\omega_m t$ for mechanical mode with $|\alpha| = 3$, $|\beta| = 2$, $g = 2\pi \times 10$ KHz and $g/\omega_m = 0.1$ (a) $n = 2$ (b) $n = 3$.....45
9. Figure 3.4: Plot of $S_{1b}(4)$ (blue solid line) and $S_{2b}(4)$ (red dotted line) with t(ms) for b mode with n=4, $|\alpha| = 3$, $|\beta| = 2$, $g = 2\pi \times 10$ KHz and $g/\omega_m = 0.1$.....46
10. Figure 3.5: Variation of compound mode field quadratures $(\Delta X_{ab})^2$ (solid line) and $(\Delta Y_{ab})^2$ (dashed line) with rescaled time $\omega_m t$. The parameters are $|\alpha| = 3$, $|\beta| = 2$ (a) $g/\omega_m = 2$, $g = 1.2$ MHz (b) $g/\omega_m = 2\,000$, $g = 1.2$ GHz.....47

11. Figure 3.6: Plot of spin squeezing factors S(x) (solid line) and S(y) (dashed line) with rescaled time $\omega_m t$. The parameters are $|\alpha| = 3, |\beta| = 2$, $g = 2\pi \times$ 0.4 KHz and $g/\omega_m = 0.004$.....48
12. Figure 3.7: Plot of (a) sum squeezing parameters $S_{1_{ab}}$ (blue solid line) and $S_{2_{ab}}$ (red dashed line) and (b) difference squeezing parameters $Q_{1_{ab}}$ (blue solid line) and $Q_{2_{ab}}$ (red dashed line) with $\omega_m t$ for $|\alpha| = 3$, $g = 2\pi \times 10$ KHz and $g/\omega_m = 0.1$.....50
13. Figure 3.8: Variation of compound mode antibunching factor $A_{ab}(n)$ with rescaled time gt with (a) $n = 6, |\beta| = 2$, $g = 2\pi \times 0.4$ KHz, $g/\omega_m = 0.004$, $|\alpha| = 3$ (solid curve) and $|\alpha| = 4$ (dashed curve) (b) $|\alpha| = 3$, $|\beta| = 2$, $g = 2\pi \times$ 0.4 KHz, $g/\omega_m = 0.004$, $n = 7$ (solid curve) and $n = 6$ (dashed curve)..... 51
14. Figure 3.9: Variation of $(\Delta u)^2 + (\Delta v)^2 - 2$ with rescaled time $\omega_m t$ for $|\alpha| = 3$ (solid curve) and $|\alpha| = 5$ (dashed curve), $|\beta| = 2$ (a) $g/\omega_m = 2$, $g = 1.2$ MHz (b) $g/\omega_m = 2\,000$, $g = 1.2$ GHz.....53
15. Figure 3.10: Variation of E_{ab} with rescaled time $\omega_m t$ for $|\alpha| = 3$ (solid curve) and $|\alpha| = 5$ (dashed curve), $|\beta| = 2$ (a) $g/\omega_m = 2$, $g = 1.2$ MHz (b) $g/\omega_m = 2\,000$, $g = 1.2$ GHz.....53
16. Figure 3.11: Variation of $E_{ab}^{\prime m,n}$ with rescaled time gt for $|\alpha| = 3, |\beta| = 2$, m = n = 2 (solid curve) and m = 2, n = 3 (dashed curve) (a) $g = 2\pi \times 0.4$ KHz, $g/\omega_m = 0.004$ (b) $g = 2\pi \times 4$ KHz, $g/\omega_m = 0.04$.....54
17. Figure 3.12: Plot of the variance of the field quadratures with normalised cavity detuning. (a) $(\Delta X_b)^2$ (solid blue line) and $(\Delta Y_b)^2$ (solid red line) for $g/\omega_m = 0.02$; $(\Delta X_b)^2$ (dashed blue line) and $(\Delta Y_b)^2$ (dashed red line) for $g/\omega_m = 0.05$ (b) $(\Delta X_{ab})^2$ (solid blue line) and $(\Delta Y_{ab})^2$ (solid red line) for

$g/\omega_m = 0.02$; $(\Delta X_{ab})^2$ (dashed blue line) and $(\Delta Y_{ab})^2$ (dashed red line) for $g/\omega_m = 0.05$. The other parameters are $k_o/\omega_m = 0.1$, $\gamma/\omega_m = 10^{-7}$, $E/\omega_m = 0.1$, $T = 100mK$ and $n_{th} = 1000$.....56

18. Figure 3.13: Variation of $(\Delta u)^2 + (\Delta v)^2 - 2$ with normalised cavity detuning for $g/\omega_m = 0.005$ (red line); $g/\omega_m = 0.08$ (blue line). The other parameters are $k_o/\omega_m = 0.1$, $\gamma/\omega_m = 10^{-7}$, $E/\omega_m = 0.1$, $T = 100mK$ and $n_{th} = 1000$ 57
19. Figure 4.1: Schematic diagram of two cavity systems coupled via photon hopping. Each cavity system consists with a fixed mirror and a movable mirror. The cavities are coupled with photon hopping interaction……..……...62
20. Figure 4.2: Quadrature variation of $(\Delta X_{a_k})^2$ (solid line) and $(\Delta Y_{a_k})^2$ (dashed line) for single cavity field mode with $\omega_m t$ for $|\alpha_k| = 3$, $|\beta_k| = 2$, $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times 0.4$ MHz.....66
21. Figure 4.3: Temporal variation of single cavity field mode squeezing parameters (a) $S_{a_j}(2)$ (blue line) and $Q_{a_j}(2)$ (red line); (b) $S_{a_j}(3)$ (blue line) and $Q_{a_j}(3)$ (red line) for $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times 0.4$ MHz, $|\alpha_j| = 5$, $|\beta_j| = 3$.....68
22. Figure 4.4: Temporal variation of mechanical field mode squeezing parameters (a) $S_{b_j}(2)$ (blue line) and $Q_{b_j}(2)$ (red line); (b) $S_{b_j}(3)$ (blue line) and $Q_{b_j}(3)$ (red line) for $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times 0.4$ MHz, $|\alpha_j| = 5$, $|\beta_j| = 3$……..…….....68
23. Figure 4.5: Qudrature variation (a) $(\Delta X_{a_k b_k})^2$ (solid line) and $(\Delta Y_{a_k b_k})^2$ (dashed line) for compound mode $a_k b_k$ with $\omega_m t$ for $|\alpha_k| = 6$, $|\beta_k| = 2$ (b) $(\Delta X_{a_k b_j})^2$

(solid line) and $(\Delta Y_{a_k b_j})^2$ (dashed line) for compound mode $a_k b_j$ with $\omega_m t$ for $|\alpha_k| = 6$, $|\beta_j| = 2$. The other parameters are $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times$ 0.4 MHz……..70

24. Figure 4.6: Variation of antibunching factor (a) $A_{a_k b_k}$ for compound mode $a_k b_k$ for $|\alpha_k| = 6$, $|\beta_k| = 2$ (solid line) and $|\alpha_k| = 8$, $|\beta_k| = 2$ (dashed line) and (b) $A_{a_k b_j}$ for compound mode $a_k b_j$ for $|\alpha_k| = 6$, $|\beta_j| = 2$ (solid line) and $|\alpha_k| = 8$, $|\beta_j| = 2$ (dashed line) with $\omega_m t$. The other parameters are $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times 0.4$ MHz……...……...72
25. Figure 4.7: Variation of antibunching factors for different compound mode with rescaled time gt (a) $A_{a_k b_k}$ for compound mode $a_k b_k$ for $|\alpha_k| = 3$, $|\beta_k| = 2$, $n = 2$, $g/\omega_m = 0.08$, $g/\xi = 2.5$, $g = 2\pi \times 0.01$ MHz and phase angle 0 (blue line), $\pi/2$ (red line), π (green line) (b) $A_{a_k b_k}$ for compound mode $a_k b_k$ for $|\alpha_k| = 3$, $|\beta_k| = 2$, $n = 4$, $g/\omega_m = 0.08$, $g/\xi = 2.5$, $g = 2\pi \times 0.01$ MHz and phase angle 0 (blue line), $\pi/2$ (red line), π (green line) (c) $A_{a_k b_j}$ for compound mode $a_k b_j$ for $|\alpha_k| = 3$, $|\beta_k| = 2$, $n = 2$, $g/\omega_m = 0.8$, $g/\xi = 4$, $g = 2\pi \times$ 0.1 MHz and phase angle 0 (blue line), $\pi/2$ (red line), π (green line) and (d) $A_{a_k a_j}$ for compound mode $a_k a_j$ for $|\alpha_k| = 3$, $|\beta_k| = 2$, $n = 2$, $g/\omega_m = 0.8$, $g/\xi = 4$, $g = 2\pi \times 0.1$ MHz and phase angle 0 (blue line), $\pi/2$ (red line)……..……75
26. Figure 4.8: Variation of entanglement parameters (a) $e_{a_k b_k}$ for compound mode $a_k b_k$ for $|\alpha_k| = 6$, $|\beta_k| = 2$ (solid line) and $|\alpha_k| = 8$, $|\beta_k| = 2$ (dashed line) and (b) $e_{a_k b_j}$ for compound mode $a_k b_j$ for $|\alpha_k| = 6$, $|\beta_j| = 2$ (solid line) and

$|\alpha_k| = 8$, $|\beta_j| = 2$ (dashed line) with $\omega_m t$. The other parameters are $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times 0.4$ MHz……...……...77

27. Figure 4.9: Variation of (a) $E_{a_k b_k}$ with $\omega_m t$ for $g/\omega_m = 0.001$, $g/\xi = 0.012$, $g = 2\pi \times 1.2$ MHz, $|\alpha_k| = 6$, $|\beta_k| = 2$ (solid line) and $|\alpha_k| = 8$, $|\beta_k| = 2$ (dashed line) and (b) $E'_{a_k b_k}$ with $\omega_m t$ for $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times 0.4$ MHz, $|\alpha_k| = 6$, $|\beta_k| = 2$ (solid line) and $|\alpha_k| = 8$, $|\beta_k| = 2$ (dashed line)……..78

28. Figure 4.10: Variation of (a) $E^{m,n}_{a_k b_k}$ with $\omega_m t$ for $g/\omega_m = 0.001$, $g/\xi = 0.012$, $g = 2\pi \times 1.2$ MHz, $m = n = 3$, $|\alpha_k| = 6$, $|\beta_k| = 2$ (solid line) and $|\alpha_k| = 8$, $|\beta_k| = 2$ (dashed line) and (b) $E'^{m,n}_{a_k b_k}$ with $\omega_m t$ for $g/\omega_m = 0.3$, $g/\xi = 4$, $g = 2\pi \times 0.4$ MHz, $m = n = 3$, $|\alpha_k| = 6$, $|\beta_k| = 2$ (solid line) and $|\alpha_k| = 8$, $|\beta_k| = 2$ (dashed line)……... …….81

29. Figure 4.11: Plot of intermodal entanglement factors with normalized time gt for phase angle $\theta_1 = 0$ and $\theta_2 = 0$ (a) ζ_1 for $a_k b_k b_j$ mode (b) ζ_2 for $a_k b_j b_k$ mode (c) ζ_3 for $a_k a_j b_j$ mode (d) ζ_4 for $a_j a_k b_j$ mode. Other parameters are $g/\omega_m = 0.0033$, $g/\xi = 0.12$ and $|\alpha_k| = 3$, $|\alpha_j| = 3$, $|\beta_k| = 2$, $|\beta_j| = 2$ (solid line); $|\alpha_k| = 4$, $|\alpha_j| = 3$, $|\beta_k| = 2$, $|\beta_j| = 1$ (dashed line)……....84

30. Figure 4.12: Variation of intermodal entanglement factors with normalized time gt for phase angle $\theta_1 = 0$ and $\theta_2 = 0$ (a) ε_1 for $a_k b_k b_j$ mode (b) ε_2 for $a_k b_j b_k$ mode (c) ε_3 for $a_k a_j b_j$ mode (d) ε_4 for $a_j a_k b_j$ mode. Other parameters are $g/\omega_m = 0.0033$, $g/\xi = 0.12$ and $|\alpha_k| = 3$, $|\alpha_j| = 3$, $|\beta_k| = 2$, $|\beta_j| = 2$ (solid line) ; $|\alpha_k| = 4$, $|\alpha_j| = 3$, $|\beta_k| = 2$, $|\beta_j| = 1$ (dashed line)……..…86

31. Figure 4.13: Temporal variation of $E_{a_1 b_1 a_2 b_2}$ for phase angle $\theta_1 = \theta_2 = 0$, $g/\omega_m = 0.0033$, $g/\xi = 0.12$ and $|\alpha_k| = 3$, $|\alpha_j| = 3$, $|\beta_k| = 2$, $|\beta_j| = 2$ (solid line) ; $|\alpha_k| = 4$, $|\alpha_j| = 3$, $|\beta_k| = 2$, $|\beta_j| = 1$ (dotted line)……...87
32. Figure 4.14: Variation of ζ's for three different field modes (a) $a_k b_k b_j$ mode (b) $a_k b_j b_k$ mode (c) $a_k a_j b_j$ mode (d) $a_j a_k b_j$ mode with cavity detuning Δ/ω_m for different values of ξ/ω_m. Other parameters are $g/\omega_m = 0.0033$, $k/\omega_m = 0.5$, $\gamma/\omega_m = 10^{-5}$, $\Omega/\omega_m = 0.02$ and $n_{th} = 20$……90
33. Figure 4.15: Variation of ε's for three different field modes (a) $a_k b_k b_j$ mode (b) $a_k b_j b_k$ mode (c) $a_k a_j b_j$ mode (d) $a_j a_k b_j$ mode with normalized cavity detuning Δ/ω_m for different values of ξ/ω_m. Other parameters are $g/\omega_m = 0.0033$, $k/\omega_m = 0.5$, $\gamma/\omega_m = 10^{-5}$, $\Omega/\omega_m = 0.02$ and $n_{th} = 20$……...91
34. Figure 4.16: Plot of $E_{a_1 b_1 a_2 b_2}$ with normalized cavity detuning Δ/ω_m for different values of ξ/ω_m. Other parameters are $g/\omega_m = 0.0033$, $k/\omega_m = 0.5$, $\gamma/\omega_m = 10^{-5}$, $\Omega/\omega_m = 0.02$ and $n_{th} = 20$ ……..92
35. Figure 5.1: Schematic diagram of the model system……..98
36. Figure 5.2: Variation of the variance of the field quadratures $(\Delta X_{a_1})^2$ (solid line) and $(\Delta Y_{a_1})^2$ (dashed line) with normalised time Ut. The parameters are $|\alpha_1| = 2$, $|\alpha_2| = 1$(blue);$|\alpha_1| = 2$,$|\alpha_2| = 2$(red) (a) $J/k_1 = 0.2$, $k_2/k_1 = -0.2$ (b) $J/k_1 = 0.5$, $k_2/k_1 = 0.2$(c) $J/k_1 = 0.5$, $k_2/k_1 = -0.2$……...104
37. Figure 5.3: Variation of amplitude squared squeezing factors S_1 (solid line) and S_2 (dashed line) with Ut. The parameters are $|\alpha_1| = 2$, $|\alpha_2| = 1$ (a) $J/k_1 = 0.2$, $k_2/k_1 = 0.2$ (b) $J/k_1 = 0.5$, $k_2/k_1 = 0.2$ (c) $J/k_1 = 0.5$, $k_2/k_1 = -0.2$ and (d) $J/k_1 = 1.0$, $k_2/k_1 = -0.2$…….……...…..105

38. Figure 5.4: Variation of $\left(\Delta X_{a_1 a_2}\right)^2$ (solid line) and $\left(\Delta Y_{a_1 a_2}\right)^2$ (dashed line) with normalised time Ut. The parameters are $|\alpha_1| = 2$, $|\alpha_2| = 1$ (blue) ; $|\alpha_1| = 2$, $|\alpha_2| = 2$ (red) (a) $J/k_1 = 0.2$, $k_2/k_1 = 0.2$ (b) $J/k_1 = 0.2$, $k_2/k_1 = -0.2$ (c) $J/k_1 = 0.5$, $k_2/k_1 = 0.2$ (d) $J/k_1 = 0.5$, $k_2/k_1 = -0.2$ (e) $J/k_1 = 1.0$, $k_2/k_1 = 0.2$ (f) $J/k_1 = 1.0$, $k_2/k_1 = -0.2$…….....106

39. Figure 5.5: Plot of (a) $P_s(a_1)$ with normalised time Ut for $k_2/k_1 = 0.5$ (Red line), $k_2/k_1 = 0.2$ (blue line), $k_2/k_1 = -0.2$ (blue dot-dashed line) and $k_2/k_1 = -0.5$ (red dot-dashed line) (b) $N_s(a_1)$ with normalised time Ut for $k_2/k_1 = 0.2$ (blue line), $k_2/k_1 = 0.5$ (red line), $k_2/k_1 = -0.2$ (blue dot-dashed line) and $k_2/k_1 = -0.5$ (red dot-dashed line). The other parameters are $|\alpha_1| = 2$, $|\alpha_2| = 1$, $J/k_1 = 0.5$……....108

40. Figure 5.6: Plot of (a) $P_s(a_1 a_2)$ with normalised time Ut for $k_2/k_1 = 0.5$ (red line), $k_2/k_1 = 0.2$ (blue line), $k_2/k_1 = -0.2$ (blue dot-dashed line) and $k_2/k_1 = -0.5$ (red dot-dashed line) (b) $N_s(a_1 a_2)$ with normalised time Ut for $k_2/k_1 = 0.2$ (blue line), $k_2/k_1 = 0.5$ (red line), $k_2/k_1 = -0.2$ (blue dot-dashed line) and $k_2/k_1 = -0.5$ (red dot-dashed line). The other parameters are $|\alpha_1| = 2, |\alpha_2| = 1, J/k_a = 0.5$……..……109

41. Figure 5.7: Variation of $\left(\Delta X_{a_1}\right)^2$ and $\left(\Delta Y_{a_1}\right)^2$ with (a-b) cavity detuning Δ/k_1 for different values of normalised photon tunnelling strength J/k_1. The gain-to-loss ratio $k_2/k_1 = 0.2$ and $k_2/k_1 = -0.2$ (c) nonlinear strength U/k_1 for different values of k_2/k_1 , $\Delta/k_1 = 0$ and $J/k_1 = 1$ (d) normalized photon tunnelling strength J/k_1 for different values of U/k_1, $\Delta/k_1 = 0$ and $k_2/k_1 = 0.1$. The normalised driving strength is $\Omega/k_1 = 0.01$….....110

42. Figure 5.8: Variation of $\left(\Delta X_{a_1 a_2}\right)^2$ and $\left(\Delta Y_{a_1 a_2}\right)^2$ with (a) cavity detuning Δ/k_1 for different values of J/k_1 and $k_2/k_1 = -0.2$ (b) nonlinear strength U/k_1 for $\Delta/k_1 = 0$ and $k_2/k_1 = 0.2$ (c) nonlinear strength U/k_1 for $\Delta/k_1 = 0$ and $k_2/k_1 = -0.2$ (d) normalised photon tunnelling strength J/k_1 for different values of U/k_1, $\Delta/k_1 = 0$ and $k_2/k_1 = 0.1$. The normalised driving strength is $\Omega/k_1 = 0.01$…….....111

43. Figure 5.9: Variation of $e_{a_1 a_2}$ with normalised time Ut for $U/k_1 = 0.1$, $\alpha_1 = 2$, $\alpha_2 = 1$ (solid line); $\alpha_1 = 3$, $\alpha_2 = 1$ (dashed line) with (a) $J/k_1 = 0.2$, $k_2/k_1 = 3$ (b) $J/k_1 = 0.2, k_2/k_1 = -3$ (c)$J/k_1 = 0.5$, $k_2/k_1 = 3$ (d)$J/k_1 = 0.5$, $k_2/k_1 = -1$ (e) $J/k_1 = 0.5$, $k_2/k_1 = -3$ (f) $J/k_1 = 1.0$, $k_2/k_1 = -3$.....113

44. Figure 5.10: Variation of $E_{a_1 a_2}$with normalised time Ut for $\alpha_1 = 2$, $\alpha_2 = 1$ (solid line); $\alpha_1 = 3$, $\alpha_2 = 1$ (dashed line) (a) $J/k_1 = 0.2$, $k_2/k_1 = 3$ (b) $J/k_1 = 0.2, k_2/k_1 = -3$ (c)$J/k_1 = 0.5$, $k_2/k_1 = 3$ (d)$J/k_1 = 0.5$, $k_2/k_1 = -1$(e) $J/k_1 = 0.5$, $k_2/k_1 = -3$ (f) $J/k_1 = 1.0$, $k_2/k_1 = -3$.....115

45. Figure 5.11: Plot of $e_{a_1 a_2}$ with cavity detuning Δ/k_1 for $\Omega/k_1 = 0.01$ (a)-(b) $U/k_1 = 0.1$ and $J/k_1 = 0.5$ (c) $k_2/k_1 = -1$ and $U/k_1 = 0.1$ and (d) $k_2/k_1 = -1$ and $J/k_1 = 0.5$ ……...……116

46. Figure 5.12:Variation of $E_{a_1 a_2}(m, n)$ with normalised time Ut for $U/k_1 = 0.1$, $\alpha_1 = 2$, $\alpha_2 = 1$, $m = 2$, $n = 1$ (solid line), $m = 2$, $n = 2$ (dot-dashed line) and $m = 1$, $n = 2$ (dotted line) (a) $J/k_1 = 0.2$, $k_2/k_1 = 3$ (b) $J/k_1 = 0.2$, $k_2/k_1 = -3$ (c)$J/k_1 = 0.5$, $k_2/k_1 = 3$ (d)$J/k_1 = 0.5$, $k_2/k_1 = -1$(e) $J/k_1 = 0.5$, $k_2/k_1 = -3$ (f) $J/k_1 = 1.0$, $k_2/k_1 = -3$……...……..117

47. Figure 5.13: Plot of $G_{a_1a_2}$ with cavity detuning Δ/k_1 for $\Omega/k_1 = 0.01$ (a)-(b) $U/k_1 = 0.1$ and $J/k_1 = 0.5$ (c) $k_2/k_1 = -1$ and $U/k_1 = 0.5$ and (d) $k_2/k_1 = -1$ and $J/k_1 = 0.5$……...…118

48. Figure 5.14: Plot of $A_{a_1a_2}$ with cavity detuning Δ/k_1 for $\Omega/k_1 = 0.01$ (a)-(b) $U/k_1 = 0.1$ and $J/k_1 = 0.5$ (c) $k_2/k_1 = -1$ and $J/k_1 = 0.5$ and (d) $k_2/k_1 = -1$ and $U/k_1 = 0.1$……..119

49. Figure 5.16: Variation of ξ_{ij} with cavity detuning Δ/k_1 for $J/k_1 = 0.5$, $U/k_1 = 0.1$ and $\Omega/k_1 = 0.01$ (a) a_1a_2 mode with $k_2/k_1 = -1$ (red solid line), $k_2/k_1 = -3$ (blue solid line) and a_2a_1 mode with $k_2/k_1 = -1$ (red dashed line), $k_2/k_1 = -3$ (blue dashed line) (b) a_1a_2 mode with $k_2/k_1 = 1$ (red solid line), $k_2/k_1 = 3$ (blue solid line) and a_2a_1 mode with $k_2/k_1 = 1$ (red dashed line) and $k_2/k_1 = 3$(blue dashed line)…… ……121

50. Figure 5.17: Plot of ξ_{ij} with nonlinearity parameter U/k_1 for $J/k_1 = 0.5$, $\Delta = 0$ and $\Omega/k_1 = 0.01$ for (a) a_1a_2 mode with $k_2/k_1 = -1$ (red solid line) and $k_2/k_1 = -3$ (blue solid line) and a_2a_1 mode with $k_2/k_1 = -1$ (red dashed line) and $k_2/k_1 = -3$ (blue dashed line) (b) a_1a_2 mode with $k_2/k_1 = 1$ (red solid line) and $k_2/k_1 = 3$ (blue solid line) and a_2a_1 mode with $k_2/k_1 = 1$ (red dashed line) and $k_2/k_1 = 3$ (blue dashed line)…… ….121

51. Figure 6.1: Schematic diagram of the PT-symmetric coupled micro-cavities……...127

52. Figure 6.2: Mean cavity photon number as a function of normalised driving strength with different values of mode coupling strength and Kerr-nonlinear strength (a) $J/k_a = 0$ and $U/k_a = 0.1$ (b) $J/k_a = 0.3$ and $U/k_a = 0.1$ (c) $J/k_a = 0.5$, $U/k_a = 0.1$ (d) $J/k_a = 0.7$, $U/k_a = 0.1$ (e) $J/k_a = 0.5$,

$U/k_a = 0.01$ (f) $J/k_a = 0.5$, $U/k_a = 0$; The other parameters are $k_b/k_a = -1$, $\Delta_a/k_a = -1$ and $\Delta_b/k_a = 0.09$……..…….130

53. Figure 6.3: Mean cavity photon number as a function of normalised cavity detuning with different gain-to-loss ratio and mode coupling strength (a) $k_b/k_a = 0.01$ (dashed red), $k_b/k_a = -0.01$(solid blue), $J/k_a = 0.1$ and (b) $k_b/k_a = 0.01, J/k_a = 1$ (c) $k_b/k_a = 0.01$, $J/k_a = 0.1$, $U/k_a = 0.1$ and (b) $k_b/k_a = -0.01, J/k_a = 1$, $U/k_a = 0.1$. The normalised driving strength is $\Omega_d/k_a = 1$…….....132
54. Figure 6.4: Plot of mean cavity photon number as a function of normalised driving strength with $J/k_a = 0.7$, $U/k_a = 0.1$, $\Delta_a/k_a = -1$, $k_b/k_a = -1$ and $\Delta_b/k_a = 0.09$……133
55. Figure 6.5: Variation of $S(\omega)$ as a function of normalized frequency ω/Δ with different mode coupling strength a) $J = 0$ b) $J = 2\pi \times 0.5 \times 5.35\ MHz$ c) $J = 2\pi \times 1.0 \times 5.35\ MHz$ d) $J = 2\pi \times 2.5 \times 5.35\ MHz$. The other parameters are U/Δ =1.07, $\Omega/\Delta = 0.107$, $\Delta = 2\pi \times 1\ MHz$ and $k_2/k_1 = 1$…… 134
56. Figure 6.6: Variation of $S(\omega)$ as a function of normalized frequency ω/Δ with different mode coupling strength a) $J = 2\pi \times 0.5 \times 5.35\ MHz$ b) $J = 2\pi \times 2.0 \times 5.35\ MHz$. The other parameters are U/Δ =1.07, $\Omega/\Delta = 0.107$, $\Delta = 2\pi \times 1\ MHz$ and $k_2/k_1 = 1$……135
57. Figure 7.1: Schematic representation of the model system with two coupled micro-cavities (passive and active). Passive cavity is driven by a strong control field and a weak probe field……141
58. Figure 7.2: Variation of forward transmission rate T_F (blue line) and backward reflection rate T_B (red line) with normalised probe detuning Δ_p/k_a for

$\Delta_a/k_a = -0.1$, $\Delta_b/k_a = 0.0$, $U/k_a = 0.01$, $|a_s| = 0.06$ (a) $k_b/k_a = 0.005$, $J/k_a = 0.2$ (b) $k_b/k_a = 0.005$, $J/k_a = 1.0$ (c) $k_b/k_a = 0.5$, $J/k_a = 0.5$ and (d) $k_b/k_a = 0.5$, $J/k_a = 1.0$……..…..…….146

59. Figure 7.3: Variation of forward transmission rate T_F (blue line) and backward reflection rate T_B (red line) with normalised probe detuning Δ_p/k_a for $\Delta_a/k_a = -0.1$, $\Delta_b/k_a = 0.0$, $U/k_a = 0.01$, $|a_s| = 0.06$, $J/k_a = 0.05$ (a) $k_b/k_a = 0.0001$ (b) $k_b/k_a = 0.001$ (c)$k_b/k_a = 0.01$ (d)$k_b/k_a = 0.05$.....147
60. Figure 7.4: Variation of forward transmission rate T_F (blue line) and backward reflection rate T_B (red line) with normalised probe detuning Δ_p/k_a for $k_b/k_a = 0.005$, $U/k_a = 0.01$, $\Delta_a/k_a = -0.1$, $\Delta_b/k_a = 0.0$, $|a_s| = 0.06$ (a) $J/k_a = 0.005$ (b) $J/k_a = 0.02$ (c) $J/k_a = 0.025$ and (d) $J/k_a = 0.1$..... 148
61. Figure 7.5: Forward transmission rate T_F as a function of probe detuning Δ_p/k_a for $U/k_a = 0.01$, $\Delta_b/k_a = 0$, $J/k_a = 0.05$,$|a_s| = 0.06$ (a) $k_b/k_a = -0.03$, $\Delta_a/k_a = -0.01$ (b) $k_b/k_a = -0.03$, $\Delta_a/k_a = -0.1$ (c) $k_b/k_a = -0.025$, $\Delta_a/k_a = -0.1$ and (d) $k_b/k_a = -0.02$, $\Delta_a/k_a = -0.3$……..…149
62. Figure 7.6: Forward transmission rate T_F as a function of probe detuning Δ_p/k_a for $U/k_a = 0.01$, $\Delta_b/k_a = 0.0$, $J/k_a = 0.05$, $|a_s| = 0.06$ (a) $k_b/k_a = -0.004$, $\Delta_a/k_a = 0.1$ b) $k_b/k_a = -0.003$, $\Delta_a/k_a = 0.1$ c) $k_b/k_a = -0.0025$, $\Delta_a/k_a = 0.1$ d) $k_b/k_a = -0.003$, $\Delta_a/k_a = -0.1$……151