Appendix D

Detailed mathematical analysis of equation (7.7)

$$\frac{d}{dt}\left(A_{-}e^{-i\Delta_p t}+A_{+}e^{i\Delta_p t}\right) = G_1\left(A_{-}e^{-i\Delta_p t}+A_{+}e^{i\Delta_p t}\right)-iJ\left(B_{-}e^{-i\Delta_p t}+B_{+}e^{i\Delta_p t}\right) - i\Omega_p e^{-i\Delta_p t} \quad (D.1)$$

$$\frac{d}{dt}\left(B_{-}e^{-i\Delta_p t}+B_{+}e^{i\Delta_p t}\right) = G_2\left(B_{-}e^{-i\Delta_p t}+B_{+}e^{i\Delta_p t}\right)-iJ\left(A_{-}e^{-i\Delta_p t}+A_{+}e^{i\Delta_p t}\right) \quad \text{(D.2)}$$

Collecting coefficients of $e^{-i\Delta_p t}$ and $e^{i\Delta_p t}$, we obtain

$$A_{-}\left(i\Delta_p+G_1\right) = -iJB_{-}+i\Omega_p$$

$$A_{+}\left(-i\Delta_p+G_1\right) = iJB_{+}$$

$$B_{-}\left(i\Delta_p+G_2\right) = iJA_{-}$$

$$B_{+}\left(-i\Delta_p+G_2\right) = iJA_{+} \quad \text{(D.3)}$$

Using above, we have the results

$$B_{-} = \frac{iJA_{-}}{i\Delta_p+G_2}$$

$$A_{-} = \frac{i\Omega_p\left(i\Delta_p+G_2\right)}{\left(i\Delta_p+G_2\right)\left(i\Delta_p+G_1\right)+J^2} \quad \text{(D.4)}$$

Using expression of G_1 and G_2, we get the followings

$$G_1G_2 = \frac{k_a k_b}{4} - \Delta_a\Delta_b - 4U\Delta_b|a_s|^2 + i\left(\frac{\Delta_a k_b}{2}+\frac{k_a\Delta_b}{2}+2Uk_b|a_s|^2\right) \quad \text{(D.5)}$$

$$i\Delta_p(G_1+G_2) = \Delta_a\Delta_p + \Delta_b\Delta_p - 4U\Delta_b|a_s|^2 - \frac{i\Delta_p}{2}(k_a+k_b) \quad \text{(D.6)}$$