Chapter 1

Introduction

1.1 History of quantum optics

The quantum physics initiated its expedition via Planck's explanation of black body spectrum [1]. His work was later elaborated through the explanation of photo-electric effect [2]. According to Einstein scheme the energy of the radiation field is not only quantized but also it passes over in the quantized form. Followed by these, Bohr atom model, Compton effect, Frank Hertz experiment, stimulated emission, de Broglie waves, Dirac's theory, Schrodinger equation etc, changed our realization in the microscopic world. Quantum mechanics basically explains light-matter interaction in three ways [3]. First, in classical optics, the light is considered as e. m. wave and atom as a dipole. This theory fails to give clear explanation about various optical phenomena. Second, in semi-classical theory where light field is considered same as in classical theory but atomic energy field is treated as quantized. This theory successfully illustrates different optical phenomena like photo-electric effect. But, to explain photoelectric effect Einstein used the concept of light quanta (photon) due to lack of the idea of quantization of energy in atomic system. Third, quantized energy of both atom and light field leads to the new branch of advanced physics i.e. quantum optics. Different optical phenomena those couldn't be explained via classical as well as semiclassical theory but these are explained via quantum optics. Details of these are discussed later in section 2.6.

In quantum mechanics the information of a system may be written in the form of a mathematical function, termed as wave function. It has a finite dimension in Hilbert space and it is presented by a column vector. According to Dirac notation [4] the quantum state is expressed by a Ket $|\varphi\rangle$ and its adjoint known as bra $\langle\varphi|$. For n-dimensional Hilbert space, any quantum state expands into n independent linear state vector called basis vector and such orthogonal basis form a basis set. Again, all quantum mechanical observables are characterized by an operator. These operators give real eigenvalues for a measurement. The information of an observable may be obtained from the interaction of the system with external mechanism.

Quantum optics has been getting serious attention for last few decades. It commences its journey via a remarkable paper by Einstein, Podolosky and Rosen (EPR) in 1935 [5] about peculiar quantum correlation. The ideas of this paper were successfully developed by Bell and Bhom with concrete prediction [6, 7]. This opens up a new topic quantum information processing (QIP). Amplitude interference of the optical field had been investigated via coherence before many years ago but this is first order correlation. In 1950s Hanbury Brown and Twiss (HBT) [8] experimentally worked out intensity correlation and this led to a new path via photon statistics. Using this concept (ref. 5) Rabi, Remsey, Kastler and others [9, 10] had been illustrated light-matter interactions in between 1950s and 60s. In early 1950s, Basov, Prokhorov, Townes and others [11, 12] had developed masers, based on stimulated emission and population inversion. The maser action was extended to optical field and achieved a revolutionized invention of laser. The progresses of the modern technology depend on various types of lasing action.

Glabuer, Sudarshan, Wolf, Mandel and others [13, 14] developed photo detection and quantum theory of coherence. This gave description of nature of light field in phase space using quasi-probability distribution as developed by Wigner and others [15-17].

In early 1970s, tunable lasers were invented and this originated from several ways such as laser spectroscopy, optical nutation, self-induced transparency etc. Stroud and co-workers [18] illustrated theoretically resonance fluorescence. Mandel, Kimble and co-workers [19] discussed antibunching effect via resonance fluorescence and this led to new field – nonclassical light. In between 1980s and 1990s laser cooling method was demonstrated. This opens up a new topic cavity optomechanics (COM). Followed by these, harmonic generation and down-conversion process were demonstrated [20]. After 1980s, squeezed light had been produced with phase sensitive noise and it is a good resource to detect weak force [21, 22].

New basic idea in QIP, leading to quantum computation, cryptography, teleportation, dense coding etc have been developed recently, by Ekert, Bennett, Deutsch and others [23-29]. Different basic experiments about above aspects have been continuing in different laboratories. Quantum gates are also designed via unitary transformation. Quantum algorithm was developed by P. Shor [30], which gives solutions of the practical problems [31-34].

So, quantum optics plays a pivotal role in between atomic physics and laser field. It behaves like a vehicle by which technology rapidly advances. But of course, field remains a trip progressively throwing up unexpected results.

1.2 Objectives

The major objectives of the thesis work are to characterize different nonlinear and nonclassical properties in optical and optomechanical systems. Possible applications of these are also reported. These are summarized below -

1. Characterization of different nonclassical and nonlinear effects in different optomechanical and optical systems.
2. Variations of different nonclassical correlation function with different system parameters have been investigated.

3. Lower as well as higher order variations of nonclassical effects have been analyzed. For higher order study the degree of nonclassical states are enhanced as compared to lower order.
4. Study of different squeezed states may have potential applications for production of low noise signal, sum and difference frequency generation.
5. Photon antibunching may have application in quantum cryptography via generation of single photon source. Entangled states are used for state transfer, quantum information and quantum communication purposes.
6. Asymmetric EPR steering shows stronger correlation as compared to entangled state.
7. Optical system with balanced loss gain shows bistable nature which may be useful for scheming all-optical switch, optical memory element, power limiter and optical sensor. The zero window intensity is tunable.
8. Power spectrum of the output field has been studied in parity-time-symmetric (PT-symmetric) micro-cavities.
9. Optically induced transparency (OIT) has been studied in coupled cavity system and transmission profile shows asymmetric tunable Fano line-shape.

1.3 Outline of the work

The total thesis work is arranged into eight chapters. Chapter 1 contains a brief introduction and fundamentals about quantum systems. A short over view of quantization of energy is presented. The used quantum dynamics for solving different system Hamiltonian are also discussed.

In chapter 2, we have reviewed different optomechanical and optical system which are reported in different earlier theoretical illustrations and experimental works. Details of the mechanism about the cavity optomechanics also have been discussed. The details of different nonclassicalities and their detection through different inequalities are also described.

In chapter 3, we have investigated different nonclassical effects in quadratic optomechanical system (OMS). The nonclassical effects are squeezing for both single and compound field mode, difference and sum squeezing, spin squeezing, quantum statistics of the particles and inseparability.

In chapter 4, we have extensively studied different nonclassical properties in two cavity OMS. Two cavities are coupled via photon tunnelling. The influence of different system parameters on the properties is also reported. Phase dependence of different higher order nonclassicalities is illustrated.

In chapter 5, nonclassicalities in PT-symmetric coupled micro-cavity system have discussed. The possibilities of different nonclassicalities are studied at unbroken and broken regime. Effects of the PT-symmetric phase transition on various nonclassicalities, are analyzed in details. The power spectrum of the output field is also described.

In chapter 6, we have analyzed optical bistability in PT-symmetric micro-cavity system. The influence of the exceptional point (EP) on the bistability effect is discussed. The bistability is explained via hysteresis of the mean cavity photon number. The usefulness of the system as efficient optical switch, optical memory and power limiter are also reported.

In chapter 7, optically induced transparency (OIT) of a probe field in coupled micro-cavity system is illustrated. The forward transmission profile (FTP) and backward reflection profile (BRP) of the output fields are analyzed for passive-passive cavity system (PPCS) and passive-active cavity system (PACS). Asymmetric Fano-line shape of FTP with application is reported.

In chapter 8, we have concluded and discussed the future scopes of present thesis work.

1.4 Quantization

Photon, a fundamental particle of an e. m. wave, has a quantized energy termed as quanta. The quantum is set up by the quantized energy of e. m. wave. Energy of a quantum is defined as $E = \hbar\omega$ where $\hbar$ is reduced Plank constant and ω is the frequency. The quantization of light field is illustrated in context of quantization of e. m. field because photon absorption and emission take place in light radiation process. In addition, e. m. wave carries the electric as well as magnetic characteristics. In next part, we present a brief description about the quantization of e. m. field.

1.4.1 Quantization of electromagnetic field

A total explanation of e. m. field can be addressed by Maxwell's equations [35]. Our focus in the work is the depiction of light field and its properties beyond wave equations. At free space, the Maxwell's equations are

$$\boldsymbol{\nabla}.\boldsymbol{E} = 0$$

$$\boldsymbol{\nabla}.\boldsymbol{B} = 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \tag{1.1}$$

Where $\boldsymbol{E}$ and $\boldsymbol{B}$ represent electric and magnetic field, respectively and speed of e. m. wave is c. From these equations one can be establish general wave equation (same for both the field)

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \tag{1.2}$$

One possible solution may be (likely as) plane wave propagation $\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E_0}\exp(\boldsymbol{K}.\boldsymbol{r} - vt)$. To quantize radiation field [4, 36, 37], we use superposition principle and obtain the expression for single mode $\boldsymbol{E}$, polarized along x-direction and propagating along z-direction is

$$E_x(z,t) = S_j q_j(t)\sin(k_j z) \tag{1.3}$$

where $q_j(t)$ denotes normal amplitude of $j-th$ mode with $k_j = \frac{j\pi}{L}$ (L –length of the cavity resonator) and $S_j = \left(\frac{2m_j v_j^2}{\epsilon_0 V}\right)^2$ having dimension of area with V –volume of the resonator.

Similarly, expression of single mode $\boldsymbol{B}$ polarised along y-direction is

$$B_y(z,t) = S_j \frac{q_j(t)}{c^2 k_j}\sin(k_j z) \tag{1.4}$$

The total energy expressed in the form of classical Hamiltonian as

$$H = \frac{1}{2}\int dV(\epsilon_0 {E_x}^2 + \frac{1}{\mu_0}{B_y}^2) \tag{1.5}$$

Using equation (1.3) and (1.4), integrating and rearranging we obtain

$$H = \frac{1}{2}\sum_j\left(\frac{{p_j}^2}{m_j} + m_j v_j^2 q_j^2\right) \tag{1.6}$$

where $p_j = m_j \dot{q}_j$ is the momentum. Introducing new set of operators by replacing variables

$$\frac{1}{\sqrt{2\hbar m_j v_j}}\left(m_j v_j q_j + i p_j\right) = a_j e^{-i v_j t}$$

$$\frac{1}{\sqrt{2\hbar m_j v_j}}\left(m_j v_j q_j - i p_j\right) = a_j^{\dagger} e^{-i v_j t} \tag{1.7}$$

Simplifying and normal ordering, it is obtained

$$H = \sum_j \hbar v_j \left(a_j^{\dagger} a_j + \frac{1}{2}\right) \tag{1.8}$$

The operators follow commutation relation $\left[a_i, a_j^{\dagger}\right] = \delta_{ij}$. For single field mode the expression of equation (1.8) is similar to harmonic oscillator which plays major role in the field of quantum optics. Another, most important operator is number operator $n_j = a_j^{\dagger} a_j$, it is Hermitian and obeys the following relations

$$\left[a_j,\, n_k\right] = a_j \delta_{jk} \qquad \left[a_j^{\dagger},\, n_k\right] = a_j^{\dagger} \delta_{jk} \tag{1.9}$$

$$a_j |\, n_j\rangle = \sqrt{n_j}\,|\, n_j - 1\rangle \qquad a_j^{\dagger} |\, n_j\rangle = \sqrt{n_j + 1}\,|\, n_j + 1\rangle \tag{1.10}$$

The importance of the quantization of energy of e. m. wave is related to interaction of particles. The interaction leads to energy exchange between the particles. The process of emission or absorption of the light field by two level atoms is related to energy exchange.

1.5 Quantum state

Quantum state implies the state of the system by a vector in a Hilbert space. We discuss different states which are interesting, useful and experimentally producible. In previous section, we have been noted that e. m. field relates to a set of uncoupled modes. For simplicity, we will address single mode and the results can be directly extended for n-modes [38]. Different types of states are thermal state, squeezed state, number state, coherent state etc. In our work we have taken initial state as number and coherent state. Details of these are as follows:

1.5.1 Number or Fock state

From the Hamiltonian of equation (1.8) and number operator $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$, which have real eigenvalues n_j for eigenkets $| n_j \rangle$. We can write the following equation

$$\hat{n}_j | n_j \rangle = n_j | n_j \rangle \tag{1.11}$$

We define $| n_j \rangle$ the number or Fock state. For e. m. field in free space, the ground state is vacuum state. It is lowered by all $\hat{a}_j$.

$$\hat{n}_j | 0_j \rangle = 0 \qquad\qquad \hat{a}_j |\phi\rangle = 0 \tag{1.12}$$

Thus, $$|\phi\rangle = \prod_j | 0_j \rangle = | 0_1 \rangle \otimes | 0_2 \rangle \otimes \ldots \tag{1.13}$$

Using above and equation (1.10), the Fock states takes the form

$$| n_j \rangle = \frac{\left(\hat{a}_j^\dagger\right)^{n_j}}{\sqrt{n_j!}} | 0_j \rangle \tag{1.14}$$

These form orthonormal basis such that $\left\langle n_j \middle| m_j \right\rangle = \delta_{mn}$ and $\sum_{n_j=0}^{\infty} | n_j \rangle\langle n_j | = \hat{I}_j$, $\hat{I}_j$ denotes unity operator.

Fock states with small photon number $|0\rangle$, $|1\rangle$, $|2\rangle, \ldots$ are recently generated in the laboratory. However it is very much challenging to generate Fock state with large photons. In our numerical simulation we use initial state as Fock state (in chapter 3, 4 and 5).

1.5.2 Coherent state

Coherent state implies superposition of Fock states. This is symbolised by $|\alpha\rangle$. For practical applications coherent state has an importance (for example, in pulsed lasers wave packets is produced by coherent state). Here, we have described coherent state as introduced by Glauber [39] (termed as Glauber-Sudarshan states).

We denote unitary displacement operator as

$$\widehat{D}(\alpha) \equiv e^{(\alpha\hat{a}^{\dagger}-\alpha^{*}\hat{a})} \tag{1.15}$$

where α is complex number. For coherent state $|\alpha\rangle$, $|\alpha\rangle \equiv \widehat{D}(\alpha)|0\rangle$, as displaced vacuum. Here, $\widehat{D}(\alpha)$ is similar to raising operator of coherent state. The important property of $|\alpha\rangle$ is

$$\widehat{D}^{\dagger}(\alpha)\hat{a}\,|\alpha\rangle = \alpha\widehat{D}^{\dagger}(\alpha)\,|\alpha\rangle$$

which implies

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle \tag{1.16}$$

In analogy

$$\langle\alpha|\hat{a}^{\dagger} = \alpha^{*}\langle\alpha| \tag{1.17}$$

The operator $\hat{a}$ is non-Hermitian, eigenvalues are complex.

In terms of Fock state representation we proceed as follows:

$$\langle n|\hat{a}|\alpha\rangle = \sqrt{n+1}\langle n+1|\alpha\rangle = \alpha\langle n|\alpha\rangle \tag{1.18}$$

$$\langle n|\alpha\rangle = \frac{\alpha^{n}}{\sqrt{n!}}\langle 0|\alpha\rangle \tag{1.19}$$

Using expansion $\widehat{D}(\alpha) = e^{-|\alpha|^{2}/2}e^{\alpha\hat{a}^{\dagger}}e^{\alpha^{*}\hat{a}}$ we obtain

$$\langle 0|\alpha\rangle = \left\langle 0\middle|\widehat{D}(\alpha)\middle|0\right\rangle = e^{-|\alpha|^{2}/2} \tag{1.20}$$

Thus, equations (1.19) and (1.20), the coherent state expressed via Fock state representation as

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{1.21}$$

In our analytical solution we have taken initial state as a product of coherent states.

1.6 Dynamics of a system

Here, we illustrate different procedures which may be employed to obtain temporal evolution or dynamics of a system for a Hamiltonian. There are various ways in quantum mechanics to obtain the equation of motion (EOM). Most general way to study different system is interaction picture. For special cases, the interaction picture reduces to Heisenberg and Schrodinger picture. The obtained EOM is solved analytically and numerically. We employed Heisenberg EOM and Heisenberg-Langevin EOM for analytical solution (discussed in 1.6.1). The numerical solution can be obtained by Lindbland master equation, Bloch-Redfield (BR) master equation, Floquet-Markov (FM) master equation and Monte-Carlo simulation approaches. In our work, we have used the Lindbland master equation approach as described in subsection 1.6.2.

1.6.1 Heisenberg-Langevin equation

Each and every optical and optomechanical system are defined by corresponding Hamiltonian in general form $H = H_s + H_B + H_I + H_d$; where H_s denotes system Hamiltonian, H_B is the bath Hamiltonian weakly connected to the system, H_I is the interaction part and H_d denotes external driving part. The Hamiltonian is solved analytically by Heisenberg EOM or Heisenberg-Langevin EOM. The corresponding equation of an operator $\hat{a}$ is given by

$$\dot{\hat{a}} = -i[\hat{a}, H] + \sum_j \left\{-\left[\hat{a}, \hat{x}_j^\dagger\right]\left(\frac{k_j}{2}\hat{x}_j + \sqrt{k_j}\hat{x}_{j,in}\right) + \left(\frac{k_j}{2}\hat{x}_j^\dagger + \sqrt{k_j}\hat{x}_{j,in}^\dagger\right)\left[\hat{a}, \hat{x}_j\right]\right\} \quad (1.22)$$

where k_j and $\hat{x}_{j,in}$ denote the decay rate and input flux of system operator $\hat{x}_j$.

The temporal evolution of an operator $a(t)$ follows the Heisenberg EOM as

$$a(t) = e^{iHt}a(0)e^{-iHt} \quad (1.23)$$

Expanding above equation (1.23) we obtain

$$a(t) = a(0) + it[H, a(0)] + (it)^2[H, [H, a(0)]\,] + \cdots \quad (1.24)$$

Using equation (1.22) and (1.24), we have solved the system Hamiltonians of chapter 3, 4 and 5.

1.6.2 Master equation

Again, to calculate numerically the different nonclassical correlation functions we have used Lindblad's master equation approach. The Lindblad's master equation is [40] given by

$$\dot{\rho} = -i[H, \rho] + L_\rho \quad (1.25)$$

Where Lindblad super-operator $L_\rho = L_{a_1}(\rho) + L_{a_2}(\rho)$ with $L_a(\rho) = \frac{k_j}{2}(n_a^T + 1)(2a\rho\, a^\dagger - a^\dagger a\rho - \rho\, a^\dagger a) + \frac{k_j}{2}n_a^T(2a\rho\, a^\dagger - a^\dagger a\rho - \rho\, a^\dagger a)$ where n_a^T is thermal photon/phonon number.