Chapter 8

Integrated-inventory model with stackelberg game*

8.1 Introduction

In many Stackelberg approach models, decision makers in the leading and following positions formulate their own decisions individually. Those Stackelberg game models considering the assumption that leader first selects his decisions and then decisions are becoming the constraint to the follower. Besides, concept of follower's decision made on the whole data of the leader's action and his decisions turn the new constraints to that leader's decision problem. Initially, Basu (1995) established a managerial delegation model in a duopoly with the fact that if an owner's decision to operate a manager. He also introduced Stackelberg solution in his model. Chen and Zadrozny (2002) obtained an inventory model which incurs continuous feedback solution for a dynamic Stackelberg game. Mukaidani (2007) considered the joint-calculation of linear closed-form Stackelberg

*A part of this work, presented in this chapter, is published in *Sustainability*, 8(12), 1244 pages, 2016.

policies along with low singular perturbation parametric quantity system. In this direction, Liu *et al.* (2013) formulated some Stackelberg game strategies, which incurs a leader-follower model to analyze decision makers, also a mathematical segment that allowed a sub-optimization problems in the form of constraints. Wang *et al.* (2016) provided a win-win outcome for his Stackelberg game model. By examining two decision makers of that game, supplier along with a threshold and partial trade-credit policy is considered to adjust that model.

Generally, an integrated-production-inventory model defines vendor-buyer or retailer-customer model. In that type of model, usually a vendor produces an item in a production batch line and the produced finished goods are transferred to several buyers. In spite of that, retailer's production cycle time is measured as an integer multiple of the function of consumers ordering time. Many research articles highlighted various integrated vendor-buyer models with several key parameters. Chang *et al.* (2009) discussed an integrated vendor-buyer inventory system under trade-credit policies. Demand is measured as the diminishing function of retail-price. Additionally, they surveyed an iterative algorithm to figure out optimal retail-price, number of buyers ordering amount, and numbers of deliveries per production run. Yang (2010) extended previous research articles by considering an integrated-inventory model with lead time crashing cost. Hoque (2013) analyzed a vendor-buyer integrated-production model, where lead time follows a normal distribution. On the other hand, setup time of a machine, maximum boundary on capacity of shipping vehicle, cost of transportation, and batch time are also inserted in his model. Jha and Shanker (2014) established an integrated inventory model with transportation for a single-vendor and multi-buyer. It is observed that the vendor manufactures products and deliveries that products to buyers in distinct locations by similar capability of some vehicles. The external demands of buyers are taken to be independent and follows a normal distribution. Lead time of buyers without transportation time are reduced by an added crashing cost. Most of the above mentioned research articles related with integrated-inventory model are formed with the assumption that all produced items are absolutely perfect. There are no defective items during production system. Sarkar *et al.* (2014) expanded former integrated-inventory models by including the concept of imperfect production. An inspection policy is given to examine defective items and also provides delay-in-payments in their model. In their model, non-defective item follows a binomial distribution and lead time demand follows a mixture of normal distribution. By highlighting the trade-credit policy, Ouyang *et al.* (2015) derived an integrated-inventory model with a capacity constraint and a permissible delay-payment system. An unit production cost is calculated as the function of rate of production.

Setup cost plays an important role in today's advanced manufacturing companies for shipment of products on time. Setup process is not measured as a value adding constraint. Setup cost need to be discussed at the time of enhancing productivity, minimizing waste, enlarging resource utilization, and satisfy deadlines. To minimize capital investment function, manufacturer are required to reduce setup cost. Researchers made various inventory models with this concept of setup cost reduction. Denizel *et al.* (1997) studied a dynamic lot-size model in which setup costs can be reduced by several amounts depending upon the level of raw-materials. They also derived a shortest path problem for these level of raw-materials. Diaby (2000) established a comprehensive model to reduce both setup time and setup cost. He also added that setup times can be reduced by contributing appropriate amounts of many resources like equipment, tooling. He determined how much to cut setup time for every product and how much of each good to manufacture to minimize total cost. Nyea *et al.* (2001) developed some inventory models to forecast optimal setup times, or optimal investment in setup reduction. In their paper, a new model based on queuing theory was formed to estimate work-in-process (WIP) levels. Freimer *et al.* (2006) established two types of process improvements which are (i) setup costs reduction and (ii) improvement in quality of the process. Annadurai and Uthayakumar (2010) analyzed a mixture-inventory model with the assumptions of setup cost reduction, backorders, and lost sales. They considered that an arrival order batch may hold some defective items. Both normally distributed demand and distribution free demand are depicted in their model. Allahverdi (2015) obtained independently addressed problems based on performance observers, shop, and setup times/costs environments.

After receiving the order from any consumer, components and materials are shifted for production line, and then finally finished goods are sent to consumers by some transportation vehicle to meet their requirements within due dates. Therefore, transportation cost is an additional charge to manufacturer. Transportation cost can be dependent on delivery path, capacity of delivery vehicle. Hill and Galbreth (2008) proposed a single-warehouse multi-retailer supply chain model, which incurs transportation discount cost functions. Kang and Kim (2010) surveyed a two-level supply chain model, where a supplier provides a set of retailers and derives a production plan for every retailer by applying the data on demands of end consumers. Transportation costs are included in their model during shipment of finished products. Deliveries are fulfilled through same capability vehicles to several retailers in a one-time trip. Chan and Zhang (2011) determined a collaborative transportation management model with a simulation approach, which are utilize to (a) analyze profits of CTM, (b) describe view-point of carriers flexibility, and also (c) examine delivery speed ability. Lee and Fu (2014) proposed a producer-buyer supply chain model in which delivery or transportation cost is added and adjusted as a power function. Shu *et al.* (2015) observed an integrated-production-delivery lot-size model with stochastic delivery time and transportation cost, which is the function of delivery quantity.

For reducing carbon-emission, production companies can monitor and enhance the emission performance of their products during its life-cycle stages. The carbon-emission assessment provides a possible mechanism to serve companies some emission reduction. During production process, manufacturer is to formulate low carbon system. There are several research papers, in which carbon-emission reduction is described briefly. To reduce global warming and carbon-emission from earth, Shi and Meier (2012) presented a hybrid carbon-emission model to meet up increasing necessities of practical low-carbon matters in manufacturing systems. Shi *et al.* (2012) described carbon-emission reduction potential model for technology disruption and structural allowance in cement factory. They determined energy consumption and also derived the effects and trends of technological advancement. Zhang *et al.* (2013) generated a both split and traffic assignment model, which assumed the low-carbon constraints. Their model analyzed in particular two hypothetical examine networks. Hammami *et al.* (2015) deduced a multi-echelon supply chain model with different outside suppliers, several manufacturing facilities, distinct distribution centers, and reducing carbon-emission. Tang *et al.* (2015) presented a periodic inventory review system by reducing carbon-emission with minimum shipment frequency. See Table 8.1 for contribution of various authors.

Table 8.1: Contribution of various authors

Author(s)	**Integrated-inventory model**	**Setup cost reduction**	**Transportation cost**	**Carbon-emission reduction**	**Stackelberg approach**
Basu (1995)					√
Freimer *et al.* (2006)		√			
Mukaidani (2007)					√

Author(s)	**Integrated-inventory model**	**Setup cost reduction**	**Transportation cost**	**Carbon-emission reduction**	**Stackelberg approach**
Hill and Galbreth (2008)			√		
Chang *et al.* (2009)	√				
Yang (2010)	√				
Annadurai and Uthayakumar (2010)		√			
Kang and Kim (2010)	√		√		
Chan and Zhang (2011)			√		
Shi and Meier (2012)				√	
Zhang *et al.* (2013)				√	
Liu *et al.* (2013)					√
Hoque (2013)	√		√		
Jha and Shanker (2014)	√		√		
Sarkar *et al.* (2014)	√				
Lee and Fu (2014)	√		√		
Shu *et al.* (2015)	√		√		
Ho *et al.* (2015)				√	

Author(s)	**Integrated-inventory model**	**Setup cost reduction**	**Transportation cost**	**Carbon-emission reduction**	**Stackelberg approach**
Hammami *et al.* (2015)	√			√	
Tang *et al.* (2015)				√	
Ouyang *et al.* (2015)	√				
Allahverdi (2015)		√			
Wang *et al.* (2016)					√
This chapter	√	√	√	√	√

In this chapter, an integrated-inventory model is derived, where an investment function is used to minimize vendor's setup cost. This chapter discussed about transportation cost with carbon-emission cost which are fixed and variable. Buyer incurs two types of inspection costs during testing the quality of received lots. This chapter also formulated two models one with Stackelberg approach and another without Stackelberg approach. Finally, some numerical examples are depicted to show the optimality of the vendor-buyer system's joint total cost.

8.2 Mathematical model

This chapter considers the following notation.

Decision variables

I investment for setup cost reduction per production run

δ rate of increasing delivery lots (positive integer)

n number of delivery lots of each batch per production (positive integer)

Q first shipment lot-size per batch throughout the production (units)

Parameters

D demand rate (units/year)

P production rate (units/year)

V_0 setup cost at the initial stage (\$/setup)

V_s vendor's setup cost after applying the investment (\$/setup)

h_v vendor's holding cost (\$/unit/year)

C_v vendor's fixed carbon-emission cost (\$/delivery)

V_v vendor's variable carbon-emission cost (\$/unit)

F vendor's fixed transportation cost (\$/delivery)

V_t vendor's variable transportation cost (\$/unit)

A_b buyer's ordering cost (\$/order)

V_i buyer's variable inspection cost (\$/delivery)

U_i buyer's unit inspection cost (\$/unit item inspected)

α inspection rate (units/year)

h_{b_1} buyer's holding cost for perfect items (\$/unit/year)

h_{b_2}	buyer's holding cost for imperfect items (\$/unit/year)
R_v	vendor's rework cost (\$/unit)
ρ	defective rate (units/year)
Th_v	vendor's total holding cost (\$/year)
Th_{b_1}	buyer's total holding cost of perfect items (\$/year)
Th_{b_2}	buyer's total holding cost of imperfect items (\$/year)
Tr_v	vendor's total transportation cost (\$/year)
CE_v	vendor's total carbon-emission cost (\$/year)
TC_b	buyer's total cost (\$/year)
TC_v	vendor's total cost (\$/year)
JTC_{vb}	vendor-buyer system's joint total cost (\$/year)

This chapter is considered on the basis of the following assumptions.

1. An integrated-inventory model is considered with single-buyer and single-vendor for single-type of items. To reduce setup cost, some discrete investment I is considered. Therefore, the expression of new setup cost becomes $V_s(I) = V_0 e^{-\kappa I}$, where κ is a known parameter.

2. Vendor transported delivery lots in a dissimilar size. Each shipment lots increases at a rate δ.

3. Fixed and variable carbon-emission costs along with fixed and variable transportation costs are associated with vendor.

4. At the moment buyer receives delivery lots from vendor, then the buyer starts an inspection process for classifying perfect and imperfect goods.

5. After classifying defective goods, buyer delivers those goods during the next lot comes from the vendor to rework.

6. It is considered that buyer does not pay any transportation cost as well as carbon-emission cost during delivery of defective goods.

7. Demand and production rates are assumed as constant.

8. Shortages are not considered as rate of production is bigger than the rate of demand i.e., $P > D$.

9. Lead time is taken as negligible.

Initially, buyer orders some products with ordering cost A_b. Vendor produced items with a fixed production rate P and initial setup cost of vendor is V_0. Vendor incorporated some investments I for reducing that setup cost. Vendor shipped first lots of each batch i.e., Q units with some fixed transportation cost F as well as variable transportation cost V_t. Vendor continues the whole delivery products in n times. Initially, first shipment lot-size per batch is Q. It is assumed that the increasing rate of delivery lots as δ. Therefore, vendor's second shipment lot-size is δQ. Vendor shipped third shipment lot-size as $2\delta Q$. In this way, it can be found that the number of quantity transferred to buyer on yth delivery is $(y-1)\delta Q$, $y > 1$. Throughout unequal delivery goods, vendor pays fixed carbon-emission cost C_v and also variable carbon-emission cost V_v. While that delivery lots placed to buyer, then buyer performs an inspecting procedure to check the quality of the received lots. Buyer incurs two types of inspection costs, which are variable inspection cost

V_i and unit inspection cost U_i, where screening rate is α. The rate of imperfect item is ρ in each lot. When the inspection has been completed, both perfect and imperfect items are separated. For holding perfect items, buyer incurs some cost h_{b_1}. In addition, buyer's holding cost for imperfect items is h_{b_2}. While next produced lot comes to buyer from vendor, buyer sent back all imperfect products of previous lot to vendor for reworking. In this case, it is assumed that buyer has not pay any delivery cost for shifting imperfect products to vendor.

Vendor's mathematical model

Vendor shipped lot of orders in n times in each production cycle by using single-setup-multi-delivery (SSMD) policy. After the shipment of lot-size Q, the number of quantity transmitted from vendor to buyer on yth delivery is $(y-1)\delta Q$, $y > 1$. For this reason, second delivery lot-size is δQ. Then, transported lot-sizes are $2\delta Q$, $3\delta Q$, and so on.

One can measured the total production batch, which shipped to buyer from vendor is formulated by adding the whole delivery lots

$$Q + \delta Q + 2\delta Q + ... + (n-1)\delta Q \quad = \quad Q + \frac{\delta Q n(n-1)}{2}$$

Total production cycle is obtained by splitting the demand with total production batch.

$$\frac{D}{Q + \delta \frac{n(n-1)}{2} Q} \quad = \quad \frac{2D}{2Q + Q\delta n(n-1)}.$$

As the number of total production cycle is $\frac{D}{Q+\delta\frac{n(n-1)}{2}Q} = \frac{2D}{2Q+Q\delta n(n-1)}$, setup cost is V_0, and investment to minimize setup cost is I.

Vendor's total setup cost is $V_0 e^{-\kappa I}\left(\frac{2D}{2Q+\delta n(n-1)Q}\right)$.

Vendor's total investment cost is $= \frac{2DI}{2Q+\delta n(n-1)Q}$.

At the beginning, while the production batch is around to commence, systems's total stock is starting with zero. On the other hand, buyer has sufficient stock to meet satisfy the demand before the first delivery lot comes. Buyer stock is $\frac{DQ}{P}$. The total stock inclined at a rate of $P-D$ while producing the batch quantity of $Q+\frac{\delta Qn(n-1)}{2}$ with the rate P and arrives the maximum level of $\frac{DQ}{P}+(P-D)\left(\frac{2Q+\delta n(n-1)Q}{2P}\right)$ when manufacturing of batch completed.

Therefore, system's average total stock is

$$= \frac{DQ}{P}+(P-D)\left(\frac{2Q+\delta n(n-1)Q}{4P}\right)$$

Then average vendor stock can be measured by deducting total quantity of perfect and imperfect products from average total stock, which is

$$= \frac{DQ}{P}+(P-D)\left(\frac{2Q+\delta n(n-1)Q}{4P}\right)-\left(\frac{Q(1-\rho)^2[2+\delta n(n-1)]}{4}\right)-\left(\frac{Q\rho D[2+\delta n(n-1)]}{2\alpha}\right)$$

Vendor's total holding cost is

$$\begin{aligned} Th_v &= H_v\Big[(P-D)\left(\frac{2Q+\delta n(n-1)Q}{4P}\right)-\left(\frac{Q(1-\rho)^2[2+\delta n(n-1)]}{4}\right)-\left(\frac{Q\rho D[2+\delta n(n-1)]}{2\alpha}\right) \\ &+ \frac{DQ}{P}\Big] \end{aligned}$$

Vendor's total transportation cost is measured by adding fixed and variable transportation costs which is

$$Tr_v = \frac{2nFD}{2Q+\delta n(n-1)Q}+V_t\rho D$$

Vendor's total carbon-emission cost can be obtained by calculating fixed and variable carbon-emission costs.

i.e., $CE_v=\frac{2nC_vD}{2Q+\delta n(n-1)Q}+V_v\rho D$.

Total rework cost for vendor is $= R_v\rho D$.

Then, the vendor's total inventory cost can be obtained by summing setup cost, holding cost, investment cost to minimize setup cost, fixed and variable transportation cost, fixed as well as variable carbon-emission cost, and rework cost.

$$\begin{aligned} TC_v(n,Q,\delta,I) &= \frac{2D}{2Q+\delta n(n-1)Q}(V_0 e^{-\kappa I} + nC_v + I + nF) + \rho D(V_v + V_t + R_v) + h_v Q\Big[\frac{D}{P} \\ &+ (2+\delta n(n-1))\left(\frac{(P-D)}{4P} - \frac{(1-\rho)^2}{4} - \frac{\rho D}{2\alpha}\right)\Big] \end{aligned}$$

Buyer's mathematical model

Buyer incurs total ordering cost for whole production cycle is $A_b\left(\frac{2D}{2Q+\delta n(n-1)Q}\right)$.

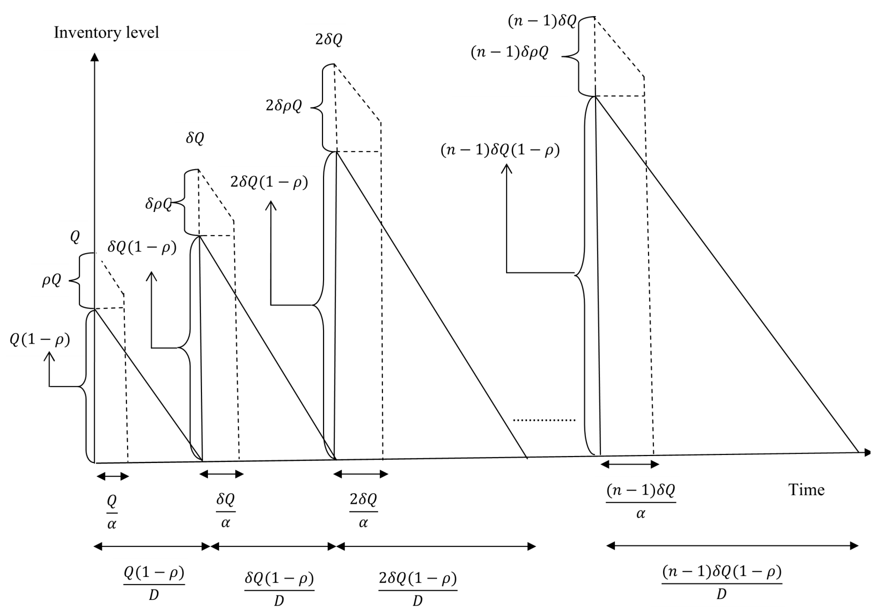

Figure 8.1: Inventory positions of buyer

During inspecting process, buyer considers two types of inspection costs i.e., unit as well as variable inspection cost.

Total inspection cost for buyer is $DU_i + \frac{2nV_iD}{2Q+\delta n(n-1)Q}$.

The total number of perfect products for whole production cycle is observed from the area of the triangle given in Figure 8.1, which is obtained as

$$\frac{Q(1-\rho)}{2}\frac{(1-\rho)Q}{D}+\frac{\delta Q(1-\rho)}{2}\frac{\delta(1-\rho)Q}{D}+...+\frac{(n-1)\delta Q(1-\rho)}{2}\frac{(n-1)\delta(1-\rho)Q}{D}$$
$$=\frac{1}{2}\frac{Q^2(1-\rho)^2}{D}\Big[1+\frac{\delta^2 n(n-1)(2n-1)}{6}\Big]$$

Hence, buyer's total holding cost for perfect products Th_{b_1} is obtained by multiplying all perfect products with production cycle i.e.,

$$\begin{aligned} Th_{b_1} &= h_{b_1}\left[\frac{1}{2}\frac{Q^2(1-\rho)^2}{D}\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)\right]\left[\frac{2D}{2Q+\delta Qn(n-1)}\right] \\ &= h_{b_1}\left[\left(\frac{Q(1-\rho)^2}{2+\delta n(n-1)}\right)\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)\right] \end{aligned}$$

The total quantity of imperfect products is calculated by the parallelogram shown in Figure 8.1.

$$Q\rho\frac{Q}{\alpha}+\delta Q\rho\frac{\delta Q}{\alpha}+...+\delta(n-1)Q\rho\frac{(n-1)\delta Q}{\alpha} = \frac{Q^2\rho}{\alpha}\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)$$

Buyer's total holding cost for imperfect products Th_{b_2} is given by multiplying total imperfect products with production cycle.

$$\begin{aligned} Th_{b_2} &= h_{b_2}\left[\frac{Q^2\rho}{\alpha}\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)\right]\left[\frac{2D}{2Q+\delta Qn(n-1)}\right] \\ &= h_{b_2}\left[\frac{2DQ\rho}{\alpha(2+\delta n(n-1))}\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)\right] \end{aligned}$$

Therefore, buyer's total inventory cost can be determined by adding ordering cost, inspection cost, holding cost of perfect products items, and imperfect products.

$$\begin{aligned} TC_b(n,Q,\delta) &= \frac{2D}{2Q+\delta n(n-1)Q}(A_b+nV_i)+DU_i+\Big((1-\rho)^2 h_{b_1}+\frac{2D\rho}{\alpha}h_{b_2}\Big)\Big(1 \\ &+ \frac{\delta^2 n(n-1)(2n-1)}{6}\Big)\frac{Q}{(2+\delta n(n-1))} \end{aligned}$$

Hence, vendor-buyer system's joint total cost JTC_{vb} is given by

$$\begin{aligned} JTC_{vb}(n,Q,\delta,I) &= \frac{2D}{2Q+\delta Qn(n-1)}(V_0e^{-\kappa I}+nC_v+I+nF+A_b+nV_i)+DU_i+\rho D(V_v+V_t \\ &+ R_v)+h_v\Big[\Big(\frac{(P-D)}{2P}-\frac{(1-\rho)^2}{2}-\frac{\rho D}{\alpha}\Big)\left(\frac{2Q+\delta n(n-1)Q}{2}\right)+\frac{DQ}{P}\Big] \\ &+ \Big(\frac{2D\rho}{\alpha}h_{b_2}+(1-\rho)^2h_{b_1}\Big)\frac{Q}{2+\delta n(n-1)}\Big(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\Big) \end{aligned}$$

The necessary conditions to minimize the vendor-buyer system's joint total cost JTC_{vb} are $\frac{\partial JTC_{vb}}{\partial I}=0$, $\frac{\partial JTC_{vb}}{\partial Q}=0$, $\frac{\partial JTC_{vb}}{\partial \delta}=0$, and $\frac{\partial JTC_{vb}}{\partial n}=0$.

The first order partial derivative of vendor-buyer system's joint total cost JTC_{vb} with respect to number of delivery lots of each batch per production n is

$$\begin{aligned} \frac{\partial JTC_{vb}}{\partial n} &= \frac{2DR_3}{2Q+\delta Qn(n-1)}-\frac{2DQ\delta(2n-1)}{(2Q+\delta Qn(n-1))^2}(V_0e^{-\kappa I}+nR_3+I+A_b)+\frac{h_vR_1\delta Q(2n-1)}{2} \\ &+ R_2\left[\frac{(2n^3-3n^2+n)Q}{6(2+\delta n(n-1))}-\left(\frac{\delta^2 n(n-1)(2n-1)}{6}+1\right)\frac{Q\delta(2n-1)}{2+\delta n(n-1)}\right] \end{aligned}$$

By calculating $\beta(n^*)=0$, where $\beta(n)=\frac{\partial JTC_{vb}}{\partial n}$, the optimal value of n (say n^*) is obtained.

See Appendix A5 for the values of R_1, R_2, and R_3. The first order partial derivative of vendor-buyer system's joint total cost JTC_{vb} with respect to first shipment lot-size per batch throughout the production Q is

$$\begin{aligned} \frac{\partial JTC_{vb}}{\partial Q} &= \frac{2D}{Q^2(2+\delta n(n-1))}(V_0e^{-\kappa I}+nC_v+I+nF+A_b+nV_i)-h_v\Big[R_1\Big(\frac{(2+\delta n(n-1))}{2} \\ &+ \frac{D}{P}\Big)\Big]+\frac{R_2}{2+\delta n(n-1)}\left(\frac{\delta^2 n(n-1)(2n-1)}{6}+1\right) \end{aligned}$$

The optimum value Q^* is given by

$$Q^* = \sqrt{\frac{2D(V_0e^{-\kappa I}+nR_3+I+A_b)}{\Big[h_v(2+\delta n(n-1))\left(R_1\frac{(2+\delta n(n-1))}{2}+\frac{D}{P}\right)+\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)R_2\Big]}}$$

Now, the first order partial derivative of vendor-buyer system's joint total cost JTC_{vb} with respect to investment for setup cost reduction per production run I is

$$\frac{\partial JTC_{vb}}{\partial I} = \frac{2D}{2Q+\delta n(n-1)Q}(1-V_0\kappa e^{-\kappa I})$$

From the equation $\frac{\partial JTC_{vb}}{\partial I} = 0$, the optimal value of I (say I^*) will be $I^* = \frac{1}{\kappa} ln(V_0\kappa)$.

Again, the first order partial derivative of vendor-buyer system's joint total cost JTC_{vb} regarding rate of increasing delivery lots δ is

$$\begin{aligned}\frac{\partial JTC_{vb}}{\partial \delta} &= -\frac{2DQn(n-1)}{2Q+\delta Qn(n-1)^2}(V_0e^{-\kappa I}+nR_3+I+A_b)+\frac{h_vR_1n(n-1)Q}{2} \\ &+ R_2\left[\frac{\delta(2n^3-3n^2+n)Q}{6(2+\delta n(n-1))} - \frac{Qn(n-1)}{(2+\delta n(n-1))^2}\left(\frac{\delta^2 n(n-1)(2n-1)}{6}+1\right)\right]\end{aligned}$$

Similarly as n, I, and Q, in this case the optimal value of δ (say δ^*) can be calculated if it satisfies $\xi(\delta^*) = 0$, where $\xi(\delta) = \frac{\partial JTC_{vb}}{\partial \delta}$.

Now, the Hessian matrix at the optimal values i.e., n^*, Q^*, δ^*, and I^* are calculated as

$$H_{ii} = \begin{bmatrix} \frac{\partial^2 JTC_{vb}(cdot)}{\partial {I^*}^2} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial I^* \partial Q^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial I^* \partial n^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial I^* \partial \delta^*} \\ \frac{\partial^2 JTC_{vb}(cdot)}{\partial Q^* \partial I^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial {Q^*}^2} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial Q^* \partial n^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial Q^* \partial \delta^*} \\ \frac{\partial^2 JTC_{vb}(cdot)}{\partial n^* \partial I^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial n^* \partial Q^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial {n^*}^2} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial n^* \partial \delta^*} \\ \frac{\partial^2 JTC_{vb}(cdot)}{\partial \delta^* \partial I^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial \delta^* \partial Q^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial \delta^* \partial n^*} & \frac{\partial^2 JTC_{vb}(cdot)}{\partial {\delta^*}^2} \end{bmatrix}$$

where $JTC_{vb}(cdot) = JTC(n^*, Q^*, \delta^*, I^*)$.

The optimal values n^*, I^*, Q^*, and δ^* for minimize vendor-buyer system's joint total cost JTC_{vb} must fulfil the conditions that all principal minors of the Hessian matrix H_{ii} are positive. As, the expressions of second order partial derivatives of JTC_{vb} are non-linear (See Appendix A6), thus each principal minors of the Hessian matrix H_{ii} are extremely non-linear. Hence, those conditions to minimize vendor-buyer system's joint total cost JTC_{vb} are determined by considering some numerical examples and graphical representations.

8.3 Stackelberg approach

Within the supply chain, it is not always possible that players of the chain are with equal power always. Sometime, retailer is more powerful like different shopping malls, sometimes manufacturer are more powerful like Microsoft. Then, it cannot be considered only joint total cost. It is required to assume someone as leader and someone as follower based on the more dominating nature of the players. For this general model, each one depicted as leader and each one as follower and optimize the cost.

Case 1 While buyer as leader and vendor as follower

Using Stackelberg approach, vendor's cost function is optimized with respect to four decision variables namely n, Q, δ, and I.

Vendor's cost function is

$$\begin{aligned} TC_v(n,Q,\delta,I) &= \frac{2D}{2Q+\delta n(n-1)Q}(V_0e^{-\kappa I}+nC_v+I+nF)+\rho D(V_v+V_t+R_v)+h_vQ\Big[\frac{D}{P} \\ &+ (2+\delta n(n-1))\left(\frac{(P-D)}{4P}-\frac{(1-\rho)^2}{4}-\frac{\rho D}{2\alpha}\right)\Big] \end{aligned}$$

The first order partial derivative of TC_v with respect to I is given by

$$\frac{\partial TC_v}{\partial I} = \frac{2D}{2Q+\delta n(n-1)Q}(1-V_0\kappa e^{-\kappa I})$$

The optimum value of I (say I^*) is $I^* = \frac{1}{\kappa}ln(V_0\kappa)$.

Equating the first order partial derivative of TC_v with respect to Q to zero, which is

$$\begin{aligned} \frac{\partial TC_v}{\partial Q} &= \frac{2D}{Q^2(2+\delta n(n-1))}(V_0e^{-\kappa I}+nC_v+I+nF)-D\rho(V_v+V_t+R_v)-h_v\Big(\frac{D}{P} \\ &+ \frac{(2+\delta n(n-1))}{2}R_1\Big) \end{aligned}$$

The optimum value Q^* is calculated as follows:

$$Q^* = \sqrt{\frac{2D(V_0e^{-\kappa I}+nC_v+I+nF)}{\left(R_4+h_v\left(\frac{D}{P}+\frac{(2+\delta n(n-1))}{2}R_1\right)\right)(2+\delta n(n-1))}}$$

[See Appendix A5 for the values of R_4.]

δ^* will be evaluated from the following equation, which is the first order partial derivative of TC_v with respect to δ.

$$\frac{\partial TC_v}{\partial \delta} = \frac{h_v Qn(n-1)R_1}{4DQn(n-1)(V_0 e^{-\kappa I} + nC_v + I + nF)} - \frac{1}{(2Q + \delta Qn(n-1))^2}$$

Therefore,

$$\delta^* = \frac{1}{Qn(n-1)}\sqrt{\frac{2DQn(n-1)(V_0 e^{-\kappa I} + nC_v + I + nF)}{h_v Qn(n-1)\frac{R_1}{2}}} - 2Q$$

Finally, the first order derivative of TC_v with respect to n is as follows:

$$\begin{aligned}\frac{\partial TC_v}{\partial n} &= \frac{2D}{2Q + \delta Qn(n-1)}\left(\frac{\delta Q(1-2n)(V_0 e^{-\kappa I} + nC_v + I + nF)}{2Q + \delta n(n-1)} + (C_v + F)\right) \\ &+ h_v Q\delta(2n-1)\frac{R_1}{2}\end{aligned}$$

Therefore, the optimum value of n (say n^*) will be obtained by equating $\phi(n^*) = \frac{\partial TC_v}{\partial n} = 0$

Substituting all optimum values i.e., I^*, Q^*, n^*, and δ^* into the buyer's cost function, the optimized buyer's cost function can be obtained as

$$\begin{aligned}TC_b(n^*, Q^*, \delta^*) &= \frac{2D}{2Q^* + \delta^* n^*(n^*-1)Q}(A_b + n^* V_i) + DU_i + \left((1-\rho)^2 h_{b_1} + \frac{2D\rho}{\alpha}h_{b_2}\right)\Big(1 \\ &+ \frac{\delta^{*2} n^*(n^*-1)(2n^*-1)}{6}\Big)\frac{Q^*}{(2 + \delta^* n^*(n^*-1))}\end{aligned}$$

Case 2 While vendor as leader and buyer as follower

In this case buyer's cost function is optimized with respect to three decision variables namely n, Q, and δ.

Buyer's cost function is

$$\begin{aligned}TC_b(n, Q, \delta) &= \frac{2D}{2Q + \delta n(n-1)Q}(A_b + nV_i) + DU_i + \left((1-\rho)^2 h_{b_1} + \frac{2D\rho}{\alpha}h_{b_2}\right)\Big(1 \\ &+ \frac{\delta^2 n(n-1)(2n-1)}{6}\Big)\frac{Q}{(2 + \delta n(n-1))}\end{aligned}$$

The first order partial derivative of TC_b with respect to Q is given by

$$\frac{\partial TC_b}{\partial Q} = \frac{2D(A_b+nV_i)}{Q^2(2+\delta n(n-1))} - \frac{DU_iR_2}{2+\delta n(n-1)}\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)$$

The optimum value of Q (say Q^*) is $Q^* = \sqrt{\frac{2(A_b+nV_i)}{U_iR_2\left(1+\frac{\delta^2 n(n-1)(2n-1)}{6}\right)}}$.

The optimal value of δ (say δ^*) will be found from the equation $\psi(\delta^*) = 0$

where

$$\begin{aligned}\psi(\delta) &= \frac{\partial TC_b}{\partial \delta} = DU_i\Big(\frac{Qn(n-1)R_2}{(2+\delta n(n-1))}\Big(\frac{\delta(2n-1)}{3} - \frac{1}{2+\delta n(n-1)}\Big(1\\ &+ \frac{\delta^2 n(n-1)(2n-1)}{6}\Big)\Big)\Big) - \frac{2Dn(n-1)(A_b+nV_i)}{Q(2+\delta n(n-1))^2}\end{aligned}$$

Putting all optimum values i.e., Q^*, δ^*, and n^* into the vendor's cost function, the optimized vendor's cost function can be obtained as

$$\begin{aligned}TC_v(n^*,Q^*,\delta^*,I) &= \frac{2D}{2Q^*+\delta^* n^*(n^*-1)Q^*}(V_0e^{-\kappa I}+n^*C_v+I+n^*F)+\rho D(V_v+V_t\\ &+ R_v)+h_vQ\left[\frac{D}{P}+(\delta^* n^*(n^*-1)+2)\left(\frac{(P-D)}{4P}-\frac{(1-\rho)^2}{4}-\frac{\rho D}{2\alpha}\right)\right]\end{aligned}$$

The optimal value of I (say I^*) will be determined from the above equation as

$$I^* = \frac{1}{\kappa} ln(V_0\kappa).$$

8.4 Numerical example without Stackelberg approach

The values of the parameters are considered by using the numerical data from Huang *et al.* (2011), Sarkar *et al.* (2015), Sarkar (2016), and Sarkar *et al.* (2016) as

$D = 1000$ units/year, $P = 4000$ units/year, $A_b = \$300$/order, $C_v = \$5$/delivery, $F = \$0.2$/shipment, $V_t = \$0.1$/unit, $V_i = \$1$/delivery, $U_i = \$0.2$/unit item inspected, $R_v = \$15$/unit, $\rho = 0.55$, $V_v = \$5$/unit, $h_{b_1} = \$35$/unit/year, $h_{b_2} = \$30$/unit/year, $h_v = \$20$/unit/year, $\alpha = 3500$ units/year,

$\kappa = 0.0014$, and $V_0 = \$1000$/setup. Therefore, after applying the investment vendor's setup cost becomes $V_s = \$714.28$/setup.

Hence, vendor-buyer system's joint total cost $JTC_{vb} = \$17809$, first shipment lot-size per batch throughout the production $Q^* = 75$ units, rate of increasing delivery lots $\delta^* = 4$ unit/year, and number of delivery lots of each batch per production $n^* = 2$, and investment for setup cost reduction per production run $I^* = \$240.34$/production run.

Figure 8.2, Figure 8.3, Figure 8.4, Figure 8.5, Figure 8.6, and Figure 8.7 indicates the optimality of the vendor-buyer system's joint total cost JTC_{vb}.

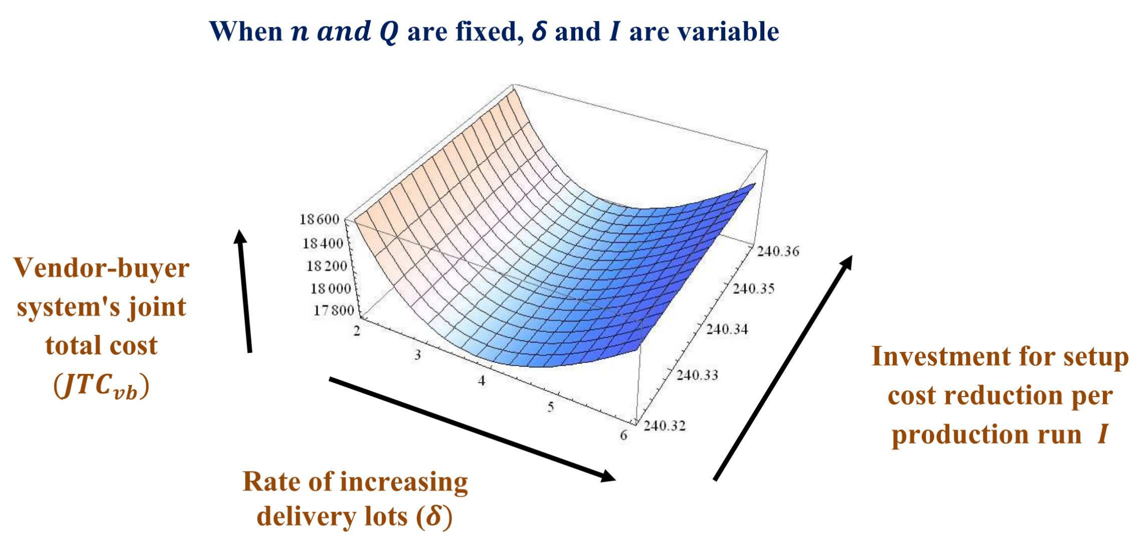

Figure 8.2: Vendor-buyer system's joint total cost (JTC_{vb}) versus rate of increasing delivery lots (δ) and investment for setup cost reduction per production run (I)

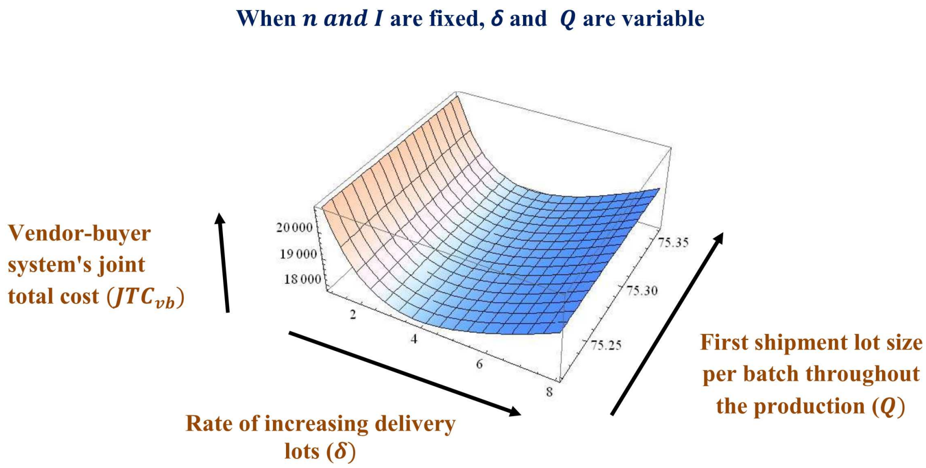

Figure 8.3: Vendor-buyer system's joint total cost (JTC_{vb}) versus rate of increasing delivery lots (δ) and first shipment lot-size per batch throughout the production (Q)

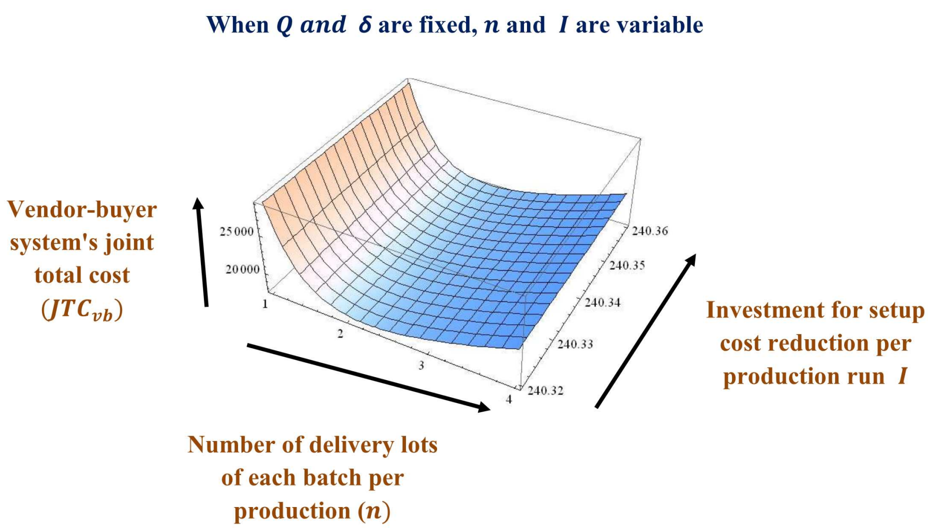

Figure 8.4: Vendor-buyer system's joint total cost (JTC_{vb}) versus number of delivery lots of each batch per production (n) and investment for setup cost reduction per production run (I)

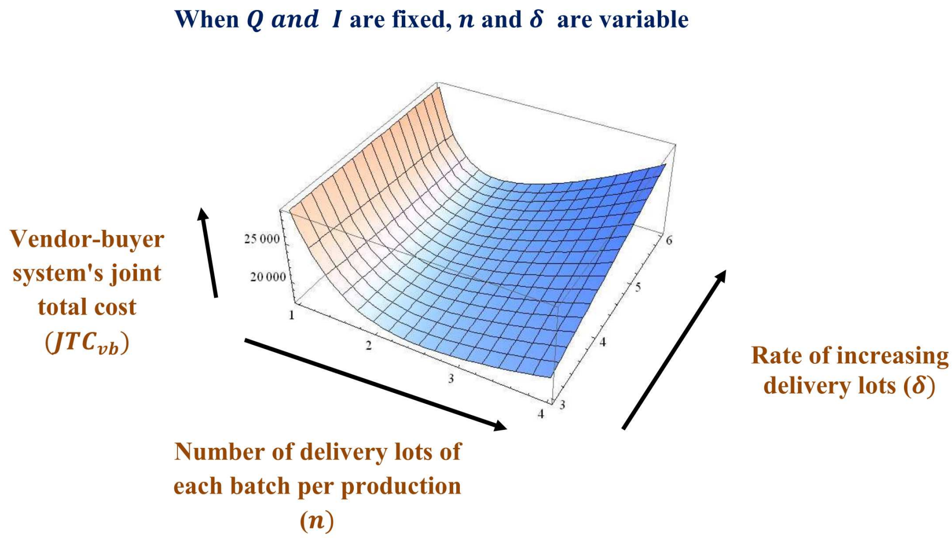

Figure 8.5: Vendor-buyer system's joint total cost (JTC_{vb}) versus number of delivery lots of each batch per production (n) and rate of increasing delivery lots (δ)

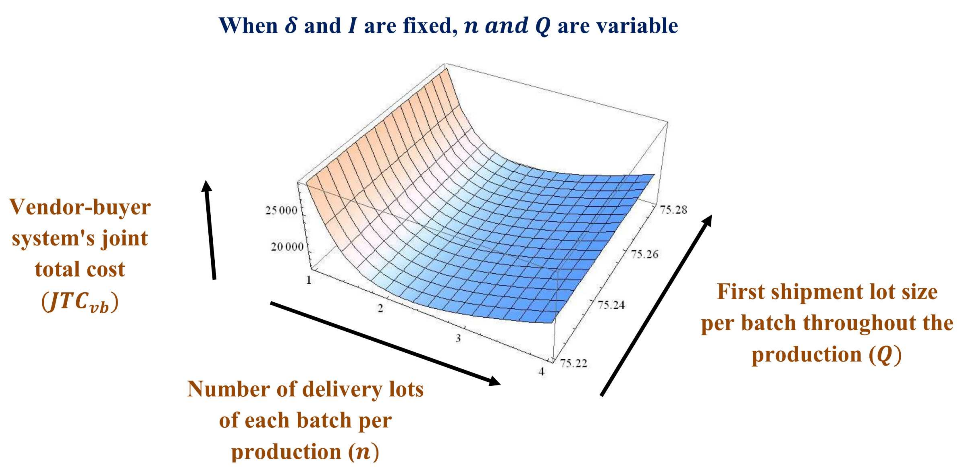

Figure 8.6: Vendor-buyer system's joint total cost (JTC_{vb}) versus number of delivery lots of each batch per production (n) and first shipment lot-size per batch throughout the production (Q)

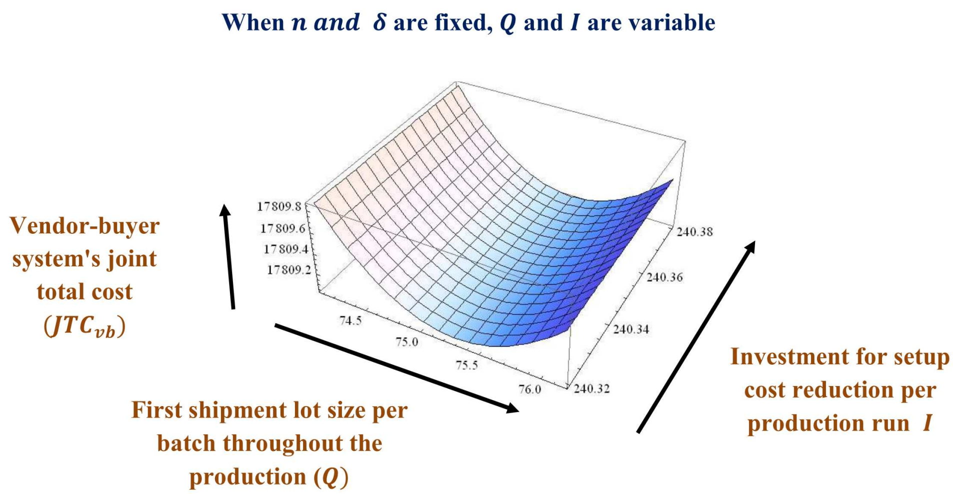

Figure 8.7: Vendor-buyer system's joint total cost (JTC_{vb}) versus first shipment lot-size per batch throughout the production (Q) investment for setup cost reduction per production run (I)

Table 8.2: Sensitivity analysis of key parameters

Parameters	Changes(in %)	JTC_{vb}	Parameters	Changes(in %)	JTC_{vb}
V_i	−50%	−0.09	A_b	−50%	−2.31
	−25%	−0.04		−25%	−1.14
	+25%	0.04		+25%	1.10
	+50%	0.09		+50%	2.17
ρ	−50%	−29.83	U_i	−50%	−0.06
	−25%	−14.98		−25%	−0.03
	+25%	15.13		+25%	0.03
	+50%	30.41		+50%	0.06

Parameters	Changes(in %)	JTC_{vb}	Parameters	Changes(in %)	JTC_{vb}
	−50%	−0.003		−50%	−3.55
	−25%	−0.001		−25%	−1.73
F	+25%	0.001	h_{b_2}	+25%	1.66
	+50%	0.003		+50%	3.25
	−50%	−3.70		−50%	−3.70
	−25%	−1.80		−25%	−1.80
h_v	+25%	1.72	V_t	+25%	1.72
	+50%	3.37		+50%	3.37
	−50%	−2.64		−50%	−0.07
	−25%	−1.30		−25%	−0.04
h_{b_1}	+25%	1.25	C_v	+25%	0.04
	+50%	2.46		+50%	0.07

'–' refers to infeasible solution.

Table 8.2 illustrates sensitivity analysis, which shows the impact of each parameters which are V_i, U_i, F, V_t, ρ, h_v, C_v, h_{b_1}, h_{b_2}, and A_b severally on vendor-buyer system's joint total cost JTC_{vb}.

- Vendor-buyer system's joint total cost JTC_{vb} increases while unit and variable inspection costs i.e., U_i and V_i increases. On the other hand, if unit transportation cost F and variable transportation cost V_t are increased, then vendor-buyer system's joint total cost JTC_{vb} is also increased.

- If defective rate ρ, vendor's holding cost h_v, and vendor's fixed carbon-emission cost C_v increases, then vendor-buyer system's joint total cost JTC_{vb} also increases.

- Increasing values in buyer's holding cost for perfect items h_{b_1}, holding cost for imperfect items h_{b_2}, and buyer's ordering cost A_b implies that vendor-buyer system's joint total cost JTC_{vb} is increased. It is found that the negative percentage changes for these three parameters h_{b_1}, h_{b_2}, A_b are much more than the positive percentage changes.

8.5 Numerical example by using Stackelberg approach

Case 1 while buyer as leader and vendor as follower

$D = 1000$ units/year, $P = 4000$ units/year, $A_b = \$300$/order, $C_v = \$5$/delivery, $F = \$0.2$/shipment, $V_t = \$0.1$/unit, $V_i = \$1$/delivery, $U_i = \$0.2$/unit item inspected, $R_v = \$15$/unit, $\rho = 0.55$, $V_v = \$5$/unit, $h_{b_1} = \$35$/unit/year, $h_{b_2} = \$30$/unit/year, $h_v = \$20$/unit/year, $\alpha = 3500$ units/year, $\kappa = 0.0014$, and $V_0 = \$1000$/setup. Hence, vendor-buyer system's joint total cost $JTC_{vb} = \$17131.36$, first shipment lot-size per batch throughout the production $Q^* = 58$ units, rate of increasing delivery lots $\delta^* = 3$ unit/year, and number of delivery lots of each batch per production $n^* = 3$, and investment for setup cost reduction per production run $I^* = \$240.34$/production run.

Table 8.3: Sensitivity analysis for buyer's cost

Parameters	Changes(in %)	TC_b	Parameters	Changes(in %)	TC_b
	-50%	-9.28		-50%	-0.09
	-25%	-4.64		-25%	-0.05
A_b	$+25\%$	4.64	V_i	$+25\%$	0.05
	$+50\%$	9.28		$+50\%$	0.09

Parameters	Changes(in %)	TC_b	Parameters	Changes(in %)	TC_b
	−50%	−0.36		−50%	−17.28
	−25%	−0.18		−25%	−8.64
U_i	+25%	0.18	h_{b_1}	+25%	8.64
	+50%	0.36		+50%	17.28

Table 8.3 discusses the effect on total cost for buyer TC_b by changing several parameters such as A_b, V_i, U_i, and h_{b_1}, respectively.

- For the parameter A_b, which is buyer's ordering cost, negative and positive percentage changes are similar. If the value of the parameter A_b increases that means buyer's total cost TC_b is also increased.

- It is observed that if unit inspection cost U_i and variable inspection cost V_i are increased, then buyer's total cost TC_b is also increased.

- When buyer's holding cost for perfect items i.e., h_{b_1} increases, then buyer's total cost TC_b is increased. Both negative percentage change and positive percentage changes are similar for this parameter.

Table 8.4: Sensitivity analysis for vendor's cost

Parameters	Changes(in %)	TC_v	Parameters	Changes(in %)	TC_v
	−50%	−9.57		−50%	−6.76
	−25%	−4.78		−25%	−3.09
V_v	+25%	4.78	h_v	+25%	2.72
	+50%	9.57		+50%	5.18

Parameters	Changes(in %)	TC_v	Parameters	Changes(in %)	TC_v
	−50%	−0.003		−50%	−0.09
	−25%	−0.002		−25%	−0.04
F	+25%	0.002	C_v	+25%	0.04
	+50%	0.003		+50%	0.09

Table 8.4 allows sensitivity analysis for evaluating the effect of several parameters such as V_v, h_v, F, and C_v, respectively on vendor's total cost TC_v.

- If vendor's variable carbon-emission cost V_v increases, then the vendor's total cost TC_v is also increased. In this case, both negative percentage change and positive percentage changes are equal.
- When vendor's holding cost h_v increases, then the vendor's total cost TC_v increases. The negative percentage change is maximum than positive percentage change for this parameter.
- While vendor's rework cost per unit Y_r increases, then the total cost for vendor TC_v is also increased. Both the positive and negative percentage changes are same.
- An increasing value in vendor's fixed carbon-emission cost per delivery S_v increases, the total cost for vendor TC_v. For this parameter, the positive percentage change is smaller than the negative percentage change.

Case 2 While vendor as leader and buyer as follower

All parameters for this model are as follows:

$D = 1000$ units/year, $P = 4000$ units/year, $A_b = \$300$/order, $C_v = \$5$/delivery, $F = \$0.2$/shipment, $V_t = \$0.1$/unit, $V_i = \$1$/delivery, $U_i = \$0.2$/unit item inspected, $R_v = \$15$/unit, $\rho = 0.55$,

$V_v = \$5$/unit, $h_{b_1} = \$35$/unit/year, $h_{b_2} = \$30$/unit/year, $h_v = \$20$/unit/year, $\alpha = 3500$ units/year, $\kappa = 0.0014$, and $V_0 = \$1000$/setup. Hence, vendor-buyer system's joint total cost $JTC_{vb} = \$17456.1$, first shipment lot-size per batch throughout the production $Q^* = 28$ units, rate of increasing delivery lots $\delta^* = 3$ unit/year, and number of delivery lots of each batch per production $n^* = 3$, and investment for setup cost reduction per production run $I^* = \$240.34$/production run.

Table 8.5: Sensitivity analysis for buyer's cost

Parameters	Changes(in %)	TC_b	Parameters	Changes(in %)	TC_b
	-50%	-0.24		-50%	-11.27
	-25%	-0.12		-25%	-5.46
V_i	$+25\%$	0.12	h_{b_1}	$+25\%$	5.18
	$+50\%$	0.24		$+50\%$	10.11
	-50%	-0.46		-50%	-15.32
	-25%	-0.23		-25%	-7.34
U_i	$+25\%$	0.23	h_{b_2}	$+25\%$	6.83
	$+50\%$	0.46		$+50\%$	13.25

Table 8.5 analyzes the effect on buyer's total cost TC_b by changing key parameters such as V_i, U_i, h_{b_1}, and h_{b_2}.

- It is found while unit inspection cost U_i and variable inspection cost V_i are increased, it implies buyer's total cost TC_b is also increased.

- The increasing values in buyer's holding cost for perfect items h_{b_1} and buyer's holding cost for imperfect items h_{b_2} increases buyer's total cost TC_b. For this parameter, negative percentage change is greater than the positive percentage change.

Table 8.6: Sensitivity analysis for vendor's cost

Parameters	Changes(in %)	TC_v	Parameters	Changes(in %)	TC_v
C_v	−50%	−0.17	h_v	−50%	−2.61
	−25%	−0.09		−25%	−1.31
	+25%	0.09		+25%	1.31
	+50%	0.17		+50%	2.61
V_v	−50%	−8.99	R_v	−50%	−26.98
	−25%	−4.50		−25%	−13.49
	+25%	4.50		+25%	13.49
	+50%	8.99		+50%	26.98

Table 8.6 gives sensitivity analysis for examining the impact of various parameters like V_v, h_v, R_v, and C_v, respectively on vendor's total cost TC_v.

- If vendor's fixed and variable carbon-emission costs i.e., C_v and V_v increases, then the vendor's total cost TC_v also increases. It is observed that the negative percentage change as well as the positive percentage changes are similar.
- For the increasing value in vendor's rework cost R_v and vendor's holding cost h_v indicates vendor's total cost TC_v increases. As C_v and V_v, both negative percentage change and positive percentage change are equal for R_v and h_v.

Table 8.7: Model without Stackelberg approach

Joint total cost	\$17809

Table 8.8: Model with Stackelberg approach

Description	**Buyer cost**	**Vendor cost**	**Joint total cost**
While buyer as leader and vendor as follower	\$2761.06	\$14370.3	\$17131.36
While vendor as leader and buyer as follower	\$2165.7	\$15290.4	\$17456.1

By analyzing the above comparison table 8.7 and table 8.8, it can be observed that the model with Stackelberg approach gives lowest vendor-buyer system's joint total cost JTC_{vb} than the model without Stackelberg approach.

Case Study

This model obtained the idea of Stackelberg game policy in an integrated inventory model. Two types of inspection costs are assumed. Fixed and variable carbon emission costs are incorporated with transportation costs. In this model, Stackelberg game policy is highlighted throughout the vendor-buyer model. Once, buyer is taken to be as leader and vendor is follower and vice-versa based on the more dominating nature of players. For example, in different shopping malls, retailers are more powerful and for Microsoft Windows companies, manufacturers are more powerful. That implies that in various shopping malls, retailers play the role of leader and manufacturers play the role of follower. While, for Microsoft Windows companies, manufacturers play the role of leader and retailers play the role of follower.

Numerical example without Stackelberg approach

The values of the parameters are considered as

$D = 900$ units/year, $P = 3000$ units/year, $A_b = \$200$/order, $C_v = \$6$/delivery, $F = \$0.1$/shipment, $V_t = \$0.2$/unit, $V_i = \$2$/delivery, $U_i = \$0.03$/unit item inspected, $R_v = \$20$/unit, $\rho = 0.7$, $V_v = \$6$/unit, $h_{b_1} = \$45$/unit/year, $h_{b_2} = \$35$/unit/year, $h_v = \$25$/unit/year, $\alpha = 3000$ units/year, $\kappa = 0.002$, and $V_0 = \$1200$/setup. Therefore, after applying the investment vendor's setup cost becomes $V_s = \$500.004$/setup.

Hence, vendor-buyer system's joint total cost $JTC_{vb} = \$23262$, first shipment lot-size per batch throughout the production $Q^* = 37$ units, rate of increasing delivery lots $\delta^* = 3$ unit/year, and number of delivery lots of each batch per production $n^* = 3$, and investment for setup cost reduction per production run $I^* = \$437.73$/production run.

Figure 8.8, Figure 8.9, Figure 8.10, Figure 8.11, Figure 8.12, and Figure 8.13 indicates the optimality of the vendor-buyer system's joint total cost JTC_{vb}.

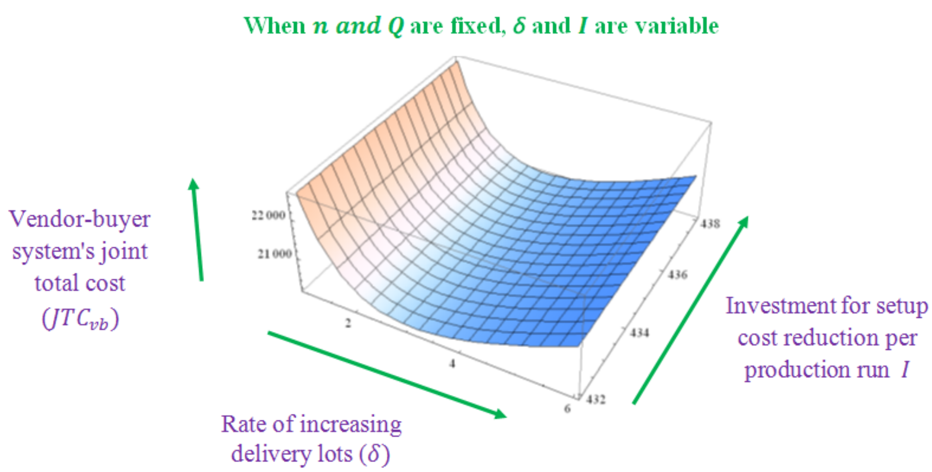

Figure 8.8: Vendor-buyer system's joint total cost (JTC_{vb}) versus rate of increasing delivery lots (δ) and investment for setup cost reduction per production run (I)

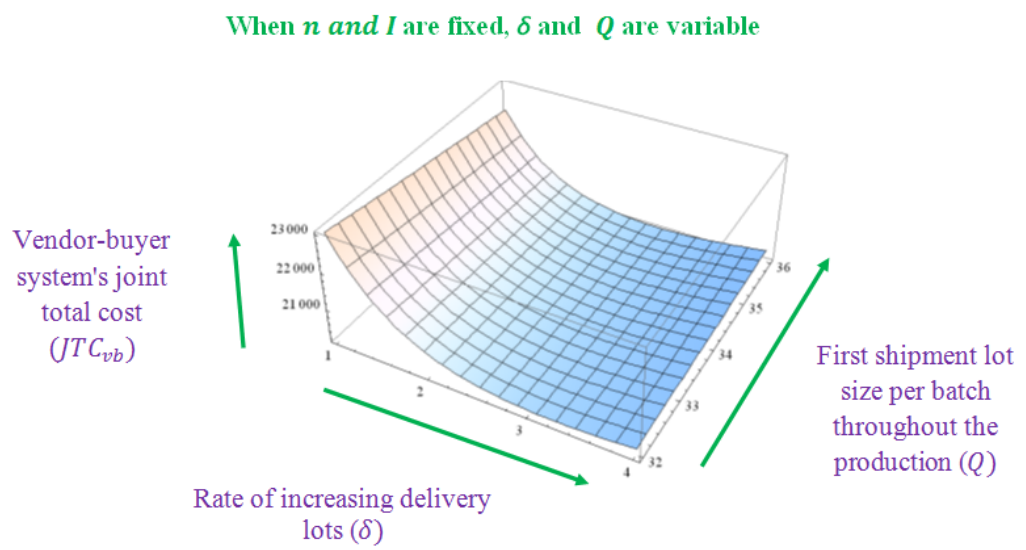

Figure 8.9: Vendor-buyer system's joint total cost (JTC_{vb}) versus rate of increasing delivery lots (δ) and first shipment lot-size per batch throughout the production (Q)

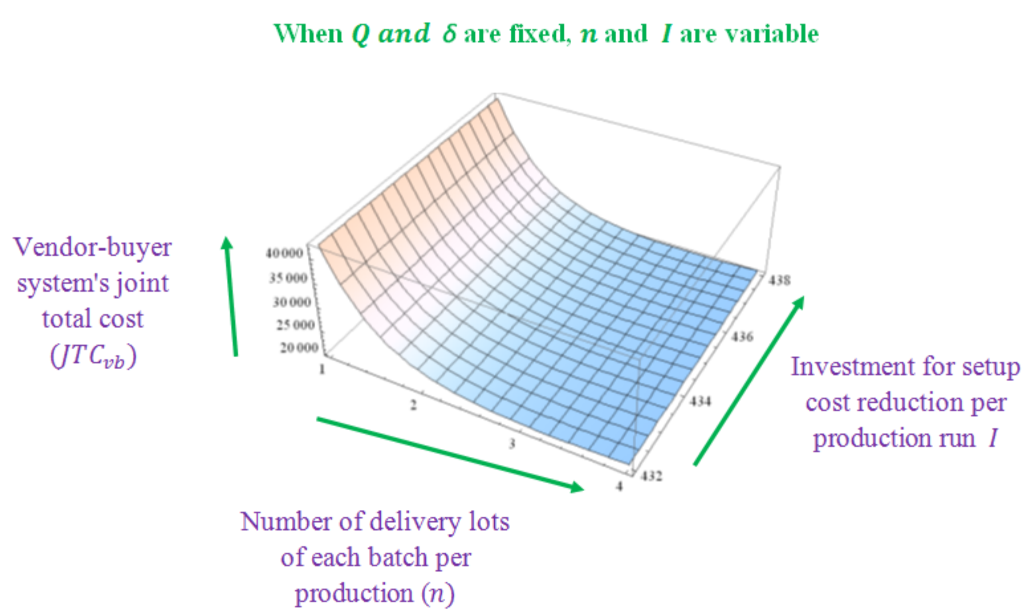

Figure 8.10: Vendor-buyer system's joint total cost (JTC_{vb}) versus number of delivery lots of each batch per production (n) and investment for setup cost reduction per production run (I)

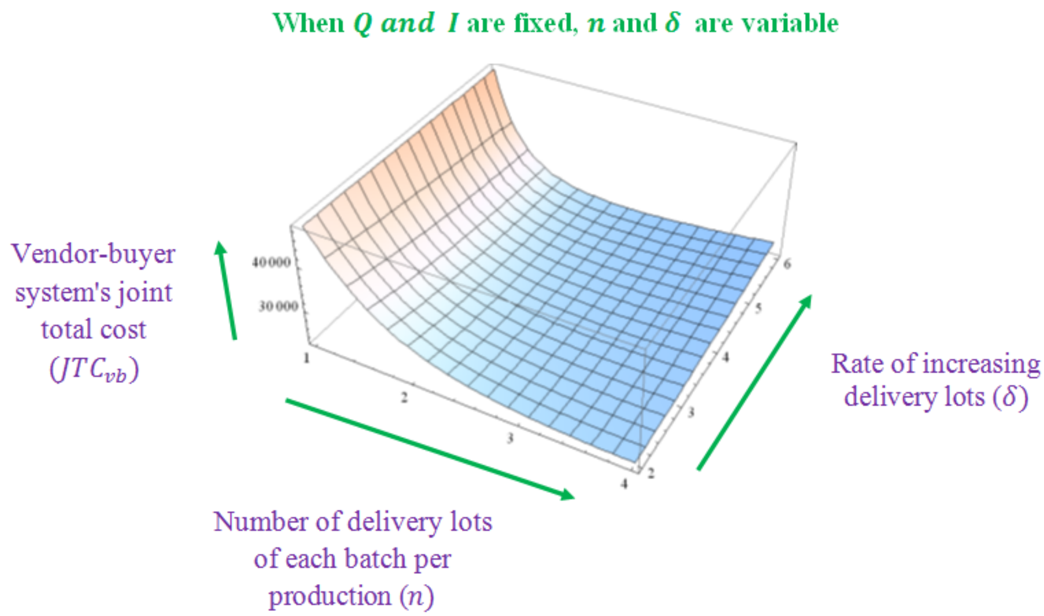

Figure 8.11: Vendor-buyer system's joint total cost (JTC_{vb}) versus number of delivery lots of each batch per production (n) and rate of increasing delivery lots (δ)

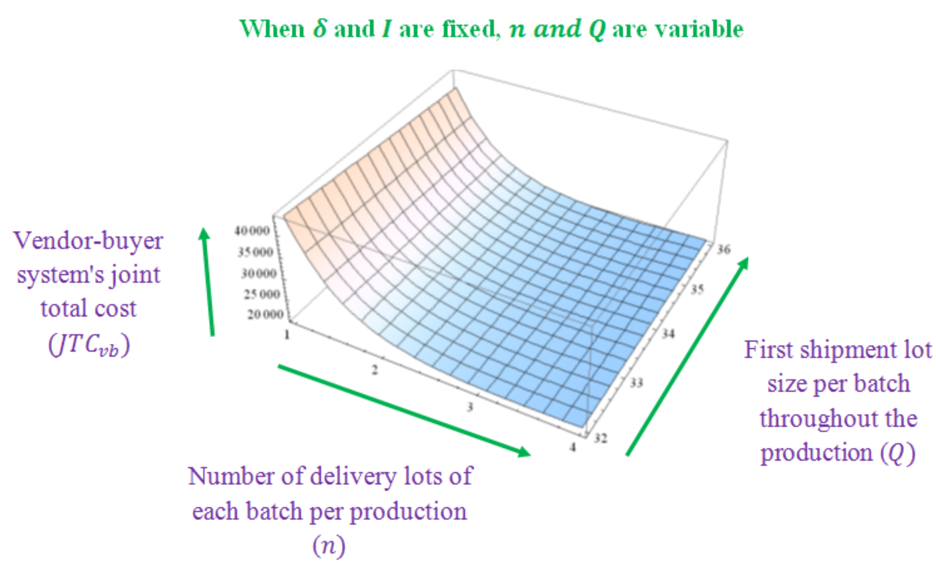

Figure 8.12: Vendor-buyer system's joint total cost (JTC_{vb}) versus number of delivery lots of each batch per production (n) and first shipment lot-size per batch throughout the production (Q)

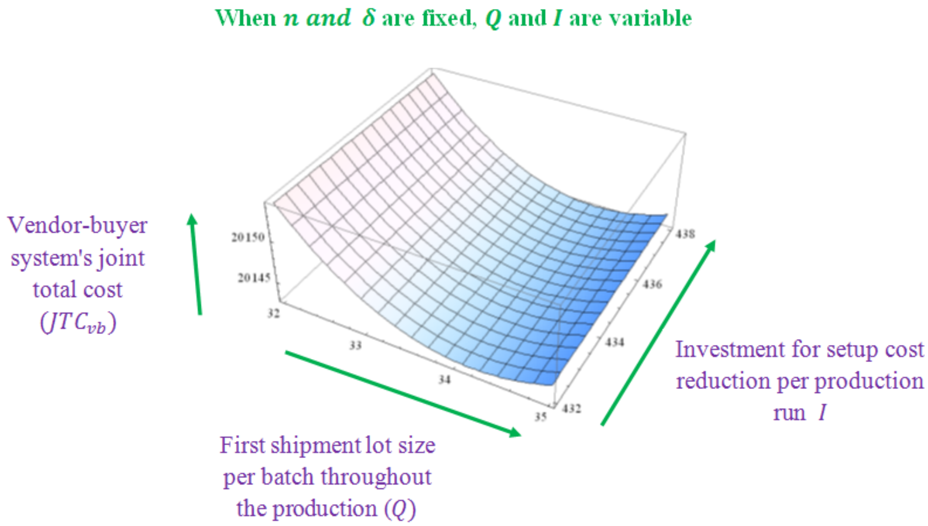

Figure 8.13: Vendor-buyer system's joint total cost (JTC_{vb}) versus first shipment lot-size per batch throughout the production (Q) investment for setup cost reduction per production run (I)

Numerical example by using Stackelberg approach

Case 1 while buyer as leader and vendor as follower

$D = 900$ units/year, $P = 3000$ units/year, $A_b = \$200$/order, $C_v = \$6$/delivery, $F = \$0.1$/shipment, $V_t = \$0.2$/unit, $V_i = \$2$/delivery, $U_i = \$0.03$/unit item inspected, $R_v = \$20$/unit, $\rho = 0.7$, $V_v = \$6$/unit, $h_{b_1} = \$45$/unit/year, $h_{b_2} = \$35$/unit/year, $h_v = \$25$/unit/year, $\alpha = 3000$ units/year, $\kappa = 0.002$, and $V_0 = \$1200$/setup. Hence, vendor-buyer system's joint total cost $JTC_{vb} = \$22098$, first shipment lot-size per batch throughout the production $Q^* = 52$ units, rate of increasing delivery lots $\delta^* = 3$ unit/year, and number of delivery lots of each batch per production $n^* = 3$, and investment for setup cost reduction per production run $I^* = \$437.73$/production run.

Case 2 While vendor as leader and buyer as follower

All parameters for this model are as follows:

$D = 900$ units/year, $P = 3000$ units/year, $A_b = \$200$/order, $V_i = \$2$/delivery, $U_i = \$0.03$/unit item inspected, $\rho = 0.7$, $h_{b_1} = \$45$/unit/year, $h_{b_2} = \$35$/unit/year, $h_v = \$25$/unit/year, $\alpha = 3000$ units/year. Hence, vendor-buyer system's joint total cost $JTC_{vb} = \$23248.5$, first shipment lot-size per batch throughout the production $Q^* = 20$ units, rate of increasing delivery lots $\delta^* = 3$ unit/year, and number of delivery lots of each batch per production $n^* = 3$, and investment for setup cost reduction per production run $I^* = \$437.73$/production run.

Model without Stackelberg approach

Joint total cost	\$23262

Model with Stackelberg approach

Description	**Buyer cost**	**Vendor cost**	**Joint total cost**
While buyer as leader and vendor as follower	\$2312.54	\$19785.5	\$22098
While vendor as leader and buyer as follower	\$1815.34	\$21433.2	\$23248.5

8.6 Concluding remarks and future works

This chapter presented a discrete investment function for vendor's setup cost reduction. Two types of transportation cost which are fixed and variable as well as carbon-emission cost are incorporated in this chapter. Stackelberg approach is introduced to illustrate the numerical findings of this chapter. Further, this research can be extended by including machine breakdown, stochastic lead time, and lead-time crashing cost.

8.7 Appendices

Appendix A5

$$
\begin{aligned}
R_1 &= \frac{(P-D)}{2P} - \frac{(1-\rho)^2}{2} - \frac{\rho D}{\alpha} \\
R_2 &= \frac{2D\rho}{\alpha} h_{b_2} + (1-\rho)^2 h_{b_1} \\
R_3 &= C_v + F + V_i \\
R_4 &= D\rho(V_v + V_t + R_v)
\end{aligned}
$$

Appendix A6

The second order partial derivatives of vendor-buyer system's joint total cost JTC_{vb} at the optimal values I^*, Q^*, δ^*, and n^* are given by

$$
\begin{aligned}
\frac{\partial^2 JTC_{vb}(n^*, Q^*, \delta^*, I^*)}{\partial I^{*2}} &= \frac{2DV_0\kappa^2}{2Q+\delta Q n(n-1)} e^{-\kappa I} > 0. \\
\frac{\partial^2 JTC_{vb}(n^*, Q^*, \delta^*, I^*)}{\partial Q^{*2}} &= \frac{4DQn(n-1)}{(2+\delta n(n-1))^2}(V_0 e^{-\kappa I} + nR_3 + I + A_b) \\
\frac{\partial^2 JTC_{vb}(n^*, Q^*, \delta^*, I^*)}{\partial \delta^{*2}} &= \frac{n(n-1)Q}{(2+\delta n(n-1))}\Big[R_2\Big(\frac{(2n-1)}{3} - \frac{2\delta(2n^3-3n^2+n)}{3(2+\delta n(n-1))} \\
&+ \frac{2(n^2-n)}{(2+\delta n(n-1))^2}\Big(1 + \frac{\delta^2 n(n-1)(2n-1)}{6}\Big)\Big) \\
&+ \frac{4Dn(n-1)(V_0 e^{-\kappa I} + nR_3 + I + A_b)}{(2Q+\delta n(n-1)Q)^2}\Big] \\
\frac{\partial^2 JTC_{vb}(n^*, Q^*, \delta^*, I^*)}{\partial n^{*2}} &= \frac{4D\delta Q}{(2+\delta n(n-1))^2}((R_3(1-2n) + V_0 e^{-\kappa I} + nR_3 + I + A_b)\delta Q(2n-1)^2) \\
&+ h_v R_1 \delta Q + \frac{R_2}{(2+\delta n(n-1))}\Big[-2\delta\left(1 + \frac{\delta^2(2n^3-3n^2+n)}{6}\right) \\
&+ Q\Big((2n-1) - \frac{(6n^2-6n+1)\delta(2n-1)}{6} + \frac{\delta(2n-1)}{(2+\delta n(n-1))^2}\Big((2n \\
&- 1) - \frac{Q\delta^2(6n^2-6n+1)}{6}\Big)\Big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial I^*\partial Q^*} &= \frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial Q^*\partial I^*} = -\frac{4D}{2Q+\delta Qn(n-1)]}(1-V_0\kappa e^{-\kappa I}) \\
\frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial I^*\partial \delta^*} &= \frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial \delta^*\partial I^*} = -\frac{2DQn(n-1)}{(2Q+\delta n(n-1)Q)^2}(1-V_0\kappa e^{-\kappa I}) \\
\frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial I^*\partial n^*} &= \frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial n^*\partial I^*} = -\frac{2D\delta Q(2n-1)}{(2Q+\delta Qn(n-1))^2}(1-V_0\kappa e^{-\kappa I}) \\
\frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial Q^*\partial n^*} &= \frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial n^*\partial Q^*} \\
&= \frac{h_vR_1\delta(1-2n)}{2} - Y\Big[\frac{(2n-1)\delta}{(2+\delta n(n-1))^2}\Big(1+\frac{\delta^2(2n^3-3n^2+n)}{6}\Big) \\
&+ \frac{\delta^2(6n^2-6n+1)}{6(2+\delta n(n-1))}\Big] - \frac{2D}{Q^2(2+\delta n(n-1))}\Big(\frac{(V_0e^{-\kappa I}+nR_3+I+A_b)}{(2+\delta n(n-1))} \\
&- R_3\Big) \\
\frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial Q^*\partial \delta^*} &= \frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial \delta^*\partial Q^*} \\
&= \frac{h_vR_1(2n-1)}{2} - \frac{2Dn(n-1)}{Q^2(2+\delta n(n-1))^2}(V_0e^{-\kappa I}+nR_3+I+A_b) \\
&- \frac{R_2n(n-1)}{(2+\delta n(n-1))^2}\Big(\frac{\delta^2(2n^3-3n^2+n)}{6}+1\Big) + \frac{2\delta(2n^3-3n^2+n)R_2}{6(2+\delta n(n-1))} \\
\frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial n^*\partial \delta^*} &= \frac{\partial^2 JTC_{vb}(n^*,Q^*,\delta^*,I*)}{\partial \delta^*\partial n^*} \\
&= (V_0e^{-\kappa I}+nR_3+I+A_b)\frac{2DQ(2n-1)}{(2Q+\delta Qn(n-1))^2}\Big(1+\frac{2\delta(n^2-n)}{(2+\delta(n^2-n))}\Big) \\
&+ \frac{h_vR_1(2n-1)Q}{2} + R_2\Big[\frac{\delta Q(6n^2-6n+1)}{3(2+\delta n(n-1))} - \frac{Q(2n-1)}{(2+\delta n(n-1))^2}\Big(1 \\
&+ \frac{\delta^2(2n^3-3n^2+n)}{6}\Big)\Big(1-\frac{2n(n-1)}{2+\delta n(n-1)}\Big) \\
&- \frac{\delta^2 nQ(n-1)}{(2+\delta n(n-1))^2}\Big(\frac{(2n-1)^2}{3}+\frac{(6n^2-6n+1)}{6}\Big)\Big]
\end{aligned}
$$