Chapter 7

An integrated-inventory model with environmental issue*

7.1 Introduction

The impact of carbon-emission cost are highly considered at the time of transporting products. Carbon emission cost can be variable as shipping of lot-size may be variable due to demand of buyers. Hence, carbon-emission cost may also treated as variable. Nag and Parikh (2005) made distinct essential matters regarding carbon-emission minimization. Butler *et al.* (2008) distinguished the contribution of several sectors to the total emissions from every city, and joint these deviation to various methods. Ma *et al.* (2011) addressed the fact of energy deviation and carbon-emissions in Tianj throughout the time interval 1995 to 2007. They also obtained the basic sources of carbon-emissions and added some suggestions on carbon-emission reduction. Wygonik and Goodchild (2011) designed an emissions minimization vehicle routing problem along with time windows. They

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showed a static relation between kilograms of carbon-dioxide and monetary cost. Their results showed suggests the most effective way to reduce cost, and therefore emissions. Hua *et al.* (2011) investigated the direction of carbon footprints in inventory management system. Bachmann and Kamp (2014) provided some technique for the benefit of environment. Zhang *et al.* (2014) developed a game model for determining the promotional impact of increasing oil price on companies behavior about carbon-emission reduction. Their research depicted the theoretical observation for proclaiming rational low carbon strategies.

Setup cost related the cost for adjusting a whole system of production. It is important for many manufacturing industry to minimize the setup cost as this cost is associated with the total cost. Most of existing literatures stated that setup cost as constant. By adding some investment, setup cost can be reduced. Hong *et al.* (1996) investigated three production policies under non-constant, deterministic demand, and dynamic setup cost reduction with an investment function. Chuang *et al.* (2004) discussed some inventory models that allows shortages, lost sales along with lead time and variable setup cost. Hou (2007) invented an production-inventory model for imperfect production to minimize setup cost. Later, Sarkar and Majumder (2013) developed a supply chain model in which vendors setup cost is reduced. Diaby *et al.* (2013) derived the fact of setup cost reduction by adding some investment with quality improvement. Sarkar and Moon (2014) discovered an imperfect production process that describes the relation among quality improvement, reorder point, setup cost, lead time, an shortages. Sarkar *et al.* (2015) represented an inventory model along with some factors like quality improvement, setup cost reduction, and service-level constraint.

It is assumed whenever buyer places an order to vendor, then vendor transports those ordered items with similar lot-sizes. The produced products may be transferred in partial lot-sizes for controlling holding cost and setup cost. Goyal and Szendrovits (1986) depicted a lot-size inventory model for determining economic lot-size and sizes of each batch during every stage. Hoque and Kingsman (1995) discussed a new heuristic solution method for fixed lot-size model with a constant sequence of production stage. Bogaschewsky *et al.* (2001) expanded earlier research articles related to this field by adding multi-stage production model with unequal sizes of shipments. Siajadi *et al.* (2006) produced an vendor-buyer model with shipment strategy to minimize joint total cost. By allowing shortages for buyer, Zhou and Wang (2007) considered a single-vendorsingle-buyer model for deterioration and delivery policy. Hoque (2013) developed a manufacturer-buyer integrated-inventory model by assuming equal/unequal-sized batches delivery. Hariga *et al.* (2014) described a mixed integer non-linear programming model in which unequal delivery policies for retailer are assumed.

Earlier, it is considered that all produced products are absolutely error free which incurs that those produced items are non defective. Comparing real life situation, this consideration does not valid. Generally, over the long-run production process defective items may arise. By utilizing some inspection process, buyer can detect imperfect as well as perfect products. By applying inspection policy, production factories are capable to give good quality products in the market. Wang and Sheu (2001) presented some inspection technique for the batch. Wang and Sheu (2003) obtained a deteriorating production system with product inspection policy. Wang (2005) represented an inventory model for production run length and product inspection policy with the help of an efficient solution process. Ben-Daya and Noman (2008) formulated an integrated inventory inspection model based on the assumption that when a lot is received, buyer uses some type of inspection policy. Konstantaras *et al.* (2010) discussed an ordering inventory model with the fact that all received products may be damaged during transportation and production processing time. Yoo *et al.* (2012) discovered an inspection process with consumer return policy. They considered production and inspection quality investment with all quality costs. In their model, there are two types of errors such as Type I and Type II inspection error are applied. Later, Sarkar and Saren (2016) expanded previous inventory models with an EPQ model by adding warranty and inspection errors.

Supply chain defines a management linking the organizations to fulfill demand across the chain as efficiently as possible. It generally minimize transportation costs of inventories and manage inventories needed across the supply chain. Supply chain satisfied competitive pressures from shorter development times, more new items. Asghari (2014) determined a supply chain model that allows how customers' orders being allocated. He also described essential aspects of strategic planning of manufacturing in supply chain. In this direction of research articles, Lin *et al.* (2014) made a hybrid approach incorporating interpretive structural modeling for constructing a hierarchical organization. In addition, their model formulated the fuzzy set theory for examining linguistic preferences. On the other hand, they provided an idea about financial view as well as life cycle assessment which are most essential performance and weighted criteria. Watanabe and Kusukawa (2014) designed a decentralized GSC (DGSC) and also an integrated GSC (IGSC) model. In their model, they deduced few mathematical models to determine collection incentive of utilized items, lower boundary of quality level for recycling impact. Chen (2014) showed that how green performances impact firms environmental operation with green invention. In addition, his model observed the positive relation among green performances, green invention, and environmental operation. Kusukawa (2014) derived a decentralized supply chain (DSC) that provides decision-making approach policy for two conditions. They added that decentralized supply chain (DSC) increases retailers profit function. Watanabe and Kusukawa (2015) surveyed a dual-sourcing supply chain (DSSC) that allows two aspects of demand which are as (i) demand distribution is known (ii) mean and variance of the demand are known. They also assumed that under a decentralized DSSC (DSC), a retailer analyzes optimal ordering policy to increase his total expected profit. Besides, under the integrated DSSC (ISC), optimal ordering policy is obtained to maximize whole systems total expected profit. See Table 7.1 for contribution of respective researchers.

Table 7.1: Contribution of the different authors

Author(s)	**Setup cost reduction**	**Unequal lot-size**	**Carbon-emission cost**	**Inspection policy**	**Supply chain management**
Goyal and Szendrovits (1986)		$\surd$			
Hong *et al.* (1996)	$\surd$	$\surd$			
Wang and Sheu (2001)				$\surd$	
Wang and Sheu (2003)				$\surd$	
Chuang *et al.* (2004)	$\surd$				
Nag and Parikh (2005)			$\surd$		
Wang (2005)				$\surd$	
Hou (2007)	$\surd$				
Zhou and Wang (2007)		$\surd$			
Ben-Daya and Noman (2008)				$\surd$	
Butler *et al.* (2008)			$\surd$		
Ma *et al.* (2011)			$\surd$		

Author(s)	**Setup cost reduction**	**Unequal lot-size**	**Carbon-emission cost**	**Inspection policy**	**Supply chain management**
Wygonik and Goodchild (2011)			√		
Hua *et al.* (2011)			√		
Yoo *et al.* (2012)				√	
Diaby *et al.* (2013)	√				
Sarkar and Majumder (2013)	√				√
Hariga *et al.* (2014)		√			
Asghari (2014)					√
Lin *et al.* (2014)					√
Chen (2014)					√
Kusukawa (2014)					√
Sarkar *et al.* (2015)	√				
Watanabe and Kusukawa (2015)					√
Sarkar and Saren (2016)				√	
This chapter	√	√	√	√	√

This chapter highlighted the impact of carbon-emission cost reduction at the time of transportation/shipping policy in business organization. At the time of shipping products, vendor and buyer incurs two types of carbon-emission costs namely fixed and variable. Vendor's setup cost is measured as variable instead of taking as constant. Delivery lot-sizes are considered as unequal and variable. After receiving the lot, buyer performs an inspection process to discover defective items. Vendor performed a reworking procedure on defective items which they received from buyer. This model mainly constructed to reduce the carbon-emission cost for both vendor-buyer system. This chapter continues with mathematical model, solution method of this model, numerical example, and sensitivity analysis.

7.2 Mathematical model

Following notation are used for developing this model.

Decision variables

A_1 vendor's setup cost (\$/setup)

λ rate of increasing shipment lot-size (positive integer)

n number of shipments per batch production (positive integer)

q first shipment lot-size of every batch at the time of production (units)

Parameters

D demand rate (units/year)

P production rate (units/year)

H_v	vendor's cost for holding products per unit per year (\$/unit/year)
S_v	fixed carbon-emission cost per delivery for vendor (\$/delivery)
Y_v	variable carbon-emission cost per unit for vendor (\$/unit)
A_2	buyer's ordering cost per order (\$/order)
A_3	buyer's delivery cost per shipment (\$/shipment)
S	buyer's inspection cost per unit (\$/unit)
x	inspection rate (units/year)
S_b	fixed carbon-emission cost per shipment for buyer (\$/shipment)
Y_b	variable carbon-emission cost per unit for buyer (\$/unit)
H_1	buyer's holding cost for non-defective items (\$/unit/year)
H_2	buyer's holding cost for defective items (\$/unit/year)
Y_r	vendor's rework cost per unit (\$/unit)
γ	defective rate
V_h	total holding cost for vendor (\$/year)
B_n	total holding cost of non-defective items for buyer (\$/year)
B_d	buyer's total holding cost of defective products (\$/year)
B_c	total carbon-emission cost for buyer (\$/year)
V_c	total carbon-emission cost for vendor (\$/year)

T_B total cost for buyer (\$/year)

T_V total cost for vendor (\$/year)

JTC joint total cost of vendor-buyer system (\$/year)

To build up this model, following assumptions are utilized.

1. For a single type of product, this model described a single-vendor single-buyer model.
2. After ordering products by buyer is done, vendor delivered lot-sizes at a rate λ.
3. Vendor's setup cost is taken as variable instead of taking as constant. An investment is added to reduce setup cost.
4. After receiving each lot, buyer starts an inspection segment through which he/she can understood which are defective items. While next lot has reached from vendor, buyer return all defective products of former lot to vendor for further reworking process.
5. During delivery of lots, carbon-emission costs are inserted for both vendor and buyer. Two types of carbon-emission costs i.e., fixed and variable are considered.
6. Deterministic demand is considered.
7. Shortages are not provided in this model as rate of production is much more than rate of demand i.e., $P > D$.
8. Lead time is considered as negligible.

At first, buyer orders a lot of their required products with some ordering cost A_2 per order. In addition, vendor manufactured that ordering lots of retailer with some production rate P per year and variable setup cost A_1. Later, vendor transported first lot-size q units with delivery cost A_3. In every production cycle, vendor shipped lot of orders in n times. After the delivery of first q items, the ordering lot has been transferred from vendor to buyer on jth delivery is $(j-1)\lambda q$, $j > 1$. Fixed carbon-emission cost S_v and variable carbon-emission cost Y_v due to variable lot size of delivery are added for vendor. After getting the lot, buyer starts an inspection process with inspection rate x incurs some cost S. It is also assumed that rate of defective product is γ in each received lot. As the inspection procedure is completed, non-defective as well as defective products are detected. Buyer stores non-defective products with some holding cost H_1 and return those defective products to vendor for further reworking procedure. At the time of transporting of these defective products, fixed and variable carbon-emission costs for buyer are added which are S_b per shipment and Y_b per unit product respectively.

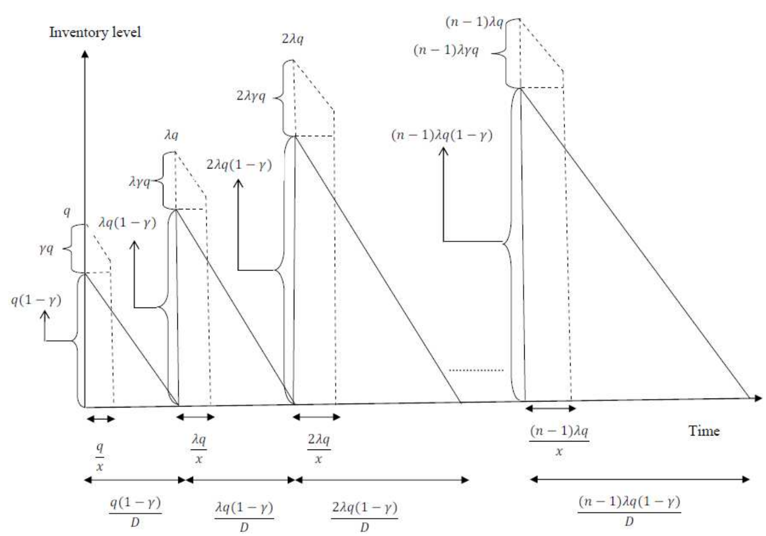

Figure 7.1: Buyers mathematical model

With the help of Single-setup-multi-delivery (SSMD) policy, vendor transferred ordering lots in n

times for each production cycle. After the delivery of lot-size q, the ordering quantity has been delivered to buyer from vendor on jth delivery is $(j-1)\lambda q$, $j > 1$ i.e, second shipment lot-size is λq. After that, delivery lot-sizes are $2\lambda q$, $3\lambda q$, and so on.

Hence the production lot-size which delivered to buyer from vendor is calculated by adding the total of shipment lots

$$q + \lambda q + 2\lambda q + ... + (n-1)\lambda q \quad = \quad q + \frac{\lambda q n(n-1)}{2}$$

The number of production cycle will be found by dividing the demand along with the production batch as follows

$$\frac{D}{q + \lambda \frac{n(n-1)}{2} q} \quad = \quad \frac{2D}{2q + q\lambda n(n-1)}.$$

Total ordering cost for buyer is $= A_2 \left(\frac{2D}{2q+\lambda qn(n-1)} \right)$.

Total delivery cost for buyer is $= nA_3 \left(\frac{2D}{2q+\lambda qn(n-1)} \right)$.

After receiving ordering lots from vendor, buyer performs an inspection procedure for detecting the defective items.

Total inspection cost for buyer is $= SD$.

The total number of non-defective items for every production cycle is calculated by the area of the triangle shown in Figure 7.1 i.e.,

$$\begin{aligned} & \frac{1}{2} q(1-\gamma) \frac{(1-\gamma)q}{D} + \frac{1}{2} \lambda q(1-\gamma) \frac{\lambda(1-\gamma)q}{D} + ... + \frac{1}{2}(n-1)\lambda q(1-\gamma) \frac{(n-1)\lambda(1-\gamma)q}{D} \\ = \quad & \frac{1}{2} \frac{q^2(1-\gamma)^2}{D} \left[1 + \frac{\lambda^2 n(n-1)(2n-1)}{6} \right] \end{aligned}$$

Therefore, the total holding cost of non-defective items B_n is obtained by multiplying all non-defective items with production cycle.

$$\begin{aligned} B_n \quad &= \quad H_1 \left[\frac{1}{2} \frac{q^2(1-\gamma)^2}{D} \left(1 + \frac{\lambda^2 n(n-1)(2n-1)}{6} \right) \right] \left[\frac{2D}{2q + \lambda qn(n-1)} \right] \\ &= \quad H_1 \left[\left(\frac{q(1-\gamma)^2}{2 + \lambda n(n-1)} \right) \left(1 + \frac{\lambda^2 n(n-1)(2n-1)}{6} \right) \right] \end{aligned}$$

The total number of defective items is obtained from the parallelogram given in Figure 7.1 i.e.,

$$q\gamma\frac{q}{x}+\lambda q\gamma\frac{\lambda q}{x}+...+\lambda(n-1)q\gamma\frac{(n-1)\lambda q}{x}=\frac{q^2\gamma}{x}\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right)$$

Buyer's total holding cost of imperfect products B_d is obtained by multiplying whole defective items with production cycle.

$$\begin{aligned}B_d &= H_2\left[\frac{q^2\gamma}{x}\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right)\right]\left[\frac{2D}{2q+\lambda qn(n-1)}\right]\\ &= H_2\left[\frac{2Dq\gamma}{x(2+\lambda n(n-1))}\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right)\right]\end{aligned}$$

At the time of delivery lots, buyer's considers two types of carbon-emission costs namely fixed as well as variable.

Buyer's fixed carbon-emission cost can be calculated for entire production cycle as

$$= nS_b\left[\frac{2D}{2q+\lambda qn(n-1)}\right]$$

Buyer's variable carbon-emission cost is measured by multiplying Y_b with demand D i.e., Y_bD.

Hence, buyer's total carbon-emission cost by highlighting both fixed and variable carbon-emission costs i.e., $B_c = nS_b\left[\frac{2D}{2q+\lambda qn(n-1)}\right]+Y_bD$.

Buyer's total inventory cost can be observed by summing cost for ordering, delivery charge, cost for inspection, carbon-emission cost, holding cost of non-defective as well as defective products.

$$\begin{aligned}T_B(n,q,\lambda) &= A_2\left(\frac{2D}{2q+\lambda qn(n-1)}\right)+nA_3\left(\frac{2D}{2q+\lambda qn(n-1)}\right)+H_2\Big[\frac{2Dq\gamma}{x(2+\lambda n(n-1))}\Big(1\\ &+ \frac{\lambda^2 n(n-1)(2n-1)}{6}\Big)\Big]+2nS_b\left(\frac{D}{2q+\lambda qn(n-1)}\right)+H_1\Big[\left(\frac{q(1-\gamma)^2}{2+\lambda n(n-1)}\right)\Big(1\\ &+ \frac{\lambda^2 n(n-1)(2n-1)}{6}\Big)\Big]+SD+Y_bD\end{aligned}$$

As the number of production cycle is

$$\frac{D}{q+\lambda\frac{n(n-1)}{2}q} = \frac{2D}{2q+q\lambda n(n-1)}$$

and setup cost is A_1.

Hence, vendor's total setup cost is $A_1\left(\frac{2D}{2q+\lambda n(n-1)q}\right)$.

Vendor's rework cost is $Y_r\gamma D$, where γ is the rate of defective.

The total capacity of stocks in production system is

$$\frac{Dq}{P}+(P-D)\left(\frac{2q+\lambda n(n-1)q}{4P}\right)$$

The average vendor stock can be obtained by subtracting total number of non-defective and defective items from vendor's total stock which is

$$\begin{aligned} &= \frac{Dq}{P}+(P-D)\left(\frac{2q+\lambda n(n-1)q}{4P}\right)-\left(\frac{q(1-\gamma)^2[2+\lambda n(n-1)]}{4}\right)\\ &- \left(\frac{q\gamma D[2+\lambda n(n-1)]}{2x}\right)\end{aligned}$$

The total holding cost for vendor is

$$\begin{aligned} V_h &= H_v\Big[\frac{Dq}{P}+(P-D)\left(\frac{2q+\lambda n(n-1)q}{4P}\right)-\left(\frac{q(1-\gamma)^2[2+\lambda n(n-1)]}{4}\right)\\ &- \left(\frac{q\gamma D[2+\lambda n(n-1)]}{2x}\right)\Big]\end{aligned}$$

As buyer's case, vendor's carbon-emission cost can be also be observed.

Vendor's total carbon-emission cost is calculated by summing fixed and variable carbon-emission costs i.e., $V_c = n\frac{2S_vD}{2q+\lambda n(n-1)q}+Y_v\gamma D$.

An investment I_{A_1} is added for setup cost reduction i.e.,

$$I_{A_1} = R\ln\left(\frac{A_0}{A_1}\right) = R(\ln A_0-\ln A_1) \qquad \text{for} \qquad 0<A_1\leq A_0$$

where A_0 is the original setup cost, $R=\frac{1}{\delta}$ and δ is the percentage decrease in A_1 per dollar increase in I_{A_1}.

Vendor's total inventory cost can be determined by summing setup cost, rework cost, fixed and variable carbon-emission cost, holding charge, and investment to reduce setup cost.

$$\begin{aligned}
T_V(n,q,\lambda,A_1) &= A_1\left(\frac{2D}{2q+\lambda n(n-1)q}\right)+Y_r\gamma D+\frac{2nS_vD}{2q+\lambda n(n-1)q}+Y_v\gamma D+H_v\Big[\frac{Dq}{P}\\
&+ (P-D)\left(\frac{2q+\lambda n(n-1)q}{4P}\right)-\left(\frac{q(1-\gamma)^2[2+\lambda n(n-1)]}{4}\right)+\alpha R(\ln A_0\\
&- \ln A_1)-\left(\frac{q\gamma D[2+\lambda n(n-1)]}{2x}\right)\Big]
\end{aligned}$$

where α is annual fractional cost for capital investment.

Therefore, the joint total cost for vendor-buyer system is formulated by

$$\begin{aligned}
JTC(n,q,\lambda,A_1) &= (A_1+A_2+nA_3+nS_b+nS_v)\left(\frac{2D}{2q+\lambda n(n-1)q}\right)+(S+Y_b+Y_r\gamma\\
&+ Y_v\gamma)D+\alpha R(\ln A_0-\ln A_1)+\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right)\Big(\frac{2D\gamma}{x}H_2\\
&+ (1-\gamma)^2H_1\Big)\frac{q}{[2+\lambda n(n-1)]}+H_vq\Big[\frac{D}{P}+(2+\lambda n(n-1))\Big(\frac{(P-D)}{4P}\\
&- \frac{(1-\gamma)^2}{4}-\frac{\gamma D}{2x}\Big)\Big]
\end{aligned}$$

7.3 Solution methodology

For minimizing the joint total cost for vendor-buyer system JTC, necessary conditions are $\frac{\partial JTC}{\partial n}=0$, $\frac{\partial JTC}{\partial q}=0$, $\frac{\partial JTC}{\partial \lambda}=0$, and $\frac{\partial JTC}{\partial A_1}=0$.

The first order partial derivative of vendor-buyer system's joint total cost JTC related to the number of shipments per batch production n is

$$\begin{aligned}
\frac{\partial JTC}{\partial n} &= \frac{2DY}{2q+\lambda n(n-1)q}-\frac{2D\lambda q(2n-1)}{(2q+\lambda n(n-1)q)^2}(A_1+A_2+nY)+(6n^2-6n\\
&+ 1)\frac{qX\lambda^2}{6(2+\lambda n(n-1))}-\frac{q\lambda X(2n-1)}{(2+\lambda n(n-1))^2}\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right)\\
&= \phi(n)
\end{aligned}$$

The optimal value of n (say n^*) can be derived if it fulfil the equation $\phi(n^*)=0$, where $\frac{\partial JTC}{\partial n}=\phi(n)$.

Now, the first order partial derivative of vendor-buyer system's joint total cost JTC regarding first delivery lot-size of every batch at the time of production i.e., q is

$$\begin{aligned}\frac{\partial JTC}{\partial q} &= \frac{2D(A_1+A_2+nY)}{q^2(2+\lambda n(n-1))} - \frac{X}{(2+\lambda n(n-1))}\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right)\\ &+ H_v\left[\frac{D}{P}+(2+\lambda n(n-1))E\right]\end{aligned}$$

(See Appendix A4 for the values of X, Y, and E.)

By equating $\frac{\partial JTC}{\partial q}=0$, the optimal value of q (say q^*) is as

$$q^* = \sqrt{\frac{2D(A_1+A_2+nY)}{\left[X\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right)+H_v(2+\lambda n(n-1))\left(\frac{D}{P}+(2+\lambda n(n-1))E\right)\right]}}$$

Again, the first order partial derivative of vendor-buyer system's joint total cost JTC related to rate of increasing shipment lot size λ is

$$\begin{aligned}\frac{\partial JTC}{\partial \lambda} &= \lambda^2\left(\frac{5nX(n-1)(2n-1)}{6}+H_vEn^2(n-1)^2\right)+\lambda(2X(2n-1)\\ &+ 4H_vEn(n-1))+\left(4H_vE-X-\frac{2D(A_1+A_2+nY)}{q^2}\right)\end{aligned}$$

Similarly as n, in this case the optimal value of λ (say λ^*) can be obtained if it satisfies $\Psi(\lambda^*)=0$, where $\Psi(\lambda)=\frac{\partial JTC}{\partial \lambda}$.

Finally, the first order partial derivative of vendor-buyer system's joint total cost JTC with respect to vendor's setup cost A_1 is

$$\frac{\partial JTC}{\partial A_1} = \frac{2D}{2q+\lambda n(n-1)q} - \frac{\alpha R}{A_1}$$

From the equation $\frac{\partial JTC}{\partial A_1}=0$, the optimal value of A_1 (say $A_1{}^*$) will be

$$A_1{}^* = \frac{\alpha R(2q+\lambda n(n-1)q)}{2D}$$

Lemma 1

Vendor-buyer system's joint total cost always consider the global minimum solution as the Hessian matrix for $JTC(n, q, \lambda, A_1)$ is positive definite at the optimal values $(n^*, q^*, \lambda^*, A_1^*)$.

Proof

The first order partial derivative of vendor-buyer system's joint total cost JTC for the variables n, q, λ, and A_1 which are as

$$\begin{aligned}
\frac{\partial JTC}{\partial n} &= \frac{1}{2+\lambda n(n-1)}\left[\frac{2DY}{q} + \frac{qX\lambda^2(6n^2-6n+1)}{6}\right] - \Big[(A_1 + A_2 + nY) \\
&+ q\lambda X\left(1 + \frac{\lambda^2 n(n-1)(2n-1)}{6}\right)\Big]\frac{(2n-1)}{(2+\lambda n(n-1))^2}, \\
\frac{\partial JTC}{\partial q} &= \frac{2D(A_1+A_2+nY)}{q^2(2+\lambda n(n-1))} - \frac{X}{(2+\lambda n(n-1))}\left(1 + \frac{\lambda^2 n(n-1)(2n-1)}{6}\right) \\
&+ H_v\left[\frac{D}{P} + (2+\lambda n(n-1))E\right], \\
\frac{\partial JTC}{\partial \lambda} &= \lambda^2\left(\frac{5nX(n-1)(2n-1)}{6} + H_v E n^2(n-1)^2\right) + \lambda(2X(2n-1) \\
&+ 4H_v E n(n-1)) + \left(4H_v E - X - \frac{2D(A_1+A_2+nY)}{q^2}\right),
\end{aligned}$$

and

$$\frac{\partial JTC}{\partial A_1} = \frac{2D}{2q + \lambda n(n-1)q} - \frac{\alpha R}{A_1}.$$

The Hessian matrix at the optimal values is given as

$$H_{ii} = \begin{bmatrix}
\frac{\partial^2 JTC(\cdot)}{\partial {A_1}^{*2}} & \frac{\partial^2 JTC(\cdot)}{\partial {A_1}^*\partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial {A_1}^*\partial n^*} & \frac{\partial^2 JTC(\cdot)}{\partial {A_1}^*\partial \lambda^*} \\
\frac{\partial^2 JTC(\cdot)}{\partial q^*\partial {A_1}^*} & \frac{\partial^2 JTC(\cdot)}{\partial {q^*}^2} & \frac{\partial^2 JTC(\cdot)}{\partial q^*\partial n^*} & \frac{\partial^2 JTC(\cdot)}{\partial q^*\partial \lambda^*} \\
\frac{\partial^2 JTC(\cdot)}{\partial n^*\partial {A_1}^*} & \frac{\partial^2 JTC(\cdot)}{\partial n^*\partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial {n^*}^2} & \frac{\partial^2 JTC(\cdot)}{\partial n^*\partial \lambda^*} \\
\frac{\partial^2 JTC(\cdot)}{\partial \lambda^*\partial {A_1}^*} & \frac{\partial^2 JTC(\cdot)}{\partial \lambda^*\partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial \lambda^*\partial n^*} & \frac{\partial^2 JTC(\cdot)}{\partial {\lambda^*}^2}
\end{bmatrix}$$

where $JTC(\cdot) = JTC(n^*, q^*, \lambda^*, A_1^*)$.

In this section, second order partial derivatives with respect to optimal values ${A_1}^*$, q^*, n^*, and λ^*

are as follows

$$
\begin{aligned}
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial {A_1}^{*2}} &= \frac{\alpha R}{{A_1}^2}\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial q^{*2}} &= \frac{4D(A_1+A_2+nY)}{q^3[2+\lambda n(n-1)]}\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial \lambda^{*2}} &= 2\lambda\left(\frac{5nX(n-1)(2n-1)}{6}+H_vEn^2(n-1)^2\right)+[2X(2n-1)\\
&+ 4H_vEn(n-1)]\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial n^{*2}} &= \frac{\lambda^2(2n-1)^2}{(2+\lambda n(n-1))^3}\Big[\frac{4D(A_1+A_2+nY)}{q^2}+2q^3X\Big(1\\
&+ \frac{\lambda^2 n(n-1)(2n-1)}{6}\Big)\Big]-\frac{\lambda}{(2+\lambda n(n-1))^2}\Big[\frac{4DY(2n-1)}{q}\\
&+ \frac{4D(A_1+A_2+nY)}{q}+2qX\Big(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\Big)\\
&+ q\lambda X(2n-1)\frac{(6n^2-6n+1)}{6}(\lambda+1)\Big]\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial {A_1}^*\partial q^*} &= \frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial q^*\partial {A_1}^*}=-\frac{2D}{q^2[2+\lambda n(n-1)]}\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial {A_1}^*\partial n^*} &= \frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial n^*\partial {A_1}^*}=-\frac{2D\lambda(2n-1)}{q[2+\lambda n(n-1)]^2}\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial {A_1}^*\partial \lambda^*} &= \frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial \lambda^*\partial {A_1}^*}=-\frac{2Dn(n-1)}{q[2+\lambda n(n-1)]^2}\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial q^*\partial n^*} &= \frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial n^*\partial q^*}\\
&= \frac{1}{2+\lambda n(n-1)}\Big[\frac{X\lambda^2(6n^2-6n+1)}{6}-\frac{2DY}{q^2}\Big]-\frac{(2n-1)\lambda X}{(2+\lambda n(n-1))^2}\Big(1\\
&+ \frac{\lambda^2 n(n-1)(2n-1)}{6}\Big)\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial n^*\partial \lambda^*} &= \frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial \lambda^*\partial n^*}\\
&= \lambda^2\left(\frac{5X(6n^2-6n+1)}{6}+H_vE(4n^3-6n^2+2n)\right)+\lambda(4X\\
&+ 4H_vE(2n-1))-\frac{2DY}{q^2}\\
\frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial q^*\partial \lambda^*} &= \frac{\partial^2 JTC(n^*,q^*,\lambda^*,A_1^*)}{\partial \lambda^*\partial q^*}=\frac{4D(A_1+A_2+nY)}{q^3}
\end{aligned}
$$

At the optimal values, principal minors of Hessian matrix are given by

$$det(H_{11})=det\left(\frac{\partial^2 JTC(.)}{\partial A^{*2}}\right)=\frac{\alpha R}{{A_1}^2}>0$$

As all terms i.e., α, R, and A_1 are positive, the first principal minor is obviously greater than zero.

Now

$$det(H_{22}) = \begin{vmatrix} \frac{\partial^2 JTC(\cdot)}{\partial {A_1^*}^2} & \frac{\partial^2 JTC(\cdot)}{\partial A_1^* \partial q^*} \\ \frac{\partial^2 JTC(\cdot)}{\partial q^* \partial A_1^*} & \frac{\partial^2 JTC(\cdot)}{\partial {q^*}^2} \end{vmatrix} = x_1 y_1 - {z_1}^2$$

where

$$x_1 = \frac{\partial^2 JTC(\cdot)}{\partial {A_1^*}^2} = \frac{\alpha R}{{A_1}^2} > 0,$$

$$y_1 = \frac{\partial^2 JTC(\cdot)}{\partial {q^*}^2} = \frac{4D(A_1 + A_2 + nA_3 + nS_b + nS_v)}{q^3[2 + \lambda n(n-1)]^2} > 0,$$

and

$$z_1 = \frac{2D}{q^2[2 + \lambda n(n-1)]} > 0.$$

It is assumed that $\xi = \frac{\alpha R}{{A_1}^2} - \frac{2D}{q^2[2+\lambda n(n-1)]}$.

$$\xi > 0 \;\; as \;\; \frac{\alpha R}{{A_1}^2} > \frac{2D}{q^2[2 + \lambda n(n-1)]} > 0.$$

On the other hand, $x_1 = det(H_{11}) > 0$.

Therefore, $x_1 - z_1 > 0$.

Similarly, it can be obtained that $y_1 > z_1$.

From two conditions $x_1 > z_1$ and $y_1 > z_1$, one can obtained that $x_1 y_1 - {z_1}^2 > 0$ which implies $det(H_{22}) > 0$.

Now

$$det(H_{33}) = \begin{vmatrix} \frac{\partial^2 JTC(\cdot)}{\partial {A_1^*}^2} & \frac{\partial^2 JTC(\cdot)}{\partial A_1^* \partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial A_1^* \partial n^*} \\ \frac{\partial^2 JTC(\cdot)}{\partial q^* \partial A_1^*} & \frac{\partial^2 JTC(\cdot)}{\partial {q^*}^2} & \frac{\partial^2 JTC(\cdot)}{\partial q^* \partial n^*} \\ \frac{\partial^2 JTC(\cdot)}{\partial n^* \partial A_1^*} & \frac{\partial^2 JTC(\cdot)}{\partial n^* \partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial {n^*}^2} \end{vmatrix} = u^2 x_1 + \zeta + w|H_{22}|$$

where

$$w = \frac{\partial^2 JTC(\cdot)}{\partial n^{*2}}.$$

$$\begin{aligned} u^2 &= \Big[\frac{(2n-1)\lambda X}{(2+\lambda n(n-1))^2}\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right) - \frac{1}{2+\lambda n(n-1)}\Big(\frac{X\lambda^2(6n^2-6n+1)}{6} \\ &- \frac{2DY}{q^2}\Big)\Big]^2 \end{aligned}$$

As $x_1 > 0$, then $u^2 x_1 > 0$.

Now

$$\begin{aligned} \zeta &= \frac{2D\lambda^2 X(2n-1)^2}{q[2+\lambda n(n-1)]^4}\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right) + \frac{8D^2\lambda(2n-1)}{q^3[2+\lambda n(n-1)]^4}\Big(\frac{X\lambda^2(6n^2-6n+1)}{6} \\ &- \frac{2DY}{q^2}\Big) + \frac{16D^3\lambda^2(4n-4n^2-1)(A_1+A_2+nY)}{[2q+\lambda n(n-1)q]^5} \\ &= \frac{2D\lambda(2n-1)}{q[2+\lambda n(n-1)]^4}\Big[\lambda X(2n-1)\left(1+\frac{\lambda^2 n(n-1)(2n-1)}{6}\right) + \frac{4D}{q^2}\Big(\frac{X\lambda^2(6n^2-6n+1)}{6} \\ &- \frac{2DY}{q^2}\Big) + \frac{8D^2\lambda(2n-1)(A_1+A_2+nY)}{[2q+\lambda n(n-1)q]}\Big] \end{aligned}$$

From the equation of ζ, $\frac{2D\lambda(2n-1)}{q[2+\lambda n(n-1)]^4} > 0$.

Therefore, $\zeta > 0$ as the expression within third bracket is also greater than zero.

As $|H_{22}| > 0$, it implies that $w|H_{22}| > 0$.

Therefore $det(H_{33}) = u^2 x_1 + \zeta + w|H_{22}| > 0$.

$$det(H_{44}) = \begin{vmatrix} \frac{\partial^2 JTC(\cdot)}{\partial {A_1}^{*2}} & \frac{\partial^2 JTC(\cdot)}{\partial {A_1}^{*}\partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial {A_1}^{*}\partial n^*} & \frac{\partial^2 JTC(\cdot)}{\partial {A_1}^{*}\partial \lambda^*} \\ \frac{\partial^2 JTC(\cdot)}{\partial q^*\partial {A_1}^{*}} & \frac{\partial^2 JTC(\cdot)}{\partial q^{*2}} & \frac{\partial^2 JTC(\cdot)}{\partial q^*\partial n^*} & \frac{\partial^2 JTC(\cdot)}{\partial q^*\partial \lambda^*} \\ \frac{\partial^2 JTC(\cdot)}{\partial n^*\partial {A_1}^{*}} & \frac{\partial^2 JTC(\cdot)}{\partial n^*\partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial n^{*2}} & \frac{\partial^2 JTC(\cdot)}{\partial n^*\partial \lambda^*} \\ \frac{\partial^2 JTC(\cdot)}{\partial \lambda^*\partial {A_1}^{*}} & \frac{\partial^2 JTC(\cdot)}{\partial \lambda^*\partial q^*} & \frac{\partial^2 JTC(\cdot)}{\partial \lambda^*\partial n^*} & \frac{\partial^2 JTC(\cdot)}{\partial \lambda^{*2}} \end{vmatrix}$$

Similarly as above, it can be prove that $det(H_{44}) > 0$.

It can be noticed that all principal minors are positive. Hence, the Hessian matrix H_{ii} is positive definite at $(n^*, q^*, \lambda^*, A_1^*)$. Therefore, the joint total cost consider the global minimum solution with respect to the optimum solution $(n^*, q^*, \lambda^*, A_1^*)$.

7.4 Numerical example

Example 1(a)

The parametric values for this model are chosen as

D = 1000 units/year, P = 4000 units/year, A_2 = \$300/order, A_3 = \$100/shipment, S_v = \$5/delivery, S_b = \$5/shipment, S = \$0.5/unit, Y_b = \$5/unit, Y_r = \$15/unit, $\gamma = 0.5$, Y_v = \$5/unit, H_1 = \$35/unit/year, H_2 = \$30/unit/year, H_v = \$20/unit/year, x = 3500 units/year, $\alpha = 0.1$, $R = 16000$, and A_0 = \$1000/setup. Hence, the vendor-buyer system's joint total cost JTC = \$24045.8, first delivery lot-size of every batch during production $q^* = 55$ units, vendor's setup cost ${A_1}^*$ = \$865.39, rate of increasing shipment lot-size $\lambda^* = 6$ unit/year, and number of shipments per batch production $n^* = 2$. Figure 7.2, Figure 7.3, Figure 7.4, Figure 7.5, Figure 7.6, and Figure 7.7 provides the optimality of the joint total cost for vendor-buyer system JTC.

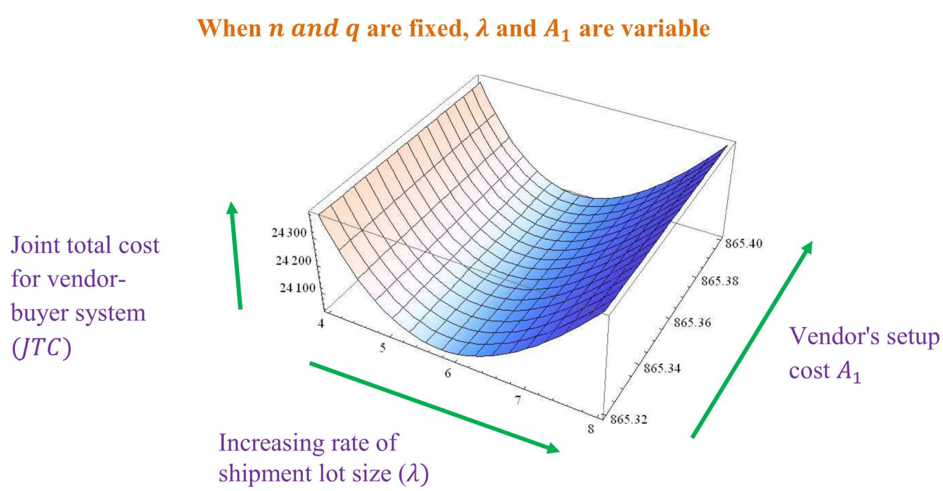

Figure 7.2: Joint total cost for vendor-buyer system (JTC) versus increasing rate of shipment lot-size (λ) and vendor's setup cost (A_1)

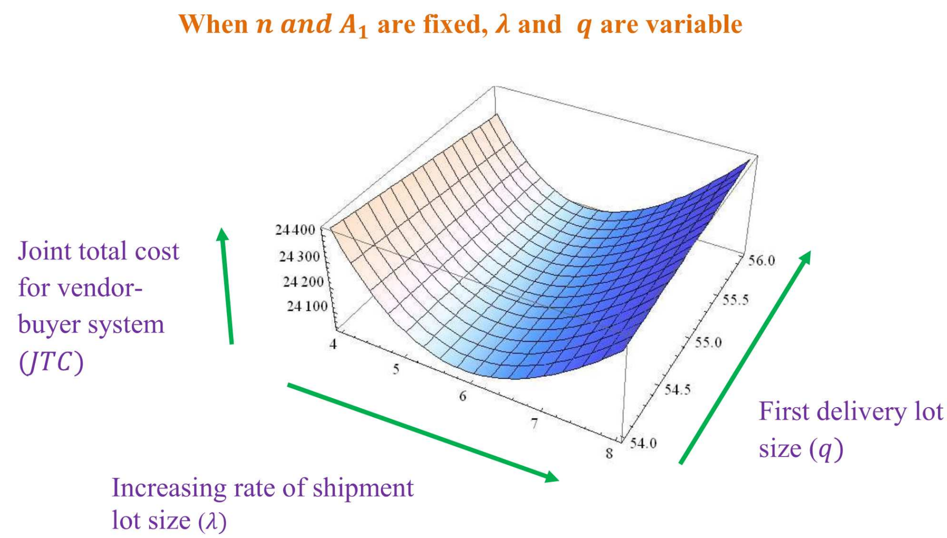

Figure 7.3: Joint total cost for vendor-buyer system (JTC) versus increasing rate of shipment lot size (λ) and first delivery lot-size (q)

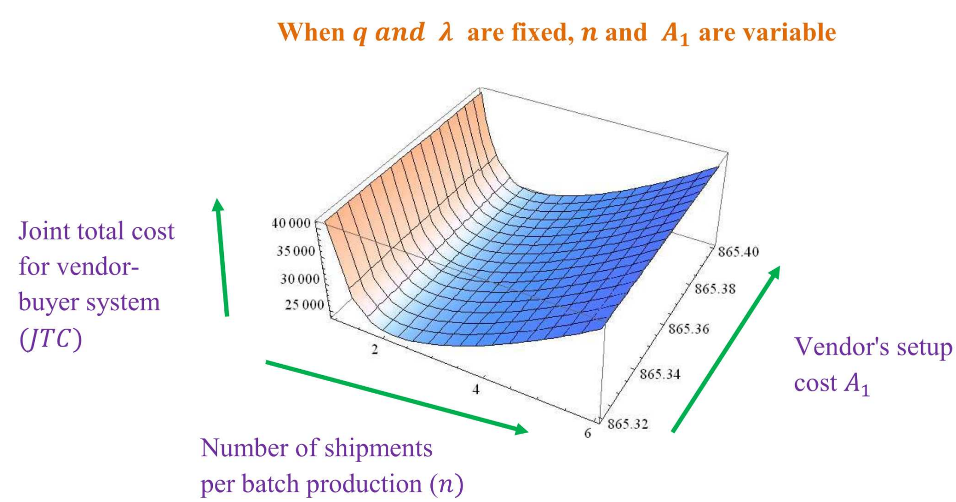

Figure 7.4: Joint total cost for vendor-buyer system JTC versus number of shipments per batch production (n) and vendor's setup cost (A_1)

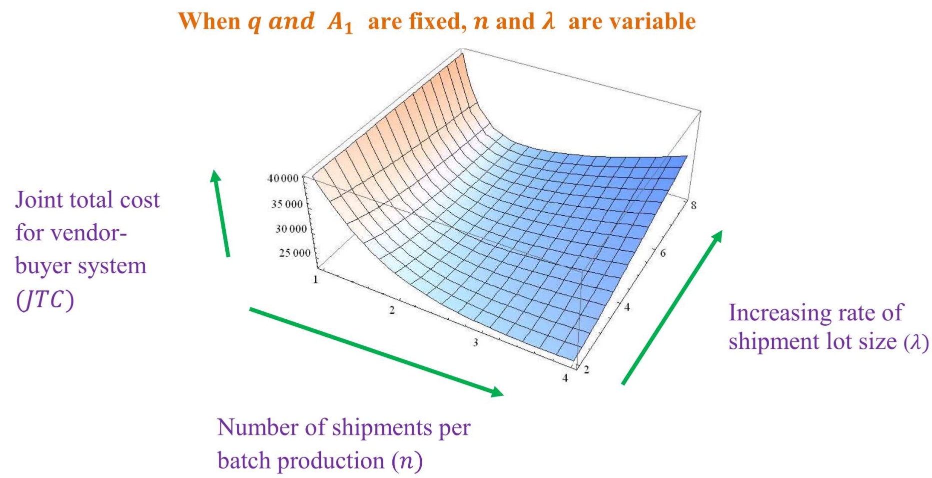

Figure 7.5: Joint total cost for vendor-buyer system JTC versus number of shipments per batch production (n) and increasing rate of shipment lot-size (λ)

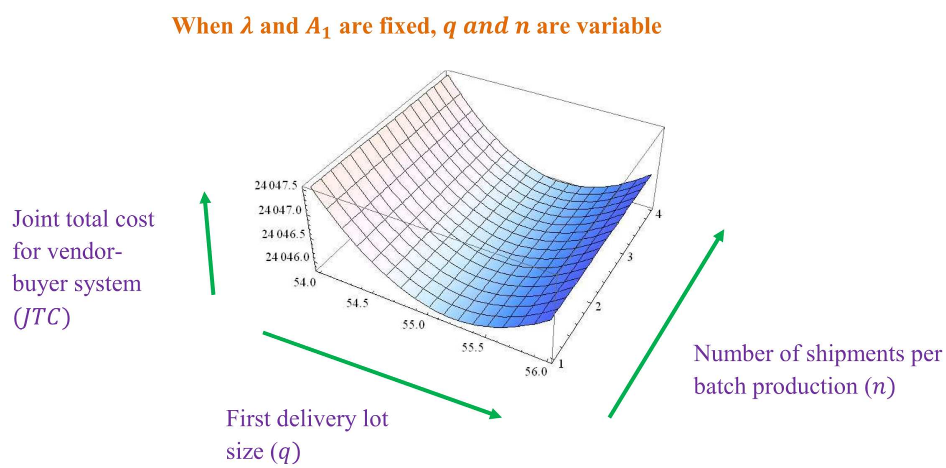

Figure 7.6: Joint total cost for vendor-buyer system (JTC) vs first delivery lot-size (q) and number of shipments per batch production (n)

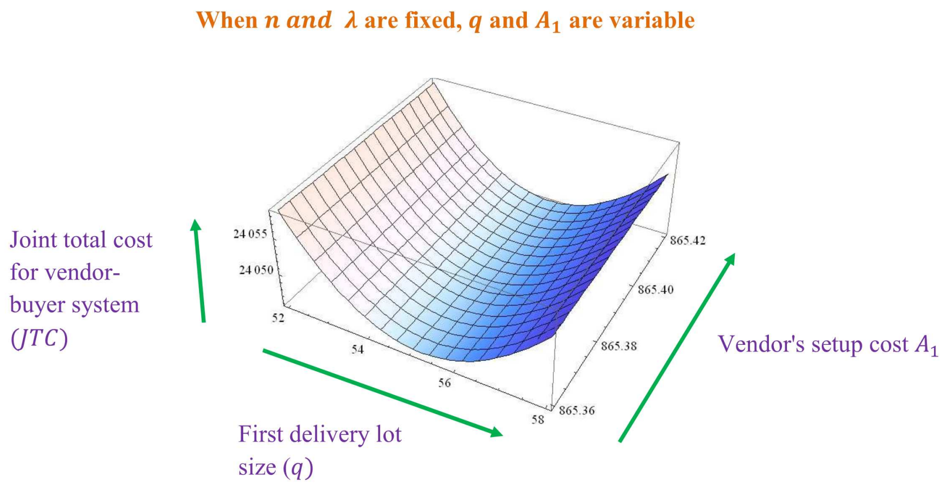

Figure 7.7: Joint total cost for vendor-buyer system (JTC) versus first delivery lot-size (q) and vendor's setup cost (A_1)

Case Study

This model considered single-setup-multiple-delivery (SSMD) policy. Vendors setup cost is reduced with some investment function. Additionally, delivery lot size is measured as unequal in each shipment of products. It is assumed that during transporting of items, both fixed and variable carbon emission costs are included to vendor as well as buyer. In this model, vital components are unequal lot sizes and carbon emission cost. Delivery truck is an example for this model. Due to SSMD policy and unequal lot sizes of products, transporting of lots increases that means more transporting vehicles i.e. delivery trucks increases. For this situation, emission of carbon dioxide from those trucks is also increases to the atmosphere.

Numerical example

Example 1(b)

The parametric values for this model are chosen as

D = 1200 units/year, P = 4500 units/year, A_2 = \$200/order, A_3 = \$80/shipment, S_v = \$4/delivery, S_b = \$6/shipment, S = \$0.6/unit, Y_b = \$6/unit, Y_r = \$20/unit, $\gamma = 0.6$, Y_v = \$6/unit, H_1 = \$40/unit/year, H_2 = \$25/unit/year, H_v = \$25/unit/year, x = 3000 units/year, $\alpha = 0.09$, $R = 15000$, and A_0 = \$500/setup. Hence, the vendor-buyer system's joint total cost JTC = \$32660.3, first delivery lot-size of every batch during production $q^* = 63$ units, vendor's setup cost $A_1{}^*$ = \$357.71, rate of increasing shipment lot-size $\lambda^* = 4$ unit/year, and number of shipments per batch production $n^* = 2$. Figure 7.8, Figure 7.9, Figure 7.10, Figure 7.11, Figure 7.12, and Figure 7.13 provides the optimality of the joint total cost for vendor-buyer system JTC.

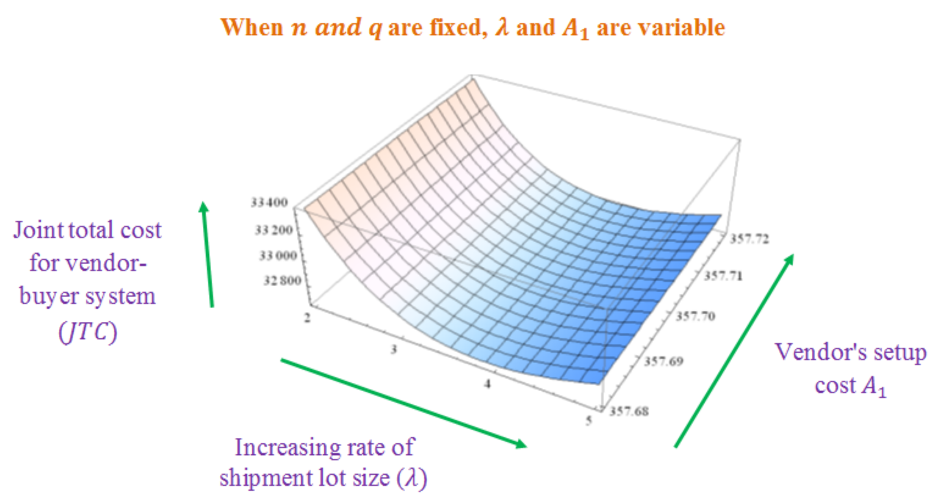

Figure 7.8: Joint total cost for vendor-buyer system (JTC) versus increasing rate of shipment lot-size (λ) and vendor's setup cost (A_1)

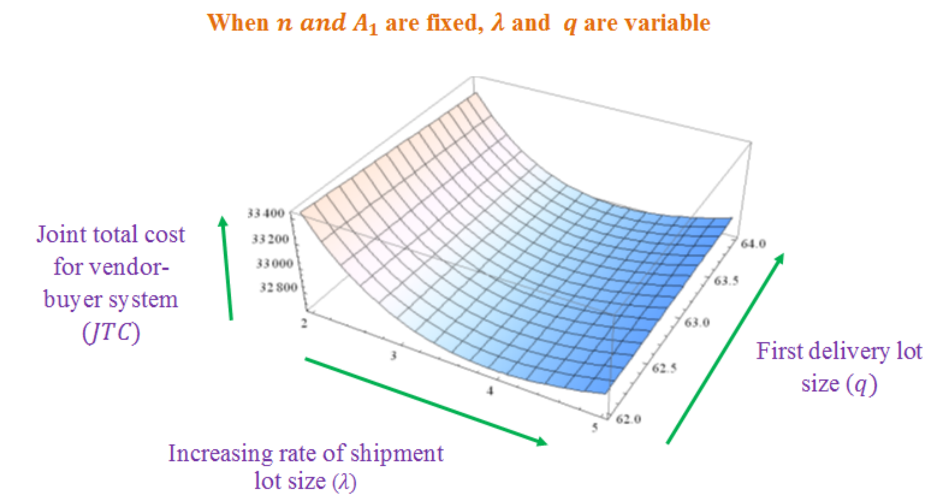

Figure 7.9: Joint total cost for vendor-buyer system (JTC) versus increasing rate of shipment lot size (λ) and first delivery lot-size (q)

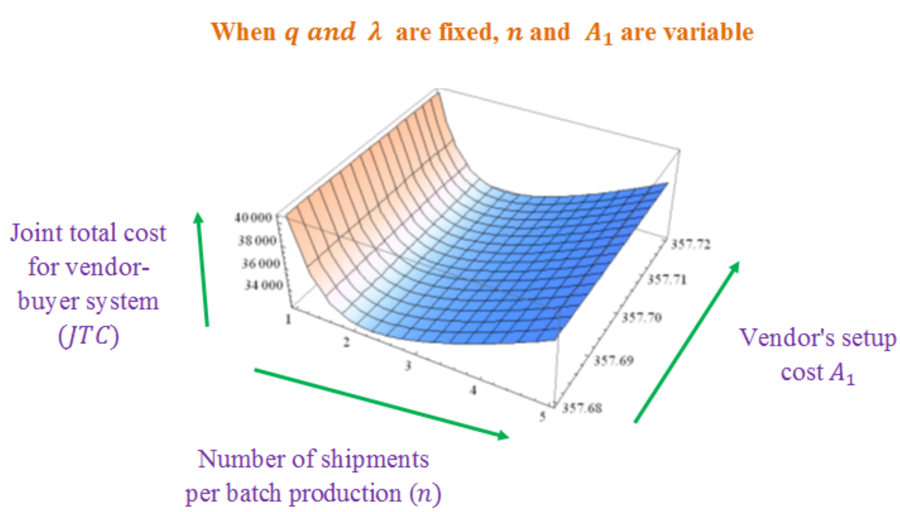

Figure 7.10: Joint total cost for vendor-buyer system JTC versus number of shipments per batch production (n) and vendor's setup cost (A_1)

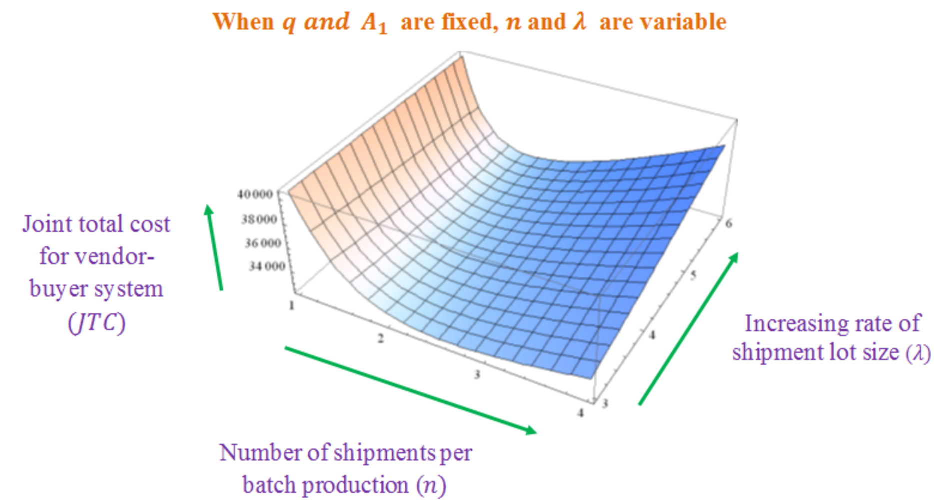

Figure 7.11: Joint total cost for vendor-buyer system JTC versus number of shipments per batch production (n) and increasing rate of shipment lot-size (λ)

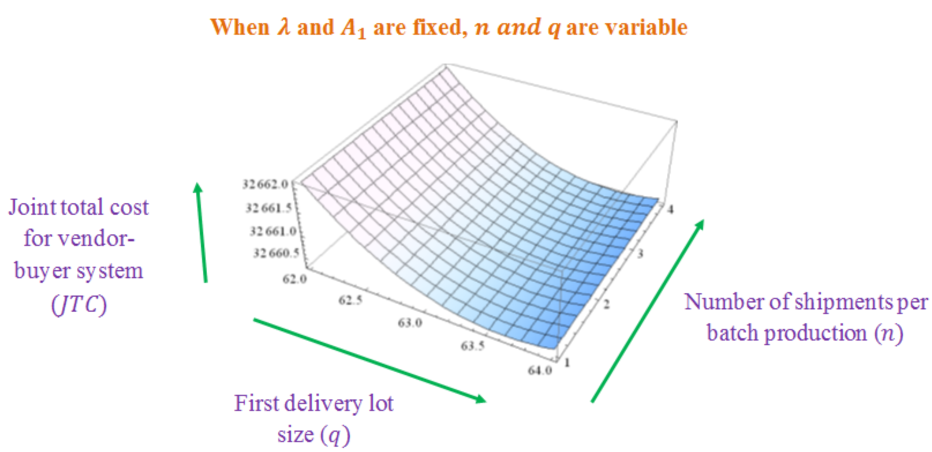

Figure 7.12: Joint total cost for vendor-buyer system (JTC) vs first delivery lot-size (q) and number of shipments per batch production (n)

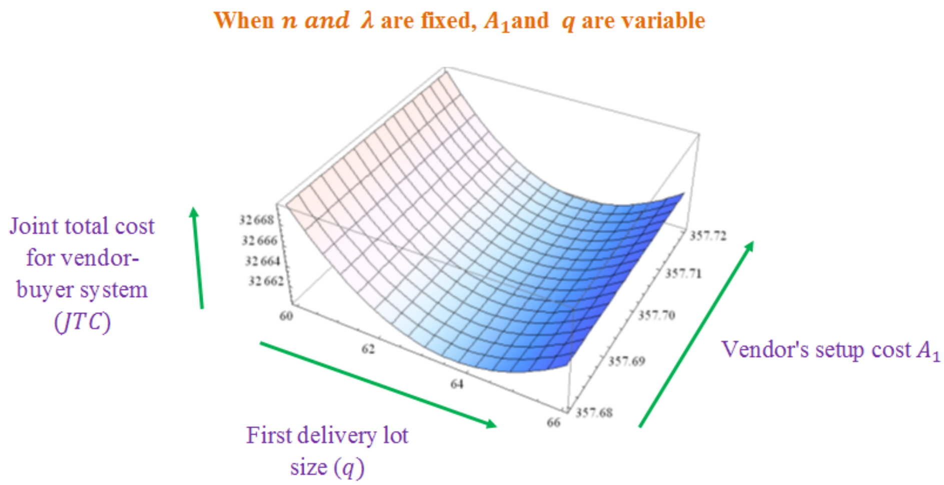

Figure 7.13: Joint total cost for vendor-buyer system (JTC) versus first delivery lot-size (q) and vendor's setup cost (A_1)

Sensitivity Analysis

The sensitivity analysis are given for the key parameters of the model in Table 7.2. This section provides sensitivity analysis to determine the impact of distinct parameters like A_2, P, S, Y_r, S_v, S_b, Y_b, Y_v, H_v, and H_1 on joint total cost for vendor-buyer system JTC.

Table 7.2: Sensitivity analysis for key parameters

Parameters	Changes(in %)	JTC	Parameters	Changes(in %)	JTC
A_2	-50%	-1.68	H_1	-50%	$-$
	-25%	-0.82		-25%	-6.76
	$+25\%$	0.8		$+25\%$	5.62
	$+50\%$	1.59		$+50\%$	10.46

Parameters	Changes(in %)	JTC	Parameters	Changes(in %)	JTC
Y_b	-50%	-1.57	P	-50%	-0.17
	-25%	-0.78		-25%	-0.04
	$+25\%$	0.78		$+25\%$	0.02
	$+50\%$	1.57		$+50\%$	0.03
S	-50%	-3.14	S_v	-50%	-0.03
	-25%	-1.57		-25%	-0.02
	$+25\%$	1.57		$+25\%$	0.02
	$+50\%$	3.14		$+50\%$	0.03
Y_r	-50%	-15.70	H_v	-50%	-0.28
	-25%	-7.85		-25%	-0.14
	$+25\%$	7.85		$+25\%$	0.13
	$+50\%$	15.70		$+50\%$	0.25
S_b	-50%	-0.03	Y_v	-50%	-0.16
	-25%	-0.02		-25%	-0.08
	$+25\%$	0.02		$+25\%$	0.08
	$+50\%$	0.03		$+50\%$	0.16

'–' means infeasible solution.

- The joint total cost for vendor-buyer system JTC raises while ordering cost A_2, production rate P, and inspection cost S are inclined. For the parameter A_2, negative percentage change and positive percentage changes are almost similar. In case P, negative percentage change is greater than the positive percentage change. The negative percentage change as well as positive percentage changes are similar for S.

- If rework cost Y_r increases, vendor-buyer system's joint total cost JTC also grows. Both positive and negative percentage changes for the parameter Y_r are same.

- For the increasing value of vendor's holding cost H_v, vendor-buyer system's joint total cost JTC also inclined.

- Buyer's fixed carbon-emission cost S_b and vendor's fixed carbon-emission cost S_v gives similar positive and negative percentage changes. An inclined value in S_b and S_v also increases vendor-buyer system's joint total cost JTC.

- Vendor-buyer system's joint total cost JTC grows while buyer's variable carbon-emission cost Y_b raises. Both the parameters Y_v and Y_b are equally sensitive in positive and negative percentage change.

- From the above sensitivity analysis table, it can be observed that vendor-buyer system's joint total cost JTC raises if buyer's holding cost for non-defective items H_1 inclined. For -50% this model gives infeasible solution.

7.5 Concluding remarks and future works

Vendor's setup cost reduction method by using an investment function is discussed in this chapter. This model assumed (SSMD) technique with variable delivery lot-sizes instead of lot-for-lot policy for minimizing vendor's holding cost. This model also reduced vendor-buyer system's joint total cost. A lemma was formulated to prove the global optimality of the solution of this model. This chapter expanded different models regarding SSMD policy excluding carbon-emission cost or SSMD policy including equal lot-sizes or SSMD policy omitting vendor's setup cost reduction. This model can be extended by adding some features such as inspection errors, shortages, and inflation.

7.6 Appendix

Appendix A4

$$
\begin{aligned}
X &= \frac{2D\gamma}{x}H_2 + (1-\gamma)^2 H_1 \\
Y &= A_3 + S_b + S_v \\
E &= \frac{(P-D)}{4P} - \frac{(1-\gamma)^2}{4} - \frac{\gamma D}{2x}
\end{aligned}
$$