Chapter 6

Inspection errors for an imperfect production system *

6.1 Introduction

For any manufacturing industry, the minimization of cost is a major consideration with product's quality. High quality products always lead to a higher cost and in-turn low quality products result to a lower cost. It is quite natural that manufactures decide the product's quality by product inspection policy. During the process inspection policy, inspection cost needs much attention which indicates the increasing value of labor cost. To reduce inspection cost, instead of full-inspection policy, only product inspection policy is utilized. For this inspection policy, defective products are detected easily with lower inspection cost. The aim of this inspection policy is to determine any quality defects that would prevent shipment of poor products. By utilizing this product inspection policy, shipments cost can be minimized by avoiding the dispatch of any imperfect quality

*A part of this work, presented in this chapter, is published in *European Journal of Operational Research*, 248(1), 263-271, 2016.

products. Chryssolouris and Patel (1987) discussed the production process where imperfect items are detected through product inspection policy. Kim and Hong (1999) considered an EMQ model in an imperfect process with an arbitrary distribution and some defective items. Wang and Sheu (2001) described the relationship between production, inventory, and inspection in a deteriorating production system. Chryssolouris (2006) developed several quality related issues for the process control. Previous research work with offline inspection policy of products was extended by Wang and Meng (2009). Bendavid and Herer (2009) introduced an optimal policy that obtains the number of units to inspect and the number of disposal units to minimize the expected cost. Hassan and Diab (2010) incorporated a visual inspection operation to examine multiple qualities characteristics. Sarkar *et al.* (2010) formulated an optimal production lot-size model which assumes safety stock, reliability parameter, and random breakdown of machinery system. They generalized the model with preventive and corrective maintenance, safety stock for repair times, and shortages. Laofor and Peansupap (2012) surveyed an innovative system for defective detection and quantification that able to augment for visual quality inspections. Chen (2013) formulated the integrated problem of production, preventive maintenance (PM), and inspection in an imperfect production process where rework and PM errors exist. Baudet *et al.* (2013) described how a sensory analysis test can be applied for the visual inspection of product surface.

The basic EPQ model assumes that all produced items are perfect. But it is almost impossible for any production system because of long-run production process. Imperfect quality or poor items may be produced during long-run production system. During the production, the system may transfer to *out-of-control* state from *in-control* state at any random time. Thus, producing items are not always perfect. Generally it depends on the condition of the production process. Usually, when the production process starts, the machine is *in-control* state and items produced are near about 100%

perfect quality. After sometimes, it transfers to *out-of-control* state and produces defective products. In this way of research, Porteus (1986) discussed about process quality by reducing the probability of the process moved to *out-of-control* state. Rosenblatt and Lee (1986) modeled the variable proportion of defective items for linear, exponential, and multi-state deteriorating processes. Tseng *et al.* (1998) examined several maintenance policies for deteriorating production systems. Ben-Daya (2002) derived an integrated model for joint determination of economic production quantity and preventive maintenance (PM) level in an imperfect process with increasing hazard rate. Chung and Hou (2003) developed a production model to obtain an optimal run time for a deteriorating production system with allowable shortages. Lee (2005) investigated an imperfect production system with imperfect product quality and supplied quantity. Lin *et al.* (2008) discussed an algebraic approach to replace the use of calculus on the cost function as well as long-run average production-inventory costs in an imperfect EMQ model. Liao *et al.* (2009) extended previous works by assuming a deteriorating production system with increasing hazard rate: imperfect repair and rework upon failure (*out-of-control* state). Ouyang and Chang (2013) discussed effects of reworking for imperfect quality items, trade-credit policy, and complete backlogging in an economic production quantity model. Tai (2013) developed two EPQ models for deteriorating/imperfect items with rework process. Sarkar and Moon (2014) established the relationship between quality improvement, reorder point, lead time, and backorder rate in an imperfect production process. Sarkar *et al.* (2014) surveyed an inventory model with improved quality, backorder price-discount policy, and controllable lead time.

Quality of any product can be obtained through product inspection policy by assuming 100% perfect inspection process. But in general, inspection process is not error free in reality as all machines are not allow 100% perfect inspections and human factors are involved. There may be a possibility of Type I error (falsely rejecting non-defective items) or Type II error (falsely accepting defective items) in any industry. Type I error generates when a perfect item is rejected and it implies manufacturer's risk. On the other hand, Type II error generates when the acceptance of an imperfect lot is considered and it implies customer's risk. It is also considered that cost of falsely accepted defective items is much greater than cost of falsely rejected non-defective items. Because, falsely accepted defective items in system may result system failure which causes loss of human lives. Raz and Bricker (1993) considered inspection errors during screening in an production process. Rentoul *et al.* (1994) studied several ways of inspection errors in manufacturing system which are made by comparing inspection points with a solid model of the desired component. Wang and Sheu (2003) determined an optimal production, inspection, and maintenance policy under the effect of process inspection errors. An inspection policy with two types of inspection errors to accept the economic production quantity for real world applications was considered by Wang (2007). An inventory model in an of imperfect production process with the preventive maintenance and inspection errors was considered by Darwish and Ben-Daya (2007). Wang *et al.* (2010) considered a partial inspection approach over commonly used policies for both full and no inspection. Lin *et al.* (2011) investigated an imperfect production system for production lot-size, maintenance, and quality with increasing hazard rates. Yoo *et al.* (2012) obtained an optimal lot-size in an imperfect production with inspection, customer return, and defective disposal. Hsu and Hsu (2013) developed an economic order quantity model with imperfect quality items, inspection errors, shortages, and sales returns. Cárdenas-Barrón *et al.* (2013) deduced an EMQ model for rework and multiple shipments. Sarkar *et al.* (2014) formulated an inventory model with inspection policy and variable lead time. Sarkar *et al.* (2014) discussed quality improvement of products, service-level constraint, and setup cost reduction for an inventory system. Sarkar *et al.* (2014) obtained an inventory model by assuming random defective rates which follow three different distribution functions such as uniform, triangular, and beta.

Warranty period is the time period in which a sale out product provides satisfactory performance to customers offered by the retailer. If any purchased product/sale out item failed to work within its warranty period, then the retailer replace it with a new item or repair one part of some parts that product. Warranty cost or post sale cost includes repair cost, parts replacement cost, and labor cost. Chun and Tang (1995) developed an inventory model with the free-replacement and fixed-period warranty policy within a given warranty period. Monga and Zuo (1998) derived a problem on reliability based design of a series-parallel system by considering burn-in, warranty, and maintenance. Wang (2004) deduced an economic production quantity problem in an imperfect production process with a free-repair warranty policy. Wang (2005) described product-inspection policy for a deteriorating production system. Chen and Lo (2006) developed an imperfect production system with allowable shortages for products with free minimal repair warranty. Giri and Dohi (2007) described an inventory model under two different inspection policies: (i) no action is taken during a production-run unless the system is discovered in an *out-of-control* state by inspection and (ii) preventive repair action is undertaken once the *in-control* state of the process is detected by inspection. Darghouth *et al.* (2012) presented an analytical model with inspection policy and warranty period. See Table 6.1 for contribution of different authors.

Table 6.1: Contribution of different authors

Author(s)	**Imperfect production system**	**Product inspection policy**	**Inspection errors**	**Warranty cost**
Porteus (1986)	$\surd$			

Author(s)	**Imperfect production system**	**Product inspection policy**	**Inspection errors**	**Warranty cost**
Rosenblatt and Lee (1986)	√			
Raz and Bricker (1993)			√	
Rentoul *et al.* (1994)			√	
Chun and Tang (1995)				√
Monga and Zuo (1998)				√
Tseng *et al.* (1998)	√			
Kim and Hong (1999)	√			
Wang and Sheu (2001)	√	√		
Wang and Sheu (2003)	√		√	
Wang (2004)	√			√
Wang (2005)	√	√		√
Chen and Lo (2006)	√			√
Wang (2007)		√	√	
Darwish and Ben-Daya (2007)	√		√	
Giri and Dohi (2007)	√			√
Wang and Meng (2009)	√	√		
Liao *et al.* (2009)	√			
Hassan and Diab (2010)		√		
Wang *et al.* (2010)	√		√	

Author(s)	**Imperfect production system**	**Product inspection policy**	**Inspection errors**	**Warranty cost**
Sarkar *et al.* (2010)	√			
Lin *et al.* (2011)	√		√	
Yoo *et al.* (2012)	√		√	
Darghouth *et al.* (2012)	√			√
Laofor and Peansupap (2012)		√		
Tai (2013)	√			
Baudet *et al.* (2013)		√		
Chen (2013)	√	√		
Hsu and Hsu (2013)	√		√	
Ouyang and Chang (2013)	√			
Sarkar and Moon (2014)	√			
Sarkar *et al.* (2014)	√			
This chapter	√	√	√	√

In this chapter, a production process in which the machinery system shifts from *in-control* state to *out-of-control* state at any random time is discussed. It is considered that once the system transfers to *out-of-control* state, it remains there until the production-run. The *out-of-control* state of the imperfect process follows a probabilistic distribution until the production stops. Defective items are detected through inspection and are reworked at some fixed cost. On the other hand, non-inspected defective items are transported to the market for sale with warranty/post sale cost.

This chapter is formulated to minimize the expected total cost per item by the product inspection policy and production-run length. This chapter ends with some numerical examples and sensitivity analysis.

6.2 Mathematical model

To develop this model some notation is used. which are as follows:

Decision variables

t production-run length (unit time)

u non-inspected fraction in a batch ($0 \leq u \leq 1$) (units)

Parameters

d annual demand per unit time (units/unit time)

p production rate per unit time (units/unit time)

k setup cost for each production-run per setup (\$/setup)

h unit inventory holding cost of a product per unit time (\$/unit/unit time)

C_m labor cost to construct a single item (\$/unit)

θ_1 percentage of defective items produced in *in-control* state

θ_2 percentage of defective items developed in *out-of-control* state, $\theta_2 > \theta_1$

X random variable during the elapsed time of system in *in-control* state

$F(x)$ distribution function of X

$\bar{F}(x)$ survival function of X i.e., $\bar{F}(x) = 1 - F(x)$

$f(x)$ probability density function of X

$E(X)$ mean lifetime of X

η fixed cost to check the process for determining the state of the system ($)

r restoration cost to transfer the process to *in-control* state if the system is in *out-of-control* state ($)

C_1 unit inspection cost ($/unit)

C_s salvaged cost per defective lot after inspection ($/defective lot)

C_w post sale (warranty) cost for non-inspected defective lots ($/non-inspected defective lot)

C_a cost of falsely accepting defective lot ($/defective lot)

C_r cost of falsely rejecting non-defective lot ($/non-defective lot)

m_1 random variable representing Type I error

m_2 random variable representing Type II error

C_d defective cost includes the costs of defective lot ($/defective lot)

$C(t, u)$ expected total cost per item ($/unit)

The following assumptions are considered to formulate this model

1. Initially, production starts from *in-control* state for a single type of product. After some time, the production system shifts to *out-of-control* state until the end of the production-run.

2. As $p > d$, there is no shortage in this model.

3. This model assumes $\theta_1 < \theta_2$ i.e., the probability of number of defective items in *in-control* state is less than the probability of number of defective items in *out-of-control* state.

4. The product inspection policy is performed to detect the defective items. These items are salvaged with some cost C_s.

5. Two types of inspection errors namely Type I and Type II errors are introduced during product inspection policy.

This model considers a manufacturing system for single type of products such as glass, food products, and electronic gadgets. Initially the production system starts from *in-control* state and shifts to *out-of-control* state at any random time and it stays in *out-of-control* state until the end of the production-run. The probability (θ_1) of the number of defective items in *in-control* state is less than the probability (θ_2) of the number of defective items in *out-of-control* state. Once the production system stops, the product inspection policy is carried out to detect defective items. This model considers negligible product inspection time. The product inspection policy starts from the (ptu)th item till the end of the production system. It indicates that $u = 0$ and $u = 1$ are full-inspection as well as no inspection, respectively. It is assumed that while screening products, the inspectors make errors as Type I error and Type II error. They separate some non-defective products as defective, i.e., $(1 - \theta_1)m_1$ in *in-control* state and $(1 - \theta_2)m_1$ in *out-of-control* state. On the other hand, they classify some defective items as non-defective, i.e., $\theta_1 m_2$ in *in-control* state and $\theta_2 m_2$ in *out-of-control* state. After product inspection with inspection cost, defective items are salvaged at some fixed cost C_s before being shipped. In addition, non-inspected defective items are taken as salvable products and those items are sent to the market with post sale (warranty) cost

C_w. Generally, it is assumed that $C_1 + C_s < C_w$.

Now the expected total cost per item $C(t, u)$ is derived as follows:

(i) Labor cost to make a single product is C_m.

(ii) From Figure 1 in ΔOAF, the inventory level starts with $p - d$ rate and in ΔFAB, the inventory level depletes with a rate $-d$. Hence, the average inventory is $\frac{(p-d)}{2d}$. Therefore, holding cost for carrying a single product during the time interval t is $\frac{h(p-d)t}{2d}$.

(iii) Setup cost for each production-run is k. During the production-run length t with the production rate p, the total produced items are pt. Therefore, setup cost per item is $\frac{k}{pt}$.

(iv) Fixed cost to check the process for obtaining the state of the system is η. Thus the process inspection cost per item is $\frac{\eta}{pt}$, where pt is total produced items.

(v) While the process shifts to *out-of-control* state, the process is transferred back to *in-control* state with some restoration cost r. Hence, the restoration cost per unit item is $\frac{rF(t)}{pt}$, where pt is total produced items.

(vi) Non-inspected fraction in every batch is assumed as u. Thus, the inspected fraction is $(1 - u)$. Therefore, product inspection cost per item is $C_1(1 - u)$.

(vii) To determine defective cost before and after sale, there are three cases which are as follows:

Case 1 $X \geq t$,

Case 2 $ut < X < t$,

and

Case 3 $0 \leq X \leq ut$.

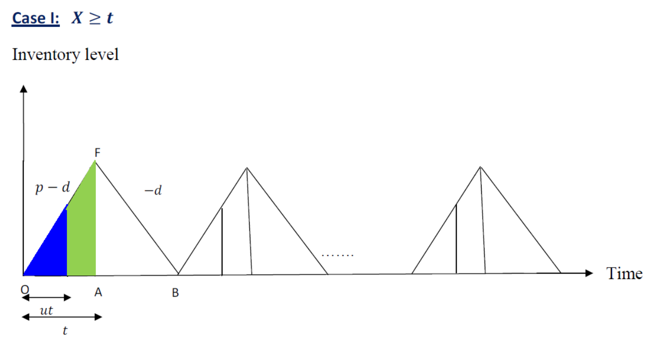

Figure 6.1: Graphical representation of inventory system when $X \geq t$

Case 1 $X \geq t$

In this case, the whole production process is in *in-control* state [See Figure 6.1]. Product inspection starts from (ptu)th item until the end of finished products. The production time interval $[0, t]$ can be divided into two sub-intervals i.e., $[0, ut]$ and $[ut, t]$, where u is the non-inspected fraction in every batch. During the time interval $[0, ut]$, non-inspected defective items i.e., $\theta_1 put$ are shipped to market with some warranty cost C_w. On the other hand, inspected defective items i.e, $\theta_1 p(t-ut)$ are salvaged at some fixed cost C_s in $[ut, t]$. Therefore, the cost of defective items is $C_d = C_w\theta_1 put + C_s\theta_1 p(t-ut)$.

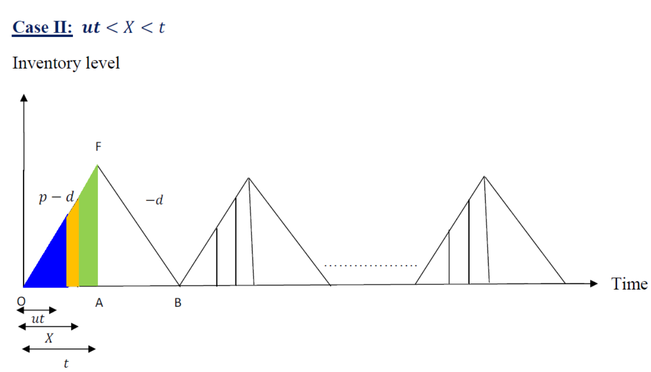

Figure 6.2: Graphical representation of inventory system when $ut < X < t$

Case 2 $ut < X < t$

During the time interval $[0, X]$, the production process is in *in-control* state and in $[X, t]$ the process is in *out-of-control* state. From figure 6.2, it is cleared that the production cycle time interval $[0, t]$ can be divided into three sub-intervals i.e., $[0, ut]$, $[ut, X]$, and $[X, t]$. Throughout the time interval $[0, ut]$, non-inspected defective items i.e., $\theta_1 put$ are shipped to market with some warranty cost C_w. Inspected defective items i.e, $\theta_1 p(X - ut)$ in $[ut, X]$ and $\theta_2 p(t - X)$ in $[t, X]$ are salvaged at some fixed cost C_s.

Therefore, the cost of defective items is $C_d = C_w\theta_1 put + C_s\theta_1 p(X - ut) + C_s\theta_2 p(t - X)$.

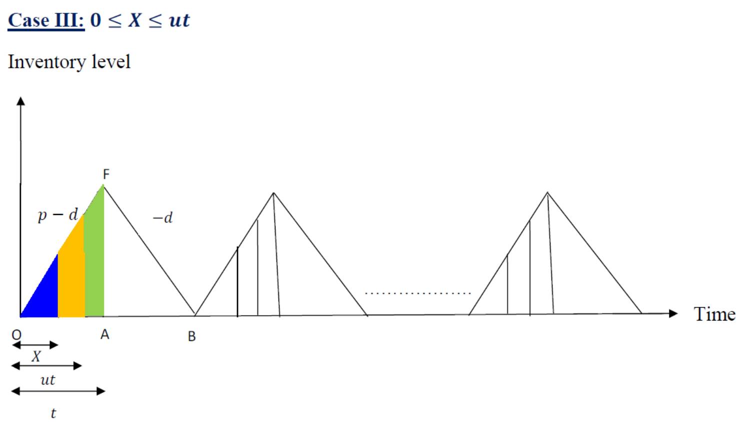

Figure 6.3: Graphical representation of inventory system when $0 \leq X \leq t$

Case 3 $0 \leq X \leq ut$

Figure 6.3 concludes that the production cycle time interval $[0, t]$ can be divided into three sub-intervals i.e., $[0, X]$, $[X, ut]$, and $[ut, t]$. Non-inspected defective items i.e., $\theta_1 pX$ in $[0, X]$ and $\theta_2 p(ut - X)$ in $[X, ut]$ are transferred to market with some warranty cost C_w. On the other hand, inspected defective items i.e, $\theta_2 p(t - ut)$ are salvaged at some fixed cost C_s throughout the time interval $[ut, t]$.

Therefore, the cost of defective items is $C_d = C_w\theta_1 pX + C_w\theta_2 p(ut - X) + C_s\theta_2 p(t - ut)$.

Hence, the defective cost C_d including the cost of defective items before and after sale is

$$C_d = \begin{cases} C_w\theta_1 put + C_s\theta_1 p(t-ut); & \text{if } X \geq t \\ C_w\theta_1 put + C_s\theta_1 p(X-ut) + C_s\theta_2 p(t-X); & \text{if } ut < X < t \\ C_w\theta_1 pX + C_w\theta_2 p(ut-X) + C_s\theta_2 p(t-ut); & \text{if } 0 \leq X \leq ut \end{cases}$$

This model considers two types of inspection errors namely Type I error and Type II error during product inspection policy.

Case 1 $X \geq t$

During product inspection in the time interval $[ut, t]$, inspectors accepts $\theta_1 p(t-ut)$ defective items in which falsely accept defective items $\theta_1 p(t-ut)m_2$ with some fixed cost C_a per unit and falsely reject non-defective items $(1-\theta_1)p(t-ut)$ with some fixed cost C_r per unit.

Therefore, the defective cost is

$C_r p(t-ut)(1-\theta_1)m_1 + C_a\theta_1 p(t-ut)(1-m_2)$.

Case 2 $ut < X < t$

During the time interval $[ut, X]$, inspectors accepts $\theta_1 p(X-ut)$ defective items in which falsely accept defective items $\theta_1 p(X-ut)m_2$ with some fixed cost C_a per unit and falsely reject non-defective items $(1-\theta_1)p(X-ut)$ with some fixed cost C_r per unit. Also, throughout the time interval $[X, t]$, inspectors accepts $\theta_2 p(t-X)$ defective items in which falsely accept defective items $\theta_2 p(t-X)m_2$ with some fixed cost C_a per unit and falsely reject non-defective items $(1-\theta_2)p(t-X)$ with some fixed cost C_r per unit.

Therefore, the defective cost for misclassification of produced items is

$p(X-ut)[C_a\theta_1(1-m_2) + C_r(1-\theta_1)m_1] + p(t-X)[C_a\theta_2(1-m_2) + C_r(1-\theta_2)m_1]$.

Case 3 $0 \le X \le ut$

In the time interval $[ut, t]$, inspectors accepts $\theta_2 p(t-ut)$ defective items in which falsely accept defective items $\theta_2 p(t-ut)m_2$ with some fixed cost C_a per unit and falsely reject non-defective items $(1-\theta_2)p(t-ut)$ with some fixed cost C_r per unit.

Therefore, the defective cost for misclassification of produced items is

$p(t-ut)[C_a\theta_2(1-m_2) + C_r(1-\theta_2)m_1]$.

After adding Type I error and Type II error, the defective cost C_d becomes

$$C_d = \left\{ \begin{array}{ll} P + C_r p(t-ut)(1-\theta_1)m_1 + C_a\theta_1 p(t-ut)(1-m_2); & \text{if } \; X \ge t \\ Q + p(X-ut)[C_a\theta_1(1-m_2) + C_r(1-\theta_1)m_1] & \text{if } ut < X < t \\ \quad +p(t-X)[C_a\theta_2(1-m_2) + C_r(1-\theta_2)m_1]; & \\ R + p(t-ut)[C_a\theta_2(1-m_2) + C_r(1-\theta_2)m_1]; & \text{if } 0 \le X \le ut \end{array} \right\}$$

where

$$\begin{aligned} P &= C_w\theta_1 put + C_s\theta_1 p(t-ut), \\ Q &= C_w\theta_1 put + C_s\theta_1 p(X-ut) + C_s\theta_2 p(t-X), \end{aligned}$$

and

$$R = C_w\theta_1 pX + C_w\theta_2 p(ut-X) + C_s\theta_2 p(t-ut).$$

The expected cost of per defective item is

$$\begin{aligned} E[C_d] &= [(1-u)C_s\theta_2 + C_a(1-E[m_2])\theta_2 + C_r E[m_1](1-\theta_2) + C_w u\theta_2] \\ &+ \frac{[(\theta_1-\theta_2)C_s + C_a(1-E[m_2]) - C_r E[m_1] \int_{ut}^{t} \bar{F}(x)dx]}{t} \\ &+ \frac{[C_w(\theta_1-\theta_2)\int_0^{ut} \bar{F}(x)dx]}{t} \end{aligned}$$

Now, the expected total cost per item i.e., $C(t,u)$ is given by

$C(t,u)$=labor cost+holding cost+setup cost+process inspection cost+restoration cost

+product inspection cost+defective cost and warranty cost.

$$
\begin{aligned}
&= C_m + \frac{h(p-d)t}{2d} + \frac{k}{pt} + \frac{\eta}{pt} + \frac{rF(t)}{pt} + C_1(1-u) + [(1-u)(C_s\theta_2 + C_a(1-E[m_2])\theta_2 \\
&+ C_rE[m_1](1-\theta_2)) + C_wu\theta_2] + \frac{[(\theta_1-\theta_2)(C_s + C_a(1-E[m_2]) - C_rE[m_1]) \int_{ut}^{t}\bar{F}(x)dx]}{t} \\
&+ \frac{[C_w(\theta_1-\theta_2)\int_0^{ut}\bar{F}(x)dx]}{t}
\end{aligned}
$$

As $C(t,u)$ is non-linear function of t and u. Thus, it is very much difficult to obtain some closed-form solutions for this model. Hence, we use numerical method to obtain the solution.

6.3 Numerical examples

Using the numerical data from Wang (2005), the expected total cost per item $C(t,u)$, production-run length t, and non-inspected fraction in a batch u are determined. Mathematica 9 as a tool is used to obtain expected total cost per item $C(t,u)$. Assuming that, the fraction of defective and the percentage of inspection errors (Type I and Type II) follow a Weibull distribution.

The density function of Weibull distribution is

$$
f(x;\alpha,\beta) = \left\{ \begin{array}{ll} \alpha\beta(\alpha x)^{\beta-1}e^{(-\alpha x)^{\beta}} & \text{if } X \geq 0 \\ 0 & \text{if } X < 0 \end{array} \right\}
$$

Example 1(a)

Let $K = k + \eta = \$50$/setup, $p = 40$ units/hour, $d = 30$ units/hour, $C_s = \$1$/defective lot, $\theta_1 = 0$, $\theta_2 = 0.05$, $h = \$0.08$/unit/hour, $C_m = 0$, $C_w = \$25$/non-inspected defective lot, $r = \$100$, $C_1 = \$0.1$/unit, $E[m_1] = 0.01$, $E[m_2] = 0.04$, $C_a = \$0.023$/defective lot, $C_r = \$0.01$/non-defective lot, $\alpha = 0.1$, and $\beta = 1$, then the expected total cost per item $C(t,u) = \$0.598$ and the expected total cost is \$400.85, production-run length $t = 16.758$ hours, and non-inspected fraction in a batch $u = 0.0005$ units. The optimum total cost $C(t, u)$ versus non-inspected fraction in a batch u and production-run length t are given by Figure 6.4.

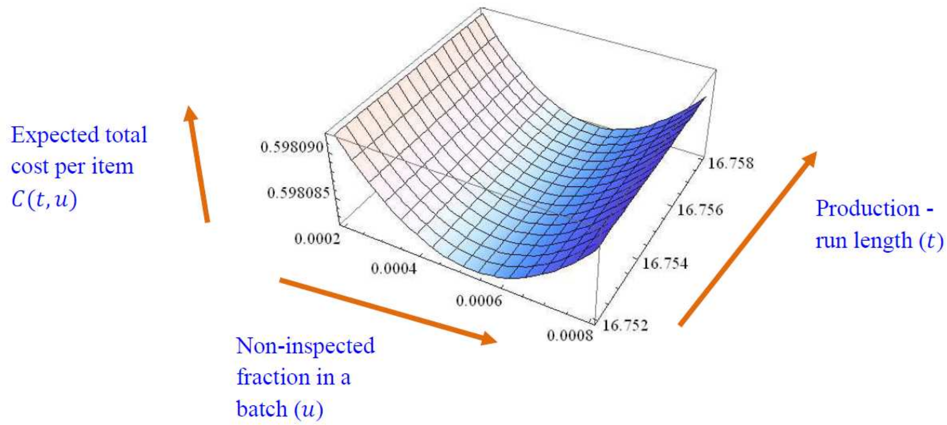

Figure 6.4: Expected total cost $C(t, u)$ versus non-inspected fraction in a batch (u) and production-run length (t)

Example 2(a)

In Wang (2005), labor cost to construct a single item C_m is 0. But in reality, the value of C_m must be non-zero. We assume the labor cost C_m as \$0.02/unit and all values are same as Example 1. Then the expected total cost per item $C(t, u) = \$0.618$ and the expected total cost is \$414.26, production-run length $t = 16.758$ hours, and non-inspected fraction in a batch $u = 0.0005$ units. The optimum total cost $C(t, u)$ versus non-inspected fraction in a batch u and production-run length t are given by Figure 6.5.

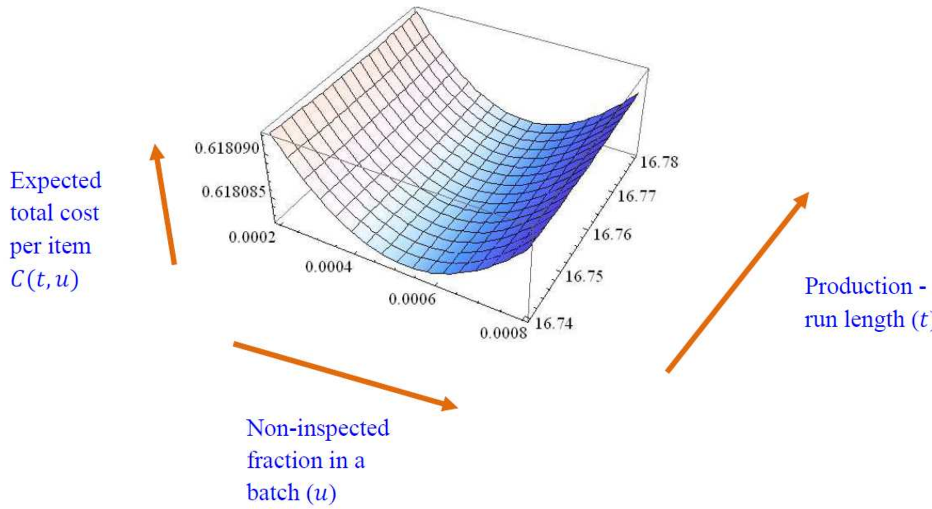

Figure 6.5: Expected total cost $C(t,u)$ versus non-inspected fraction in a batch (u) and production-run length (t)

Example 3(a)

In Wang (2005), percentage of defective items produced in *in-control* state i.e., θ_1 is zero. But, in reality during *in-control* state of production process, any machine may produce defective items. Therefore, the value of θ_1 must be non-zero. We assume defective items produced in *in-control* state i.e., θ_1 as 0.002 and the labor cost C_m as \$0.02/unit and all values are same as Example 1. Then the expected total cost per item $C(t,u) = \$0.618$ and the expected total cost is \$414.28, production-run length $t = 16.759$ hours, and non-inspected fraction in a batch $u = 0.0002$ units. The optimum total cost $C(t,u)$ versus production-run length t and non-inspected fraction in a batch u are given by Figure 6.6.

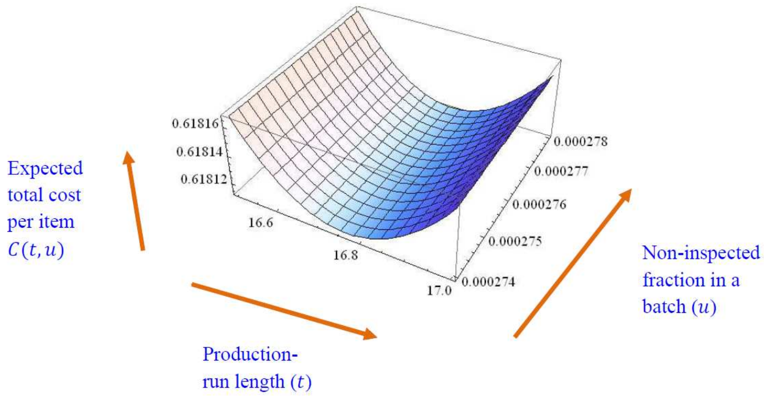

Figure 6.6: Expected total cost $C(t,u)$ versus production-run length (t) and non-inspected fraction in a batch (u)

Case Study

This model inserted product inspection policy for imperfect production system. Besides, non-inspected items are sold out to market with some warranty cost. Two types of inspection errors i.e. Type 1 error and Type 2 error are discussed in this model. Here, major factors of this model are product inspection policy, warranty cost, and inspection errors. Any electronic gadgets are real example of this concept which is mentioned above. One can consider popular electronic gadget like Mobile Phone. Each mobile phones are comes to the market with their own warranty period. Generally, warranty period of mobile phones is 1 year provided by several reputed manufacturing companies.

Numerical examples

Example 1(b)

Let $K = k + \eta = \$45$/setup, $p = 50$ units/hour, $d = 25$ units/hour, $C_s = \$1.5$/defective lot, $\theta_1 = 0$, $\theta_2 = 0.04$, $h = \$0.04$/unit/hour, $C_m = 0$, $C_w = \$20$/non-inspected defective lot, $r = \$90$, $C_1 = \$0.2$/unit, $E[m_1] = 0.02$, $E[m_2] = 0.05$, $C_a = \$0.03$/defective lot, $C_r = \$0.02$/non-defective lot, $\alpha = 0.2$, and $\beta = 1$, then the expected total cost per item $C(t, u) = \$0.598$ and the expected total cost is \$416.88, production-run length $t = 11.58$ hours, and non-inspected fraction in a batch $u = 0.005$ units. The optimum total cost $C(t, u)$ versus non-inspected fraction in a batch u and production-run length t are given by Figure 6.7.

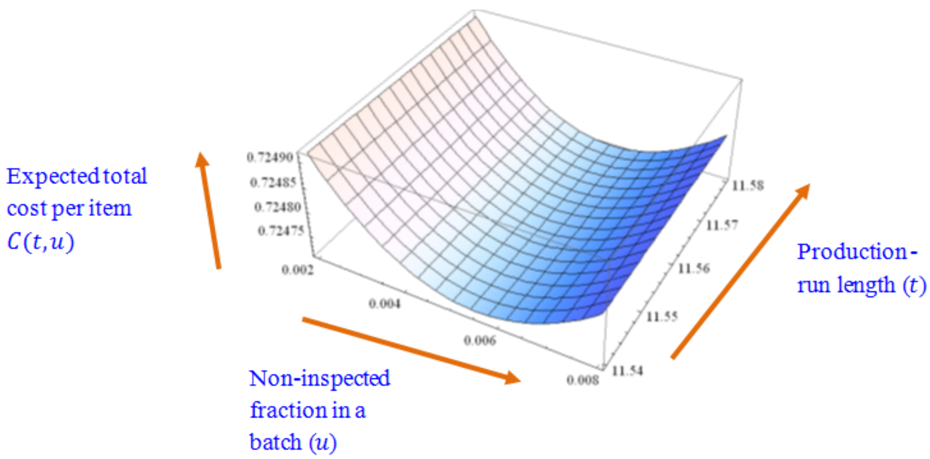

Figure 6.7: Expected total cost $C(t, u)$ versus non-inspected fraction in a batch (u) and production-run length (t)

Example 2(b)

By considering the labor cost C_m as \$0.1/unit and all values are same as Example 1(b). Then the expected total cost per item $C(t, u) = \$0.82$ and the expected total cost is \$474.78, production-run length $t = 11.58$ hours, and non-inspected fraction in a batch $u = 0.005$ units. The optimum total cost $C(t, u)$ versus non-inspected fraction in a batch u and production-run length t are given by Figure 6.8.

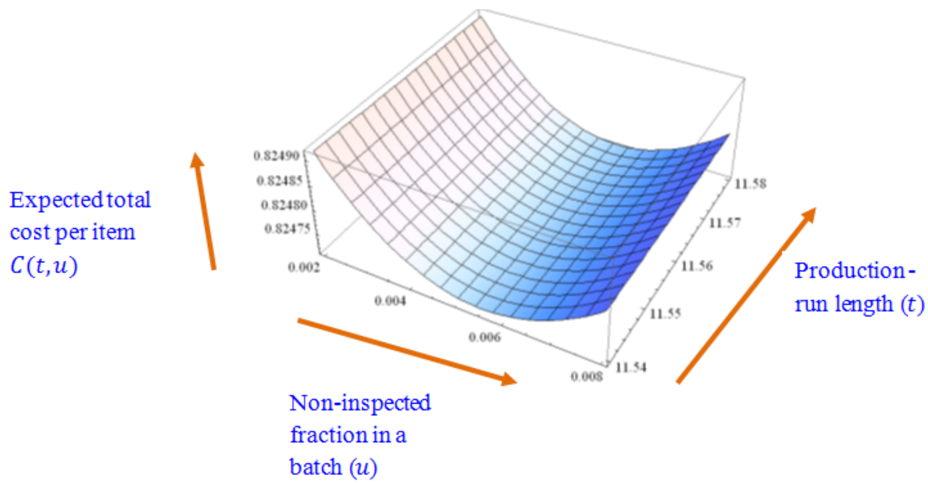

Figure 6.8: Expected total cost $C(t, u)$ versus non-inspected fraction in a batch (u) and production-run length (t)

Example 3(b)

By taking the defective items produced in *in-control* state i.e., θ_1 as 0.009 and the labor cost C_m as \$0.1/unit and all values are same as Example 1(b). Then the expected total cost per item $C(t, u)$ = \$0.82 and the expected total cost is \$476.76, production-run length $t = 11.6$ hours, and non-inspected fraction in a batch $u = 0.001$ units. The optimum total cost $C(t, u)$ versus production-run length t and non-inspected fraction in a batch u are given by Figure 6.9.

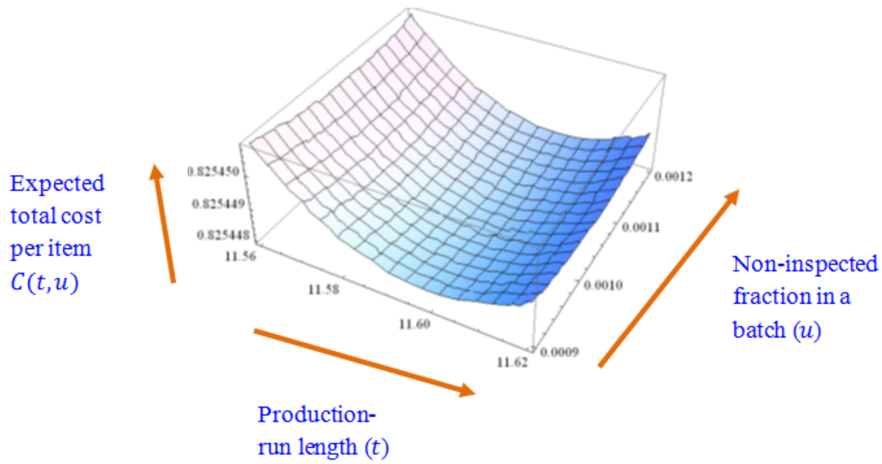

Figure 6.9: Expected total cost $C(t,u)$ versus production-run length (t) and non-inspected fraction in a batch (u)

Sensitivity Analysis

The sensitivity analysis for parameters h, C_w, r, and C_1 are given based on Example 1(a) in Table 6.2.

Table 6.2: Sensitivity analysis of h, C_w, r, and C_1 based on Example 1(a)

Parameters	Changes(in %)	$C(t,u)$	Parameters	Changes(in %)	$C(t,u)$
	−50%	−21.88		−50%	−13.73
	−25%	−10.01		−25%	−6.52
h	+25%	8.82	r	+25%	5.99
	+50%	16.79		+50%	11.57
	−50%	−0.005		−50%	−8.36
	−25%	−0.0015		−25%	−4.18
C_w	+25%	0.0009	C_1	+25%	4.18
	+50%	0.0015		+50%	8.35

- If unit inventory holding cost of a product per unit time h increases then expected total cost per item $C(t, u)$ increases. This model is more sensitive with the percentage change in h. As produced products are sent for inspection procedure one by one, therefore holding cost for produced items incurs less amount. The negative percentage change is greater than the positive percentage change in h.

- The expected total cost per item $C(t, u)$ increases if post sale (warranty) cost for non-inspected defective lots C_w increases. The parameter C_w is less sensitive as non-inspected fraction in every batch i.e., u is very small. Therefore non-inspected defective items are very few. These items are sale out with some warranty cost C_w.

- The expected total cost per item $C(t, u)$ also increases while restoration cost to transfer the process to *in-control* state if the system is in *out-of-control* state r increases. This model is more sensitive in negative than positive percentage change for r.

- The expected total cost per item $C(t, u)$ increases if the parameter C_1 increases. The unit inspection cost C_1 is almost similar for both positive and negative percentage changes.

The sensitivity analysis for parameters of this model is given based on Example 2(a) in Table 6.3.

Table 6.3: Sensitivity analysis for C_m, r, C_w, C_1, and h with respect to Example 2(a)

Parameters	Changes(in %)	$C(t, u)$	Parameters	Changes(in %)	$C(t, u)$
	-50%	-21.18		-50%	-8.09
	-25%	-9.69		-25%	-4.04
h	$+25\%$	8.53	C_1	$+25\%$	4.04
	$+50\%$	16.25		$+50\%$	8.08

Parameters	Changes(in %)	$C(t,u)$
C_m	-50%	-1.62
	-25%	-0.81
	$+25\%$	0.81
	$+50\%$	1.62
C_w	-50%	-0.005
	-25%	-0.0015
	$+25\%$	0.0009
	$+50\%$	0.0015

Parameters	Changes(in %)	$C(t,u)$
r	-50%	-13.29
	-25%	-6.31
	$+25\%$	5.80
	$+50\%$	11.20

Sensitivity analysis are considered to determine the effect of several parameters such as h, C_m, C_w, r, and C_1, respectively on the expected total cost per item $C(t,u)$.

- The expected total cost per item $C(t,u)$ increases if unit inventory holding cost of a product per unit time h increases. This model is more sensitive with the percentage change in h. As produced products are sent for inspection procedure one by one, therefore holding cost for produced items incurs less amount.

- It is observed that in labor cost to construct a single item C_m is equal for both positive and negative percentage changes. The expected total cost per item $C(t,u)$ increases if the parameter C_m increases.

- The parameter C_w is less sensitive as non-inspected fraction in every batch i.e., u is very small. Therefore non-inspected defective items are very few. These items are sale out with some warranty cost C_w. An increasing value in post sale (warranty) cost for non-inspected defective lots C_w increases the expected total cost per item $C(t,u)$.

- While restoration cost to transfer the process for *in-control* state if the system is in *out-of-control* state r increases, the expected total cost per item $C(t, u)$ also increases. This values for r are more sensitive in negative than positive percentage change.

- As the parameter C_1 increases then the expected total cost per item $C(t, u)$ also increases. The values of unit inspection cost C_1 are almost equal for both positive and negative percentage changes.

The sensitivity analysis for parameters of this model is given based on Example 3(a) in Table 6.4.

Table 6.4: Sensitivity analysis for parameters based on Example 3(a)

Parameters	Changes(in %)	$C(t, u)$
h	-50%	-21.18
	-25%	-9.69
	$+25\%$	8.53
	$+50\%$	16.25
C_m	-50%	-1.62
	-25%	-0.81
	$+25\%$	0.81
	$+50\%$	1.62
C_w	-50%	-0.004
	-25%	-0.001
	$+25\%$	0.0006
	$+50\%$	0.0009

Parameters	Changes(in %)	$C(t, u)$
C_1	-50%	-8.09
	-25%	-4.04
	$+25\%$	4.04
	$+50\%$	8.08
r	-50%	-13.29
	-25%	-6.31
	$+25\%$	5.80
	$+50\%$	11.20

The effect of various parameters as h, C_m, C_w, r, and C_1, respectively on the expected total cost per item $C(t, u)$ are discussed in this section.

- It can be concluded that this model is more sensitive with the percentage change in h. As produced products are sent for inspection procedure one by one, therefore holding cost for produced items incurs less amount. The expected total cost per item $C(t, u)$ increases if unit inventory holding cost of a product per unit time h increases.

- The expected total cost per item $C(t, u)$ increases if the parameter C_m increases. The percentage change in labor cost to construct a single item C_m is same for both positive and negative percentage changes.

- While post sale (warranty) cost for non-inspected defective lots C_w increases, the expected total cost per item $C(t, u)$ also increases. The parameter C_w is less sensitive as non-inspected fraction in every batch i.e., u is very small. Hence non-inspected defective items are very few which are sale out with some warranty cost C_w.

- This model is more sensitive in negative than positive percentage change for r. While restoration cost to transfer the process to *in-control* state if the system is in *out-of-control* state r increases, the expected total cost per item $C(t, u)$ also increases.

- It is observed that percentage change in the unit inspection cost C_1 is almost similar for both positive and negative percentage changes. The expected total cost per item $C(t, u)$ increases if the parameter C_1 increases.

6.4 Concluding remarks and future works

This chapter reduced the expected total cost per item by finding the non-inspected fraction of batch and production-run length. Some numerical examples are given to prove more savings from the existing literature. Product inspection policy and two types of errors during inspection are highlighted in this chapter to make this model more realistic. In future, some extensions can be done by considering machine breakdown. It will be a nice contribution if this model will assume some finite inspection time in production-run.