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2018

2nd Semester

STATISTICS

PAPER—C4T

(Honours)

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group-A

Answer any *five* out of eight questions : 5×2

1. Let x be a continuously distributed random variable with pdf

$$f(x) = \begin{cases} 1/x^2; & \text{if } 1 < x < \infty \\ 0; & \text{otherwise} \end{cases}.$$

Show that E(x) does not exist.

(Turn Over)

2. Prove that $\beta_2 \geq \beta_1 + 1$; where β_1 and β_2 have their usual meaning.

3. Show that E(x) does not exist for Cauchy distribution.

4. For a continuous random variable defined in the range $0 < x < \infty$, the probability distribution of X is given by

$$P(X \leq x) = 1 - e^{-\beta x} \; ; \; \beta > 0$$

Find the median of the distribution.

5. If x be a non-negative random variable with mean μ (exists) and c.d.f. F(x). Show that $\mu = \int_0^\infty (1 - F(x))\, dx$.

6. If a random variable X assumes only two values 2 and 3 with

$$P(X = 2) = 2\, P(X = 3), \text{ find } V(X).$$

7. Suppose two random variables X and Y are independent. Then find E(XY), where E(X + Y) = 3 and E(X − Y) = 5.

8. Suppose x is a non-negative random variable such that E(x) and $E\left(\frac{1}{x}\right)$ exist. Show that $E\left(\frac{1}{x}\right) \geq \frac{1}{E(x)}$.

Group-B

Answer any *four* out of sight questions : 4×5

9. Suppose (X, Y) follows BN(0, 0, 1, 1, ρ). Show that the correlation coefficient between X^2 and Y^2 is ρ^2.

10. Let X be distributed with pdf

$$f(x) = \begin{cases} 1 ; & 0 < x < 1 \\ 0 ; & \text{otherwise} \end{cases}$$

Then, using Chebysher's lemma find a lower bound of

$$p\left[1 \times \frac{1}{2} \right| \leq \sqrt{\left(\frac{1}{3}\right)} \right].$$

11. For a normal distribution with standard deviation σ, show that the central moment of order 2r is

$$\mu_{2r} = (2r - 1)\cdot(2r - 3) \ldots\ldots 5 \cdot 3 \cdot 1 \cdot \sigma^{2r}.$$

12. Find the Arithmatic Mean (AM) and Geometric Mean (GM) of the distribution

$$f(x) = \begin{cases} \frac{1}{B(p,q)} (1-x)^{p-1} x^{q-1}; & 0 \leq x \leq 1 \\ 1; & p > q > 0 \end{cases}$$

where $B(p, q) = \int_0^1 x^{q-1}(1-x)^{p-1} dx$

$$= \frac{\overline{|p}\,\overline{|q}}{\overline{|p+q}}.$$

13. Prove that the lognormal distribution is unimodal.

14. For a continuously distributed random variable X with pdf f, show that the mean deviation

$$E|X-c| = \int_{-\infty}^{\infty} |x-c| f(x)dx$$

is a minimum when c is the median of the distribution.

Group-C

Answer any *one* out of two questions : 1×10

15. Find the lines of regression of X on Y and of Y on X when the joint distribution of X and Y has the density function

$$f(x,y) = 3x^2 - 8xy + 6y^2 ; \quad 0 \le x \le 1, \; 0 \le y \le 1$$

16. Suppose F(x) is a cumulative distribution function (cdf). Examine whether G(x) given below is also a cdf.

$$G(x) = \frac{1}{2a}\int_{x-a}^{x+a} F(t)dt \text{ when } a > 0 .$$

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