2018

CBCS

1st Semester

STATISTICS

PAPER—C2T

(Honours)

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Probability and Probability Distributions – I

1. Answer any *five* questions : 5×2

(a) Given the probability space (Ω, f, P) show that if $P(A)=\frac{3}{4}$ and $P(B)=\frac{3}{8}$, for $A, B \in f$, then $\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$.

(b) Let A, B are two independent events defined on probability space (Ω, f, P) with $P(A)=\frac{1}{3}, P(B)=\frac{3}{4}$. Find $P(A \mid A \cup B)$ and $P(B \mid A \cup B)$.

(Turn Over)

(c) Suppose 5 accidents occur in Haldia in a given week. What is the probability that they all accurred in different days of that week?

(d) A fair die is rolled n times. What is the probability of obtaining at least one 6?

(e) Suppose a coin is tossed n times. If the coin is unbiased, what is the probability that first toss gives a head and n^{th} toss gives a tail?

(f) If a coin is tossed infinitely many times, what will be the sample space?

(g) Can two events A and B be simultaneously mutually exclusive and mutually independent?

(h) An unbiased Rice is thrown and X be the random variable which gives the face value. Obtain probability generating function of X.

2. Answer any *four* questions : 4×5

(a) Let $A_1, A_2 \ldots A_N$ be n events defined on the sample space Ω.

Show that $P\left(\bigcap_{i=1}^{N} A_i\right) = T_1 - T_2 + T_3 \ldots + (-1)^{N-1} T_N$

where $T_K = \sum\sum\ldots\sum_{i_1 < i_2 < \ldots < i_k} P(A_{i_1} \cup \ldots \cup A_{i_k})$.

(b) Let x_1, x_2 represent the number of ones and twos in n throws of a fair dice. Find joint distribution of x_1 and x_2.

(c) Derive the pmf of Hypergeometric distribution from suitable random experiment. How can the experiment be changed to get pmf of Binomial distribution ?

(d) The out comes of an experiment are equally likely to be one of the four points (1, 0, 0), (0, 1, 0), (0, 0, 1) and (1, 1, 1). If A, B, C denote the events x-co-ordinate 1, y-co-ordinate 1, z co-ordinate 1 respectively. Show that A, B, C are pairwise independent but not mutually independent.

(e) Show that the e.d.f of Binomial distribution can be expressed as an Incomplete Beta function.

(f) Suppose two decks of N cards numberre 1, 2 ... N are matched against each other. What is the probability of getting

(i) at least one match

(ii) exactly 'm' matches ?

3. Answer any *one* question : 1×10

(a) (i) Define probability function on a given probability space.

(ii) Show that classical definition of probability is a special case of axiomatic definition of probability.

(iii) Let P be a probability on (Ω, f), where f is the σ-field of Ω. Show that if $A_n \uparrow A$ then $P(A_n) \to P(A)$

where $A = \bigcup_{n=1}^{\infty} A_n$.

(iv) Show that countable additivity of P implies finite additivity. When is the converse also true ?

2+2+3+3

(b) (i) Derive the pmf of Negative binomial distribution from a suitable random experiment.

(ii) Let X be a non-negative integer valued random variable satisfying $P(X > m + 1 \mid X > m) = P(X \geq 1)$ for any non-negative integer m. Then show that X must have a geometric distribution.

(iii) Suppose a man wants to open a door with a bunch of 'n' keys of which only one fits the door. Each key is tried at random. Then find the expected number of trials needed to open the door. Obtain the variance.

2+3+5
