2017

STATISTICS

[Honours]

(CBCS)

[First Semester]

PAPER – C2T

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

1. Answer any *five* questions : 2×5

(*a*) What is the probability that 2 of the 4 children have same birthday ?

(*b*) *A* can solve 75% of problems of a mathematics book while *B* can solve 70% of problems of the book. What is the chance that a problem selected at random will be solved when both A and B try independently ?

(c) Four friends say A, B, C, D stand at random in a queue. Show that the two events "A precedes B" and "C precedes D" are independent.

(d) Let X and a positive integer valued random variable. Then

$$E(X) = \sum_{n=1}^{\infty} P(X \geq n)$$

(e) Let $P(X = C) = 1$. Draw the distribution function of X.

(f) The probability of hitting a target is 0·001 for each shot. Find the probability of hitting a target with two or more bullets if number of shots is 5000. $[e^{-5} = 0 \cdot 00674]$

(g) A lot consisting of 50 bulbs, 10 of which are defective, is inspected. What is the probability of selecting at most 1 defective bulb ?

(h) Show that two independent random variables are always uncorrelated.

GROUP – B

Answer any **four** questions : 5 × 4

2. Derive the probability mass function of negative binomial distribution from a random experiment consisting of a infinite sequence Bernoulli trials with success probability p. Show that

$$\binom{r+k-1}{k} p^2 (1-p)^k \to e^{-\lambda} \frac{\lambda^k}{k!}.$$

as $p \to 1, r \to \infty$ such that $r(1-p) = \lambda$ remains fixed.

3. (*a*) Show that if $P(B) > 0$, then

$$P(A_1 \cup A_2 \mid B) = P(A_1 \mid B) + P(A_2 \mid B) - P(A_1 \cap A_2 \mid B)$$

A_1, A_2, B are defined on same probability space.

(*b*) If $P(A \cap B) > 0$ show that

$$P(B \cap C \mid A) = P(B \mid A)\, P(C \mid A \cap B)$$

Here A, B, C are events in same sample space.

4. Consider a random experiment of tossing two fair dice. Define

$$X = \text{sum of the two dice}$$

$$Y = |\text{ difference of the two dice }|$$

Obtain the joint distribution table of (X, Y).

5. Each of n urns has a white and b black balls. One ball is chosen at random from 1st urn and transferred to the second, then one ball is chosen at random from second and transferred to third, and so on. At the end of this operation, if a ball is taken at random from last urn, what is the probability of its being white ?

6. State and prove the Baye's theorem for conditional probability.

7. Define the cumulative distribution function (c.d.f) of a random variables. Show that it is monotone non-decreasing and also right continuous.

GROUP – C

Answer any **one** questions : 10 × 1

8. Derive the prit of binomial distribution from a sequence of '*n*' Bernoulli trials with success probability *p*. If μ_r be the *r*th order central moment, show that

$$\mu_{r+1} = pq\left(rn\,\mu_{r-1} + \frac{d\mu_r}{dp}\right)$$

comment on the skewness and Kurtosis of the distribution. Obtain the mean derivation about mean for binomial distribution. 1 + 3 + 2 + 4

9. (*a*) Derive the moment generating function (m.g.f.) of a bivariate normal distribution.

(*b*) If the correlation coefficient between two random variable *X* and *Y* jointly following a bivariate normal distribution is zero, judge within you can say that *X* on *Y* are independently distributed. 7 + 3