2019

B.Sc. (Hons.)

4th Semester Examination

STATISTICS

Paper—C9T

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *five* out of *eight* questions : 5×2=10

(a) Distinguish between a parameter and a statistic. Explain with examples.

(b) What are the two types of Errors that arise in statistical inference ?

(c) Derive the $100(1-\alpha)\%$ confidence interval for the mean of a normal distribution with variance 4.

(d) Find the pdf of $X(n) = \max(X_1, X_2, \ldots, X_n)$, where $X_1, X_2, \ldots, X_n$ are *iid* $U(0, \theta)$.

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(e) Distinguish between the size of a test and the level of significance.

(f) Let X be a random variable ~Rec(0, 1) then find the p.d.f. of $Y = -2\ln X$.

(g) If your test always accepts Ho, what may be the possible values of α and β, where α and β have their usual meaning ?

(h) Derive the moment generating function of χ^2 distribution with n degrees of freedom.

2. Answer any *four* out of *six* quetions : 4×5=20

(a) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a normal distribution with unknown mean μ and known variance σ^2. Describe a test for testing the null hypothesis $H_o : \mu = \mu_o$ against the alternative $H1 : \mu > \mu_o$. Further, obtain $100(1 - \alpha)\%$ lower confidence interval estimator for μ.

(b) Let y denote the median of a random sample of size $n = 2k + 1$ from $N(\mu, \rho^2)$. Show that the p.d.f of y is symmetrical about μ.

(c) Derive the test procedure for testing the equality of two population means based on two independent random samples drawn from two independent normal populations when the variances are known.

(d) If Y_1 and Y_2 are independent χ^2-variables, each with n degrees of freedom, show that

$$\frac{\sqrt{n}(Y_1 - Y_2)}{2\sqrt{Y_1 Y_2}}$$

has the t-distribution with n degrees of freedom.

(e) If X_1 and X_2 are independently distributed random variables, each in the form R(0, 1), show that $U_1 = \sqrt{-2\log_e X_1} \cdot \cos 2\pi X_2$ and $U_2 = \sqrt{-2\log_e X_1} \cdot \sin 2\pi X_2$ are independently distributed N(0, 1) variables.

(f) Let r be the sample correlation coefficient for sampling from a bivariate normal distribution. Explain how this statistic can be used to test the hypothesis $H_0 : \rho = 0$.

3. Answer one question out of two questions.

$1 \times 10 = 10$

(a) Let a random sample of size n is taken from an exponential distribution with p.d.f. f(x) $= e^{-x}$; $x > 0$. Find the p.d.f of the sample range.

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(b) (i) Let $x_1, x_2, ..., x_n$ be a random sample of size n drawn from a finite population using with replacement method, where

$$x_i = \begin{cases} 1, & \text{if the individual appeared in the } i^{th} \text{ drawing is having a particular character A;} \\ 0, & \text{o.w.} \end{cases}$$

let p be the population proportion of individuals having the character A. Describe an exact test procedure for testing $H_0 : p = p_0$ against $H_1 : p < p_0$.

(ii) How do you test for the ratio of two variances in case of a bivariate normal distribution.