UG/2nd Sem/Stat/H/19

2019

B.Sc. (Hons.)

2nd Semester Examination

STATISTICS

Paper—C4T

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer any *five* out of *eight* questions : 5×2=10

   (a) Obtain the relationship between the probability generating function and the moment generating function for a count random variable X.

   (b) Show that for the rectangular distribution

$$f(x) = \frac{1}{b} \ ; a \le x \le a + b$$

the m.g.f. is $\dfrac{e^{at}\left(e^{bt}-1\right)}{bt}$

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(c) If a random variable X has cumulative distribution function

$$F(x)=\begin{cases}0 & \text{if } x<0\\ \frac{1}{3} & \text{if } x=0\\ \frac{1+x}{3} & \text{if } 0<x<1\\ 1 & \text{if } x\geq 1\end{cases}$$

Calculate E(X).

(d) Obtain the median of a normal distribution.

(e) If the joint probability density function of (x, y) is given by $f(x,y)=\frac{1}{y}e^{-x/y}$, $x > 0$, $0 < y < 1$ then find E(Y).

(f) Let X denote the tangent of an angle θ (measured in radians) chosen at random form $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Find the distribution of X.

(g) Find the median of the distribution

$$f(x)=\frac{\alpha\beta x^{\alpha-1}}{\left(1+\beta x^{\alpha}\right)^2} \;;\; \beta > 0,\ \alpha > 1,\ x \geq 0.$$

(h) Let X be the number of tosses required to obtain a head in tossing an unbiased coin. Find p.g.f. of X.

Group-B

2. Answer any *four* out of *six* questions : 4×5=20

(a) Let X be a normal random variable with mean 0 and variance 1. Show that

$$P(x \geq c) \leq e^{-ct+\frac{t^2}{2}}$$

(b) The joint density function of X and Y is given by

$$f(x, y) = \begin{cases} 2e^{-x-2y}, & 0 < x < \infty,\ 0 < y < \infty \\ 0 & , \text{otherwise} \end{cases}$$

Compute (a) P (X < Y) and (b) P (X < a) 3+2

(c) Suppose (X, Y) follows BN (μ_1, μ_2, σ_1^2, σ_1^2, ρ). Obtain the conditional distribution of Y given X = x. Show that the regression of Y on X is linear. 4+1

(d) A random variable X has probability density function $f(x) = \alpha x e^{-\beta^2 x^2}$, $x > 0$, $\alpha > 0$, $\beta > 0$. If $E(x) = \frac{\sqrt{\pi}}{2}$, determine α and β. 5

(e) Let X and Y be two random variables with joint probability density function

$$f(x, y) = \begin{cases} e^{-y}\ ; & 0 \leq x \leq y < \infty \\ 0\ ; & \text{otherwise} \end{cases}$$ 5

Examine whether X an Y are independent.

(f) Show that for any $x > 0$,

$$\frac{1}{x}-\frac{1}{x^2} \le \frac{1-\Phi(x)}{\phi(x)} \le \frac{1}{x},$$

where ϕ and Φ have their usual meaning. 5

Group-C

3. Answer any *one* out of *two* questions : 1×10=10

(a) Derive the moment generating function of Bivariate Normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ and ρ 10

(b) (i) If E (Y/X) and var (Y/X) exist for almost all values of X, show that

Var (Y) = E {Var (Y/X)} + Var {E(Y/X)}

(ii) If $x_1, x_2, \ldots, x_n$ are iid lognormal (μ, σ^2), Show that

$$\prod_{i=1}^{n} x_i^{\frac{1}{n}} \text{ follows lognormal } \left(\mu, \frac{\sigma^2}{n}\right)$$ 4+6