UG/3rd Sem/STAT(H)/T/19

2019

B.Sc.

3rd Semester Examination

STATISTICS (Honours)

Paper - C 5-T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any *ten* questions : $2\times10=20$

(a) If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, find A^2-5A. 2

(b) Find the value of α such that the matrix $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ is orthogonal. 2

(c) State Weierstrass polynomial approximation theorem. 2

[Turn Over]

(d) Prove that $E = 1 + \Delta$, where symbols have their usual meaning. 2

(e) Write down the order of convergence of fixed point iteration and Newton Raphson method to find a real root of a non-linear equation $f(x) = 0$. 2

(f) Determine the value of k so that the set $S = \{(0, 1, k), (1, k, 1), (k, 1, 0)\}$ on $\mathbb{R}^3(\mathbb{R})$ is linearly independent. 2

(g) Give the geometrical significance of trapezoidal rule of integration. 2

(h) Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$. 2

(i) Show that

$W = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 5\}$ is not a subspace of $\mathbb{R}^3$. 2

(j) Examine whether in $\mathbb{R}^3$, the vector (1, 0, 7) is in the span of S = {(0, −1, 2), (1, 2, 3)} 2

(k) Let $T:\mathbb{R}^2 \to \mathbb{R}$, where T(1, 1) = 3 and T(0, 1) = –2. Find the linear transformation T(x, y). 2

(l) If A be a 2×2 matrix, then prove that the characteristic equation is $\lambda^2 - tr(A) + \det A = 0$ 2

(m) Write down the Lagrange interpolating polynomial for the following table

x	x_0	x_1
y	y_0	y_1

2

(n) Write two applications of linear algebra in statistics. 2

(o) The trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 4 and Simpsons rule gives the value 2, what is f(1) ? 2

2. Answer any *four* questions : 5×4=20

(a) Find Lagrangin polynomial for the function $\sin \pi x$, when $x = 0, \frac{1}{6}, \frac{1}{2}$. Also compute the value of $\sin\frac{\pi}{3}$ with estimate of error. 2+2+1=5

[Turn Over]

(b) If α, β, γ are the roots of

$x^2(px+q) = r(x+1)$, prove that

$$\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix} = 0$$ 5

(c) Using Lagrange interpolating polynomial, derive the following quadratic formula

$$\int_{-a}^{b} f(x)dx = \frac{a+b}{6ab}\Big[b(2a-b)f(-a) + (a+b)^2 f(0) + a(2b-a)f(b)\Big]$$ 5

(d) Show that $W = \{(x, y, z) \in \mathbb{R}^3 : 2x - y + 3z = 0 ;$ and $x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$. Find the basis of W. What its dimension ? 2+2+1=5

(e) Show that the matrix $\begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & 2 \\ 2 & -2 & 1 \end{pmatrix}$ is orthogonal and hence solve the system of equations $x + 2y + 2z = 2$, $-2x - y + 2z = 1$, $2x - 2y + z = 7$. 2+3=5

(f) Reduce the quadratic form $x^2 + 2y^2 + 3z^2 - 2xy + 4yz$ to the normal form. Hence find rank. 4+1=5

3. Answer any *two* questions : 10×2=20

(a) Given any real valued function f(x) and (n + 1) distinct points $x_0, x_1, ..., x_n$, prove that there exists unique polynomial of maximum degree n which interpolates f(x) at the point $x_0, x_1, ..., x_n$. Write down the expression of error in approximating f(x) by a polynomial. Hence show that, for linear interpolation, for the case of equally spaced tabular data, the error does not exceed $\frac{1}{8}$ of the second difference.

5+2+3=10

(b) Find the associated matrix A corresponding to the quadratic

$Q(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 3x_3^2 - 4x_2x_3 + 4x_3x_1 - 2x_1x_2$. Find the characteristic roots and

[*Turn Over*]

corresponding characteristic vectors of the matrix A. Hence find the nature of the quadratic.

2+3+3+2=10

(c) Describe fixed-point iteration method to find a real root of the non linear equation f(x) = 0. Give the geometrical interpretation. The equation $x^2 + ax + b = 0$ has two real roots α, β. Show that the iteration method $x_{k+1} = -\frac{1}{a}\left(x_k^2 + b\right)$ is convergent near $x = \alpha$ if $2|\alpha| < |\alpha + \beta|$

4+3+3=10

(d) Find A^{-1}, when adj A = $\begin{pmatrix} 1 & 3 & -4 \\ -2 & 2 & -2 \\ 1 & -3 & 4 \end{pmatrix}$. Find the row rank and cloumn rank of the matrix A.

5+5=10
