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UG/3rd Sem/STAT(H)/T/19

2019

B.Sc.

3rd Semester Examination

STATISTICS (Honours)

Paper - C 5-T

Full Marks: 60

Time: 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers

in their own words as far as practicable.

Group - A

1. Answer any ten questions:

2×10=20

(a) If
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
, find $A^2 - 5A$.

(b) Find the value of α such that the matrix

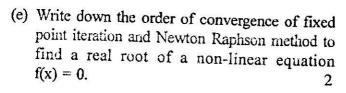
$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$
 is orthogonal.

(c) State Weierstrass polynomial approximation theorem.

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(d) Prove that $E = 1 + \Delta$, where symbols have their usual meaning.



(f) Determine the value of k so that the set $S = \{(0, 1, k), (1, k, 1), (k, 1, 0)\}$ on $IR^3(IR)$ is linearly independent.

(g) Give the geometrical significance of trapezoidal rule of integration.

(h) Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$.

(i) Show that

$$W = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 5\} \text{ is not a}$$

subspace of \mathbb{R}^3 .

(j) Examine whether in \mathbb{R}^3 , the vector (1, 0, 7) is in the span of $S = \{(0, -1, 2), (1, 2, 3)\}$

- (k) Let $T: \mathbb{R}^2 \to \mathbb{R}$, where T(1, 1) = 3 and T(0, 1) = -2. Find the linear transformation T(x, y).
 - (1) If A be a 2×2 matrix, then prove that the characteristic equation is $\lambda^2 tr(A) + det A = 0$
- (m) Write down the Lagrange interpolating polynomial for the following table

x	\mathbf{x}_0	\mathbf{x}_1
y	У0	y _l

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- (n) Write two applications of linear algebra in statistics.
- (o) The trapezoidal rule applied to $\int_0^2 f(x) dx$ gives the value 4 and Simpsons rule gives the value 2, what is f(1)?
- 2. Answer any four questions:

(a) Find Lagrangin polynomial for the function $\sin \pi x$, when $x = 0, \frac{1}{6}, \frac{1}{2}$. Also compute the

value of $\sin \frac{\pi}{3}$ with estimate of error. 2+2+1=5

[Turn Over]

 $5 \times 4 = 20$

(b) If α , β , γ are the roots of

$$x^{2}(px+q)=r(x+1)$$
, prove that

$$\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix} = 0$$

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(c) Using Lagrange interpolating polynomial, derive the following quadratic formula

$$\int_{-a}^{b} f(x) dx = \frac{a+b}{6ab} \left[b(2a-b) f(-a) + (a+b)^{2} \right].$$

$$f(0)+a(2b-a)f(b)$$
 5

(d) Show that $W = \{(x, y, z) \in \mathbb{R}^3 : 2x - y + 3z = 0 :$ and $x + y + z = 0\}$ is a subspace of \mathbb{R}^3 . Find the basis of W. What its dimension ? 2+2+1=5

(e) Show that the matrix
$$\begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & 2 \\ 2 & -2 & 1 \end{pmatrix}$$
 is

orthogonal and hence solve the system of equations x + 2y + 2z = 2, -2x - y + 2z = 1, 2x - 2y + z = 7. 2+3=5

- (f) Reduce the quadratic form $x^2 + 2y^2 + 3z^2 2xy + 4yz$ to the normal form. Hence find rank. 4+1=5
- 3. Answer any two questions:

10×2=20

(a) Given any real valued function f(x) and (n + 1) distinct points x₀, x₁, ..., x_n, prove that there exists unique polynomial of maximum degree n which interpolates f(x) at the point x₀, x₁, ..., x_n. Write down the expression of error in approximating f(x) by a polynomial. Hence show that, for linear interpolation, for the case of equally spaced tabular data, the error does not

exceed $\frac{1}{8}$ of the second difference.

5+2+3=10

(b) Find the associated matrix A corresponding to the quadratic

 $Q(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + 3x_3^2 - 4x_2x_3 + 4x_3x_1 - 2x_1x_2$. Find the characteristic roots and

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corresponding characteristic vectors of the matrix A. Hence find the nature of the quadratic.

(c) Describe fixed-point iteration method to find a real root of the non linear equation f(x) = 0. Give the geometrical interpretation. The equation $x^2 + ax + b = 0$ has two real roots α , β . Show that the iteration method $\chi_{k+1} = -\frac{1}{a} \left(\chi_k^2 + b \right)$ is convergent near $x = \alpha$ if $2 |\alpha| < |\alpha + \beta|$ 4 + 3 + 3 = 10

(d) Find A⁻¹, when adj A =
$$\begin{pmatrix} 1 & 3 & -4 \\ -2 & 2 & -2 \\ 1 & -3 & 4 \end{pmatrix}$$
. Find

the row rank and cloumn rank of the matrix A. 5+5=10