UG/5th Sem/STAT(H)/T/19

2019

B.Sc. (Honours)

5th Semester Examination

STATISTICS

Paper - DSE-2T

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Survival Analysis

Group - A

1. Answer any *five* out of eight questions : 5×2=10

(a) What do you mean by censoring ? 2

(b) Define mean residual time. 2

(c) Derive the hazard function of an exponential distribution. 2

(d) Why is competing risk analysis done ? 2

[Turn Over]

(e) Give two examples of random censoring. 2

(f) Derive the survival function of Weibull distribution. 2

(g) When is Type II censoring used ? 2

(h) What is cumulative incidence function ? 2

Group - B

2. Answer any *four* out of six questions : 4×5=20

(a) Describe the Kaplan-Meier method for estimation of survival function. 5

(b) Distinguish between Type I censoring and Type II censoring. 5

(c) Derive the hazard rate function of lognormal distribution. 5

(d) Derive the expression for mean residual life of an item aged t units of time. 5

(e) State any two indices for measurement of probability of death under competing risks. Derive their inter-relations. 2+3

(f) If f(t) denotes p.d.f. of lifetime distribution of an item and r(t) denotes its hazard rate at time t, $t \geq 0$ then show that $f(t) = r(t)\, e^{-\int_0^t r(t')dt'}$. 5

Group - C

3. Answer any *one* out of two questions : 1×10=10

(a) How can you estimate the mean survival time based on a Type I censored data from an exponential distribution ? Also derive the variance of the estimator. 6+4

(b) Describe the method of maximum likelihood estimation of the probability of death under competing events. Why can Kaplan-Meier method not be used for this estimation ? 6+4

[Turn Over]

Stochastic Processes and Queuing Theory

Full Marks : 40 Time : 2 Hours

1. Answer any *five* questions : 5×2=10

(a) What do you mean by a stationary or time-homogeneous stochastic process ? 2

(b) When a Markov chain is said to be irreducible? 2

(c) What is an absorbing state ? 2

(d) When is a state called persistent and transient ? 2

(e) What is a homogeneous Markov chain of the first order ? 2

(f) When is a counting process said to be a Poisson process ? 2

(g) Differentiate inter-arrival times and the waiting time until the *n*th event. 2

(h) What is a non-stationary Poisson process ? 2

2. Answer any *four* questions : 4×5=20

(a) Consider a Markov chain with two states, E_1 and E_2, and transition probability matrix

$$\begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$$

where $0 < p < 1$.

Determine the *n*-step transition probabilities $p_{ij}^{(n)}$. 5

(b) Let $\{X_i(t), t \geq 0\}$, for i = 1, 2, be two independent Poisson processes with rates λ_i. Show that if $X(t) = X_1(t) + X_2(t)$, then $\{X(t), t \geq 0\}$ is also a Poisson process with rate $\lambda_1 + \lambda_2$. 5

(c) For a Poisson process, show that if $s < t$, then

$$P\left[X(s)=k/X(t)=n\right]=\binom{n}{k}\left(\frac{s}{t}\right)^{k}\left(1-\frac{s}{t}\right)^{n-k},$$

$k = 0, 1, 2, ..., n.$ 5

(d) For a birth-and-death process show that if initially the population has i members, then the mean size of the population at time t is i exp $\left[(n-\mu)t\right]$, where each member gives birth at an exponential rate λ and each dies independently of the other members at an exponential rate μ. 5

(e) Suppose illegal immigration to India occurs at a Poisson rate of 8 per week (say). The probability that an illegal immigrant is Bengali speaking is $\frac{1}{6}$. Find the probability that no Bengali-speaking illegal immigrant will arrive in India during a period of 14 days. 5

(f) Describe a population growth model. 5

3. Answer any *one* question : 1×10=10

(a) Show that for a Poisson process $\{X(t), t \geq 0\}$ having rate $\lambda > 0$, the inter-arrival times T_n (n = 1, 2, ...) are iid exponential random variables with mean $\frac{1}{\lambda}$. Also show that the waiting time until the *n*th event has the gamma distribution with scale parameter λ and shape parameter *n*.

6+4=10

(b) Define the following processes :

(i) Pure birth process 3

(ii) A single-server queuing system (an M/M/I queue) 3

(iii) Markov Chain 2

(iv) Yule-Fury Process 2