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2019

B.Sc. (Honours)

5th Semester Examination

STATISTICS

Paper - C11T

(Statistical Inference - II)

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *five* out of eight. 5×2=10

   (a) State Weak law of large numbers. 2

   (b) Suppose $X_1, X_2, ..., X_n$ are a random sample from a negative binomial (r, p) distribution. Find the distribution of $\dfrac{\sqrt{n}\left(\bar{X} - r(1-p)/p\right)}{\sqrt{r(1-p)/p^2}}$ 2

   (c) State Neyman-Fisher's factorization theorem. 2

[Turn Over]

(d) What do you mean by a complete statistic ? 2

(e) Write down two properties of maximum likelihood estimators. 2

(f) Define level of significance and size of a test. 2

(g) Write two properties of likelihood ratio test. 2

(h) What is variance stabilizing transformation? Give an example. 2

2. Answer any *four* questions : 4×5=20

(a) A random sample $X_1, X_2, ..., X_n$ is drawn from a population with pdf

$$f(x/\theta) = \frac{1}{2}(1+\theta x), \quad -1 < x < 1, \quad -1 < \theta < x.$$

Find a consistent estimator of θ and check it is unbiased or not. 3+2

(b) Let $X_1, X_2, ..., X_n$ be a random sample from a $N(\mu, \sigma^2)$ population, σ^2 known.

An LRT of $H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1$ is a test that rejects H_0 if $|\bar{X} - \mu_0| \big/ \frac{\sigma}{\sqrt{n}} > C$. Find an expression, in terms of standard normal probabilities, for the power function of the test. 5

(c) Obtain the likelihood ratio tests, based on random sample $X_1, X_2, ..., X_n$ for the following hypothesis.

(i) $H_0 : \theta = \theta_0$ when $f(n) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \; 0 < x < \infty$ 2.5

(ii) $H_0 : \theta = \theta_0$ when $f(n) = \frac{1}{\theta}, \; 0 \le x \le \theta$. 2.5

(d) Suppose X is one observation from a population with beta $(\theta, 1)$ pdf. Find the most powerful level α test of $H_0 : \theta = 1$ vs $H_1 \theta = 2$. 5

(e) Find the maximum likelihood estimation of 1/p for the observation X from the discrete distribution with pmf.

$f(x) = (1-p)^{x-1} p$, x = 1, 2, ..., ∞ 5

(f) Let $X_1, X_2, \ldots, X_n$ be a random sample from the pdf

$$f(x/\mu, \sigma) = \frac{1}{\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)}, \quad \mu < x < \infty, \quad 0 < \sigma < \infty,$$

$-\infty < \mu < \infty$

Find a two dimensional sufficient statistic for (μ, σ). 5

3. Answer any *one* question : 1×10=10

(a) Let $X_1, X_2, \ldots, X_n$ be iid $N\left(\theta, \sigma^2\right)$, with both θ and σ^2 unknown. Find the MLE of θ and σ^2. 5+5

(b) Let $t_1, t_2, \ldots, t_k$ be mutually independent and unbiased estimators of μ with variance $V_1, V_2, \ldots, V_k$ respectively. Consider a linear function $T = a + \sum_{i=1}^{k} b_i t_i$.

Choose the constants $a_1 b_1, b_2, \ldots, b_k$ in such a way that T is unbiased and has the smallest variance among all unbiased linear estimators. 10
