2018

CBCS

1st Semester

PHYSICS

PAPER—C1T

(Honours)

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Mathematical Physics

Group—A

Answer any *five* questions : 5×2

1. Solve : $\frac{dz}{dx} - xz = -x$.

2. Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C \rho^2 d\phi$ in case of polar coordinates (ρ, ϕ).

3. Prove that $\iint_S \hat{n}\, ds = \vec{0}$ for any closed surface S.

4. Two solutions of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ are e^x *and* xe^x. Is the general solution $y = c_1e^x + c_2xe^x$? Check by the Wronskian.

5. Rolling a dice three times evaluate the probability of having at least one six.

6. Determine the Jacobian for spherical polar co-ordinates.

7. Prove using the property of Dirac delta function $\delta(x-a)=\delta(a-x)$

8. From a deck of 52 cards, two cards are drawn in succession. Find the chance that the first is a king and the second a queen if the first card in not replaced.

Group—B

Answer any *four* questions : 4×5

9. Give a rough plot of the force function $F(x)=x^2-4x+3$. What are the equilibrium points? Are they stable or unstable and why? 1+1+3

10. Initially at rest, a body of man m is falling under action of gravity and air resistance (R) proportional to the square of the velocity. (*i.e.* $R \propto v^2$)

Prove that $\frac{2kx}{m} = \log \frac{a^2}{a^2 - v^2}$

where v is the velocity achieved by the body after falling a distance x, g in the acceleration due to gravity and mg $= ka^2$. What is the maximum velocity the body can attain ? 5

11. (a) If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$ 3

(b) Evaluate : $\int_0^3 x^2 \delta(x+1) dx$ 2

12. Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$; $y(1) = 0$, $\frac{dy}{dx} = 1$. 5

13. (a) The Gaussian probability distribution is given by

$P_G(x:\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$; $-\infty < x < \infty$ in usual notation. Show that it has two points of inflexion at $x=\mu\pm\sigma$. 3

(b) Define the Dirac delta function $\delta(x)$. Mention the properties of $\delta(x)$. (1+1)

14. (a) What is Baye's theorem in the theory of Probability ? 2

(b) Consider three bags. The first one contains 3 white, 1 red, 2 green balls, the second one contains 2 white, 3 red, 1 green balls and the 3rd one contains 1 white, 2 red and 3 green balls. Two balls drawn out of a randomly chosen bag, are found to be one white and one red. Find the probability that the balls so drawn came from the second bag. 3

Group—C

Answer any *one* question : 1×10

15. (a) Find a unit vector perpendicular to the surface $(x-2)^2+5y^2+2z^2=8$ at the point $(1,1,1)$. 3

(b) If $\vec{A}=6z\hat{i}+(2x+y)\hat{j}-x\hat{k}$, calculate $\iint_S \vec{A}\cdot\hat{n}\ ds$ over the entire surface S of the region bounded by the cylinder $x^2+z^2=9$, $x=0$, $y=0$ and $y=8$. 5

(c) An LIC agent sells on the average 3 insurance policies per week. Use Poisson probability distribution to calculate the probability that in a given week he will sell some policies. 2

16. (a) Using Lagrange's method of undetermined multiplier, find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ 4

(b) Solve the differential equation :

$$y''-2y'+y=e^x \log x$$ 4

(c) Find $\frac{du}{dt}$ if $u=\sin\left(\frac{x}{y}\right)$, $x = e^t$ and $y = t^2$. 2

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