# 2018

## CBCS

## 3rd Semester

## PHYSICS

### PAPER—C5T

### (Honours)

*Full Marks* : 40

*Time* : 2 Hours

*The figures in the right-hand margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

## *Mathematical Physics II*

**1.** Answer any *five* quesitons : 5×2

(a) Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$ 2

(b) Prove that $erf(-x) = -erf(x)$ 2

(c) Write down the Dirichlet's conditon 2

(d) $f(x) = -\left(\frac{x}{l}\right) + l$ in range $0 \leq x \leq l$ find $a_n$. 2

(e) Find regular singular points of the differential equation $2x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + (x^2 - 4)y = 0$ 2

(f) A Lagrangian is in the form $L = \left(\frac{1}{2}\alpha\dot{q}^2 - \frac{1}{2}\beta q^2\right)$ where α, β are constant. Find Hamiltonian of the system. 2

(g) State the type of following PDES

(i) $\frac{\partial^2 u}{\partial x^2} - K^2 \frac{\partial^2 u}{\partial y^2} = 0$

(ii) $\frac{\partial^2 u}{\partial x^2} - K \frac{\partial u}{\partial x} = 0$ 2

(h) Explain cyclic co-ordinates. 2

**2.** Answer any *four* questions : 4×5

(a) Prove $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(b) Obtain the complex form of the Fourier series of the function : $f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$

(c) Prove $xJ'_n = nJ_n - xJ_{n+1}$, symbols are in usual meaning.

(d) $L = \frac{1}{2}e^{at}\left(\dot{x}^2 - p^2x^2\right)$, where p is a constant. Using Lagranges' equation prove that $\ddot{x} + a\dot{x} + p^2x = 0$.

(e) Using Hamilton's canonical equations, derive the equation of motion of a particle moving in a force field in which the potential is given by $V = -\frac{k}{r}$, where K is prositive.

(f) Solve the Laplace's equation in the three dimensional cylindrical form.

**3.** Answer any *one* question : 1×10

(a) (i) Solve $x(x-1)y'' + (3x-1)y' + y = 0$ by Frobenius method.

(ii) Prove $nP_n = xP'_n - P'_{n-1}$ 6+4

(b) (i) A pendulum with a bob of mass m and length $l$, is suspended from a massless spring of constant $k$. The spring has only verticle motion. Find the Lagrangian and Lagranges' equation.

(ii) Find the solution of the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{in the region} \quad 0 \le x \le a,$$

$0 \le y \le \infty$, satisfy the conditions

$u(0, y) = 0$, $u(a, y) = 0$

$u(x, 0) = A\left(a - \frac{x}{a}\right)$ and $u(x, \infty) = 0$ 5 + 5

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