UG/4th Sem/PHY/19

2019

B.Sc. (Hons)

4th Semester Examination

PHYSICS

Paper - C8T

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *five* questions : 5×2=10

(a) What is the Fourier transform of $\delta(x-a)$, where a is a constant ? 2

(b) Evaluate $\oint_c \frac{dz}{z}$ where c denotes a simple closed curve that encloses the origin. 2

(c) If F(k) be the Fourier transform of f(x), then show that

$$F[f(x)\cos ax] = \frac{1}{2}[F(k+a)+F(k-a)]$$

[Turn Over]

(d) If λ be an eigen value of matrix A(non-zero matrix), show that λ^{-1} is an eigen value of A^{-1}.

(e) Prove that $F_s\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}}\left(\frac{s}{a^2+s^2}\right)$

(f) Expand $f(z) = \cosh z$ about πi.

(g) What is meant by similarity transformation ?

(h) Find the poles and residues of the function
$$f(z) = \frac{3-2z}{(z-2)(z-1)^2}$$

2. Answer any *four* questions : 4×5=20

(a) Consider
$$f(x) = \frac{1}{2L} \quad \text{for } |x| < L$$
$$= 0 \quad \text{for } |x| > L$$

Calculate the Fourier transform of f(x) and state its limiting value as $L \to 0$. 4+1

(b) Show that $|\sin(z)| \geq |\sin(x)|$, where $z = x + iy$.

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(c) If $\lambda_1, \lambda_2, ...\lambda_n$ are the eigen values of a matrix A, find the eigen values of the matrix $(A-\lambda I)^2$. Here I is the unit matrix. 5

(d) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$ by using contour integration.

(e) Find a matrix P which diagonalizes the matrix $A=\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, verify $P^{-1}AP = D$ where D is the diagonal matrix.

(f) Obtain the Cauchy-Riemann equations in connection with analyticity of a function of complex variables.

3. Answer any *one* question : 10×1=10

(a) (i) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ using Fourier transform under the condition

$u = 0$ at $x = 0$

$$u = \begin{cases} 1, & 0 < x < 1 \\ 0 & x \geq 1 \end{cases} \quad \text{when } t = 0$$

and u is bounded 6

[Turn Over]

(ii) Show that —

$$\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta} = \frac{\pi}{2}$$ 4

(b) (i) Find the characteristic equation of the symmetric matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Apply Cayley-Hamilton theorem to obtain A^{-1}. 5

(ii) State and prove convolution theorem of Fourier transform. 5
