UG/3rd Sem/PHS(H)/T/19

2019

B.Sc.

3rd Semester Examination

PHYSICS (Honours)

Paper - C 5-T

Full Marks : 40 Time : 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer *five* questions from Group - A, *four* from Group - B and *one* from Group - C.

Group - A

Answer any *five* questions of the following : 2×5=10

1. State the type (parabolic, elliptic or hyperbolic) of the following partial differential equation.

(i) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

[*Turn Over*]

(ii) $9\frac{\partial^2 u}{\partial x^2}+6\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}=3x+4y+1$ 2

2. Show that complex Fourier coefficient of odd function is purely imaginary. 2

3. What is the nature of singularity of the following differential equation ?

$$y''-\frac{6}{x^2}y=0$$ 2

4. A Lagrangian $L(q,q,t)=\frac{1}{2}mq^2-\frac{1}{2}k(q-vt)^2$ 2

Find the generalised momentum and Hamiltonian of the system.

5. Prove that $erf(x)+erf_c(x)=1$ 2

6. Fourier expansion of $f(x)$ in the interval $0<x<l$

is : $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{l}$ show that

$$\int_0^l\left[f(x)^2\right]dx=\frac{1}{4}a_0^2+\frac{1}{2}\sum_{n=1}^{\infty}a_n^2$$ 2

7. If L is the Lagrangian of a system, then show that $L_1 = L \pm \frac{dF}{dt'}$ where F is a function of the generalized coordinates, momenta and time; will also satisfy Lagranege's equations. 2

8. Potential energy of a particle are given by :

$$V = \frac{A}{\sqrt{\left(x^2 + y^2 + z^2\right)}} - Bz^2 \ln\left(x^2 + y^2\right).$$

Find its generalised momenta p_x and p_z. 2

Group - B

Answer any *four* questions of the following : 4×5=20

9. Prove that $\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+n)^{m+1}}$, when n is a positive interfer and $m > -1$. 5

10. A differential equation is given by :

$$\left(1-x^2\right) y'' - 2xy' + ny = 0.$$ 2

[Turn Over]

(a) Find the singular points.

(b) Check whether the singular points are essential or non-essential. Comment whether series solution of this equation is possible or not.

2+1

11. Prove $\int_0^{\infty} e^{-(x+a)^2} dx = \frac{\sqrt{\pi}}{2}\left[1-\text{erf}(a)\right]$ 5

12. Write the integral $\int_0^1 \frac{x^3}{\sqrt{1-x^2}} dx$ in the form of Beta function and hence evaluate it. 5

13. Calculate the Legendre transform of (i) $F(x) = x^2$ (ii) $F(x) = \ln x$. State the geomtrically meaning of Legendre transform. 5

14. Derive Euler's equation of motion for couple oscillators. 5

Group - C

Answer any *one* question of the following :

10×1=10

15. (a) Solve the following boundary value problem by the method of separation of vaiaboles "

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}; \text{ Given } u(0, y) = 8e^{-3y}.$$ 4

(b) (i) Obtain the expression of kinetic energy of a particle in terms of generalized coordinates. 2

(ii) Show that in the absence of rheonomic constraints the Hamiltonian of a system is equal to the sum of kinetic and potential energies of the system. 2

(iii) Show that if the Lagrangian of a system does not depend on time explicitly then the Hamiltonian of this system remains conserved. 2

16. (a) Define error function $erf(x)$. Find $erf(0)$ and $erf(\infty)$. Draw the graph ofo error function. 1+2+1

(b) Find the solution of the following differential equation by the method of Frobenius :

$y'' - 2xy' + 2ny = 0$; where n is the non-negative integer. 6