 UG/5th Sem/Math(H)/T/19

2019

B.Sc. (Honours)

5th Semester Examination

MATHEMATICS

Paper - C11T

(Partial Differential Equations and Applications)

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Unit - I

(1st Order PDE)

1. Answer any *four* questions : 2×4

(a) Define Quasi-Linear PDE of first order and give an example.

(b) Form a PDE by eliminating the arbitrary function φ from $Z = e^{ny}\phi(x-y)$.

[*Turn Over*]

(c) Find the partial differential equation of all spheres of constant radius and having centre on the xy-plane.

(d) Find the characteristic curve of the PDE :

$$yz\frac{\partial z}{\partial x}+xz\frac{\partial z}{\partial y}=xy.$$

(e) Reduce the equation $u_x - xu_y = 0$ in canonical form.

(f) Let $z(x, y)$ be the solution of

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=4z$ satisfying the condition $z(x, y)=1$ on the circle $x^2+y^2=1$. Then find the value of $z(2, 2)$.

2. Answer any *one* question : 5×1

(a) Using the method of separation of variables solve

$$4\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=3z \text{ where } z(0, y)=3e^{-y}-e^{-5y}.$$

(b) Find the integral surface of the linear PDE :

$$x\left(y^2+z\right)\frac{\partial z}{\partial x}-y\left(x^2+z\right)\frac{\partial z}{\partial y}=\left(x^2-y^2\right)z$$

which contains the straight line $x+y=0,\ z=1$.

Unit - II

(2nd Order PDE)

3. Answer any *one* question : 2×1

(a) Determine the region in which the given equation is hyperbolic, parabolic or elliptic

$u_{xx}+xyu_{xyy}=0$.

(b) Find the characteristics of the PDE

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}+\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}-e^x(2y-3)+e^y=0.$$

4. Answer any *one* question : 10×1

(a) (i) Reduce the equation

$3u_{xx}+10u_{xy}+3u_{yy}=0$ to its canonical form and hence solve it.

[Turn Over]

(ii) Derive the one dimensional wave equation.

6+4

(b) (i) Use the polar co-rodinates r and $\theta(x = r\cos\theta, y = r\sin\theta)$ to transform the Laplace equation $u_{xx} + u_{yy} = 0$ into the polar form

$$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$ 5

(ii) Reduce the Tricomi equation $u_{xx} + xu_{yy} = 0$ to the canonical form when $x > 0$. 5

Unit - III

(Applications of PDE)

5. Answer any *one* question : 5×1

(a) Find a solution of the following non-homogeneous boundary value problem

$$u_{tt} = c^2 u_{xx} + F(x),\ 0 < x < l,\ t > 0$$

$$u(x,\ 0) = f(x),\ \ 0 \le x \le l$$

$u_t(x, 0) = g(x), \quad 0 \le x \le l$

$u(0, t) = A, \ u(l, t) = B, \ t > 0.$ 5

(b) Find the solution of Laplace's equation $\nabla^2 \psi = 0$ in the semifinite region bounded by $x \ge 0$, $0 \le y \le 1$ subject to the boundary conditions

$\left.\frac{\partial \psi}{\partial x}\right)_{x=0} = 0, \ \left.\frac{\partial \psi}{\partial y}\right)_{y=0} = 0$ and

$\psi(x, 1) = f(x).$ 5

6. Answer any *one* question : 10×1

(a) Obtain the D'Alembert's solution of the Cauchy problem

$u_{tt} - c^2 u_{xx} = 0, \qquad x \in \mathbb{R}, \ t > 0$

$u(x, 0) = f(x), \qquad x \in \mathbb{R}$

$u_t(x, 0) = g(x), \qquad x \in \mathbb{R}$

Define the term 'domain of dependence', 'range of influence'. Hence find the solution when

[Turn Over]

$$f(x) = |\sin x|, \quad x > 0$$
$$= 0 \qquad , \quad x < 0$$
$$g(x) = 0 \qquad , \quad x \in \mathbb{R}$$

(b) Find the temperature distribution of a rod for the following initial boundary value problem

$$u_t = Ku_{xx}, \quad 0 < x < l, \quad t > 0$$

$$u(0, t) = 0, \quad t \geq 0$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq l$$

$$u(l, t) = 0, \quad t \geq 0$$

Hence find the solution when $f(x) = x(l - x)$, $0 \leq x \leq l$.

Unit - IV

(Particle Dynamics)

7. Answer any *five* questions : 2×5

(a) Derive the relation $w = \frac{v \sin \varphi}{r} = \frac{vp}{r}$.

(b) Prove that the acceleration of a particle moving in a plane curve with uniform speed is $\rho\dot{\psi}^2$.

(c) Write the Kepler's Law of planetary motion.

(d) For a particle moving in a central orbit under the inverse square law $\left(\frac{\mu}{r^2}\right)$, prove that the velocity (v) at any distance r is given by

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right).$$

(e) Write the significance of $h = r^2\dot{\theta}$.

(f) Write the differential equation of the central orbit in Pedal form.

(g) A particle describes the parabola $p^2 = ar$ under a force. which is always directed towards its focus. Find the law of force.

(h) A point moves along the arc of a cycloid in such a manner that the tangent as it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

[Turn Over]

8. Answer any *two* questions : 5×2

(a) A particle describes an ellipse under a force which is always directed towards the centre of the ellipse. Find the law of force.

(b) A machine gun of mass M_o stands on a horizontal plane and contains a shot of total mass m which is fired horizontally at a uniform rate with constant velocity u relative to the gun.

(c) A particle is projected along the inner surface of a rough sphere and is acted on by no force. Show that it will return to the point of projection at the end of time $a/\mu V\left(e^{2\pi\mu}-1\right)$ where a is the radius of the sphere, V is the velocity of projection and μ is the coefficient of friction.
