2019

B.Sc.

# 3rd Semister Examination

# MATHEMATICS (Honours)

Paper - C 7-T

Full Marks: 40

Time: 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practiable.

Illustrate the answers wherever necessary.

#### Unit - I

1. Answer any two questions:

 $2 \times 2 = 4$ 

- (a) Given  $f(x, y, z) = \frac{5xy^2}{z^2}$ , find the relative maximum error in the evaluation of f(x, y, z) at x = y = z = 1, if x, y, z have absolute error  $\Delta x = \Delta y = \Delta z = 0.1$ .
- (b) Define the terms:
  - (i) Computational error, and
  - (ii) Relative percentage error

[Turn Over]

(c) Three approximate values of the number  $\frac{1}{3}$  are given as 0.30, 0.33 and 0.34. Which of these three is the best approximation?

## Unit - II

2. Answer any one question:

 $2\times1=2$ 

- (a) Discuss the condition of convergence of Newton-Raphson method.
- (b) Prove that the order of convergence of iteration method is linear.
- 3. Answer any one question:

5×1≔5

- (a) Explain the method of Iteration for computing a real root of an equation f(x) = 0. Let the iteration function φ(x) maps the interval [a, b] into itself and is differentiable there. Further there exists a non negative constant k < 1 such that ∀x in [a, b], |φ'(x)| ≤ k then prove that φ(x) has exactly one fixed point α on [a, b] and the sequence {x<sub>n</sub>} converges to α.
- (b) (i) Show that the square root of N = AB is given by  $\sqrt{N} \simeq \frac{S}{4} + \frac{N}{S}$  where S = A + B.

(ii) Derive the expression for Secant method to find the root of an equation. 2+3

## Unit - III

4. Answer any one question:

 $2 \times 1 = 2$ 

- (a) What is called pivoting? Why pivoting is necessary to solve a system of equations using Gaussian ellimination method?
- (b) State the difference between direct and iterative methods.
- 5. Answer any one question:

 $5 \times 1 = 5$ 

- (a) Describe Gauss Jacobi method for solution of a system of linear equation. State the sufficient condition for convergene of this method.
- (b) Solve the following equations by Gauss-Jordan elimination method:

$$x_1 + x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + x_3 = 6$$

$$x_1 - x_2 - x_3 = -3$$

### Unit - IV

6. Answer any one question:

 $10 \times 1 = 10$ 

- (a) (i) Given that f(0)=2, f(1)=4, f(2)=6, f(3)=10 and 3rd difference being constant. Find f(5).
  - (ii) Prove that a divided difference is symmetric function of its arguments. If  $f(x) = x^2$  then prove that  $f[x_0 \ x_1 \ x_2]$  is constant and all higher order difference are zero.
  - (iii) In a country school going children of a certain age group is given for different years as follows:

| Year           | 1995 | 2000 | 2005 | 2010 | 2015 |
|----------------|------|------|------|------|------|
| No. of student | 304  | 329  | 357  | 387  | 421  |
| (in thousand)  |      |      |      |      |      |

Estimate the number in the year 2020.

(b) Establish Lagrange's interpolation formula. Show that the Lagrangian functions are invariant under a linear transformation. 7+3

#### Unit - V

7. Answer any one question:

 $2 \times 1 = 2$ 

- (a) Why does one need to use numerical method instead of analytical method or integration.
- (b) What is degree of precision. What is the degree of precision of Weddle's rule?
- 8. Answer any one question:

 $5 \times 1 = 5$ 

(a) Using power method, find the largest eigen value in magnitude and corresponding eigen vector of

the matrix 
$$A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 2 \\ 3 & 4 & 5 \end{pmatrix}$$
.

(b) Derive Trapezoidal rule from general quadrature formula and discuss its geometrical significance.

## Unit - VI

9. Answer any one question:

 $5 \times 1 = 5$ 

(a) Define single step and multistep methods. Use R-K method of order 2 to approximate y when

$$x = 0.1$$
, 0.2, 0.3 given that  $\frac{dy}{dx} = y - x$ ,  $y(0) = 2$ .

(b) Write down the working rule of modified Euler's method for solving first order differential equation with initial condition. Comments on accuracy of Euler's method in solving a differential equation.