2019

B.Sc.

1st Semester Examination

MATHEMATICS (Honours)

Paper - C 1-T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

Unit - I

1. Answer any *three* of the following questions : 3×2=6

(a) If $y = c^{ax}\cos^2 bx$, find $y_n\left(a, b > 0\right)$.

(b) Find the oblique asymptotes of the curve

$$y = \frac{3x}{2}\log\left(e - \frac{1}{3x}\right)$$

[Turn Over]

(c) If $y = x^{n-1} \log x$, then prove that $y_n = \frac{(n-1)!}{x}$.

(d) What is reciprocal spiral? Sketch it.

(e) The parabolic path is given by

$$y = x \tan\theta - \frac{x^2}{4h\cos^2\theta}$$

what will be the asymptote of parabolic paths ?

2. Answer any *one* questions : $1\times10=10$

(a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

(ii) Let $P_n = D^n\left(x^n \log x\right)$.

Prove that $P_n = nP_{n-1} + \underline{|n-1}$. Hence show that $P_n = n!\left(\log x + 1 + \frac{1}{2} + \ldots + \frac{1}{n}\right)$. 5

(b) (i) Prove that the envelope of circles whose centres lie on the rectangular hyperbola $xy = c^2$ and which pass through its centre is $\left(x^2+y^2\right)^2 = 16c^2xy$. 5

(ii) Find the point of inflexion on the curve $\left(\theta^2-1\right)r = a\theta^2$. 5

Unit - II

3. Answer any *two* questions : 2×2=4

(a) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \sin x \, dx, n > 2$. Prove that $2(n-1)I_n = 1+(n-2)I_{n-1}$.

(b) Find the length of the curve

$x = e^\theta \sin\theta$ and $y = e^\theta \cos\theta$

between $\theta = 0$ to $\theta = \frac{\pi}{2}$.

(c) Find the reduction formula for

$\int \cos^m x \sin(nx)\, dx$.

[Turn Over]

4. Answer any *two* questions : $2\times5=10$

(a) Prove that the volume of the solid obtained by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line is $\frac{1}{2}\pi a^3 \left\{ \frac{1}{\sqrt{2}} \log\left(\sqrt{2}+1\right) - \frac{1}{3} \right\}$.

(b) If $I_{m,n} = \int_0^1 x^m (1-x)^n \, dx$,

where m and n are positive integers, then prove that $(m+n+1) I_{m,n} = n I_{m,n-1}$ and deduce that

$$I_{m,n} = \frac{m!n!}{(m+n+1)!}.$$

(c) Evaluate the surface area of the solid generated by revolving the cycloid

$x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ about the line $y = 0$.

Unit - III

5. Answer any *three* questions : $3\times2=6$

(a) Find the centre and foci of the conic

$$x^2 - 2y^2 - 2x + 8y - 1 = 0$$

(b) Find the equation of the sphere of which the circle $xy+yz+zx=0, x+y+z=3$ is a great circle.

(c) Find the condition that the line

$\frac{1}{r}=A\cos\theta+B\sin\theta$ may touch the conic

$\frac{1}{r}=1-e\cos\theta.$

(d) For what angle must the axes be turned to remove the term xy from $7x^2+4xy+3y^2$.

(e) Find the equation of cone whose vertex is origin and the base curve is $x^2+y^2=4, z=2$.

6. Answer any *one* question : $1\times5=5$

(a) If r be the radius of the circle

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$,

$lx+my+nz=0$ then prove that

$\left(r^2+d\right)\left(l^2+m^2+n^2\right)=\left(mw-nv\right)^2+\left(nu-lw\right)^2$

$+\left(lv-mu\right)^2$ and find the centre.

[Turn Over]

(b) Show that the feet of the normals from the point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the intersection of the ellipsoid and cone

$$\frac{\alpha a^2\left(b^2-c^2\right)}{x}+\frac{\beta b^2\left(c^2-a^2\right)}{y}+\frac{\gamma c^2\left(a^2-b^2\right)}{z}=0$$

7. Answer any *one* question : $10\times1=10$

(a) (i) Show that the plane $3x - 2y - z = 0$

cuts the cones $21x^2-4y^2-5z^2=0$ and

$3yz-2zx+2xy=0$

in the same pair of perpendicular lines. 5

(ii) Find the equation of the cylinder, whose generators are parallel to the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and which passes through the conic $z=0, 3x^2+7y^2=12$. 5

(b) (i) Find the locus of the point of intersection of the perpendicular generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$. 4

(ii) Reduce the equation

$$x^2+3y^2+3z^2-2xy-2yz-2zx+1=0$$

to its canonical form and determine the type of quadratic represented by it. 6

Unit - IV

8. Answer any *two* questions : 2×2=4

(a) Find the integrating factor of the differential equation

$$\left(2xy+3x^2y+6y^3\right)dx+\left(x^2+6y^2\right)dy=0$$

(b) Show that the general solution of the equation $\frac{dy}{dx}+Py=Q$ can be written in the form $y=k(u-v)+v$, where k is a constant and u and v are its two particular solutions.

(c) Solve : $\frac{dy}{dx}+y\cos x=xy^n$.

[Turn Over]

9. Answer any *one* question : 1×5=5

(a) The population of a country increases at the rate of proportional to the number of inhabitants. If the population doubles in 30 years, in how many years will it triple?

(b) Solve : $\left(px^2+y^2\right)\left(px+y\right)=\left(p+1\right)^2$

$\left[u=xy, v=x+y\right]$