2019

B.Sc.

2nd Semester Examination

MATHEMATICS (Honours)

Paper - GE2T

(Algebra)

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Unit - I

(Classical Algebra)

[Marks - 22]

1. Answer any *one* question : 1×2

(a) Apply Descartes' rule of signs to find the number of imaginary roots of

$$3x^4 + 4x^2 - 3x - 12 = 0.$$

[Turn Over]

(b) Z is a variable complex number such that $|Z| = 2$. Show that the point $Z + \frac{1}{Z}$ lies on an ellipse.

2. Answer any *two* questions : 2×5

(a) Show that the solutions of the equation $(1+x)^n - (1-x)^n = 0$ are $x = i \tan \frac{r\pi}{n}$,

where $r = 0, 1, 2, \ldots, n-1$, if n be odd.

$$= 0, 1, 2, \ldots, \frac{n}{2} - 1, \frac{n}{2} + 1, \ldots, n-1$$

if n be even.

(b) Prove that $\sum_{i=1}^{8} a_i \geq 8 \left(\prod_{i=1}^{8} a_i \right)^{\frac{1}{8}}$, where

$a_i \geq 0 \ \forall i = 1, 2, \ldots, 8$.

(c) Solve the equation $2x^4 + x^3 + 2x^2 + 3x + 18 = 0$, given that the product of two of the roots is equal to the product of the other two. 5

3. Answer any *one* question : 1×10

(a) (i) Solve the following equation by Cardan's method,

$$x^3 - 6x - 9 = 0.$$

(ii) If a, b, c be the sides of the triangle, then show that

$$\frac{1}{2} < \frac{bc + ca + ab}{a^2 + b^2 + c^2} \leq 1$$ 5+5

(b) (i) Find the relation among p, q, r, s so that the product of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ is unity.

(ii) Find the products of all the values of

$$(1+i)^{\frac{4}{5}}.$$

(iii) Prove that the minimum value of $x^2 + y^2 + z^2$ is $\left(\frac{C}{7}\right)^2$ where x, y, z are positive real numbers subject to the condition $2x + 3y + 6z = C$, C being a constant. 4+3+3

[Turn Over]

Unit - II

(Sets and Integers)

[Marks - 15]

4. Answer any *five* questions : 5×2

(a) Prove that $10^{n+1}+10^{n}+1$ is divisible by 3 $\forall\, n \in N$.

(b) State the second principle of mathematical induction.

(c) If A, B and C be three subsets of a universal set S prove that

$$A-(B \cup C)=(A-B) \cap (A-C).$$

(d) Define equivalance relation.

(e) Give an example to establish that the union of two transitive relations may not be a transitive relation.

(f) Let $A=\{x \in \mathbb{Z}: 0 \leq x \leq 10\}$ and

$B=\{x \in \mathbb{Z}: 5 \leq x \leq 15\}$ be two sets. Prove that the cardinality of two sets A and B are equal.

(g) Find $f \circ g$, if $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x)=|x|+x$, $x \in \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x)=|x|-x$, $x \in \mathbb{R}$.

(h) If a is prime to b, prove that a^2 is prime to b.

5. Answer any *one* question : 1×5

(a) Show that $\gcd(a, a+2)=1$ or 2 for every integer. 5

(b) (i) State the fundamental theorem on arithmetic.

(ii) Prove that the cube of any integer is of the form $9K$ or $9K\pm1$. 1+4

Unit - III

(System of Linear Equations)

[Marks - 9]

6. Answer any *two* questions : 2×2

(a) For what value of a the following system of equations have no solution

$$ax+2y=3$$
$$2x+ay=5-a$$

(b) Is any solution of $AX=B$ (where $B\neq\underline{0}$) linearly dependent ? Justify ? 1+1

[*Turn Over*]

(c) Find the solution of the system of equations in rational numbers.

$$x+4y+z=0$$
$$4x+y-z=0$$

7. Answer any *one* question : 1×5

(a) Investigate for what values of λ and μ the following equations

$$2x+3y+4z=10$$
$$y+2z=4$$
$$x+2y+\lambda z=\mu$$

have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions. 2+2+1

(b) (i) Find a row echelon matrix which is row equivalent to

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$$

(ii) For what value of K the planes $x-4y+5z=K$, $x-y+2z=3$ and $2x+y+z=0$ intersect in a line ? 3+2

Unit - IV

(Linear Transformation and Eigen Value)

[Marks - 14]

8. Answer any *two* questions : 2×2

(a) Find the rank of the matrix

$$\begin{bmatrix} 4 & 6 & 8 \\ 5 & 5 & 6 \\ 2 & 1 & 1 \end{bmatrix}$$

(b) If λ be an eigen value of an $n \times n$ matrix A, then show that $\frac{1}{\lambda}(i\lambda \neq 0)$ is an eigen value of A^{-1}.

(c) Let $T : R^3 \to R^3$ defined by

$$T(x, y, z) = (yz, zx, xy), \ (x, y, z) \in R^3$$

Examine whethr T is linear or not.

9. Answer any *one* question : 1×10

(a) (i) Verify Cayley-Hamilton theorem for the matrix A. Express A^{-1} as a polynomial in A and then compute A^{-1}.

[Turn Over]

$$A=\begin{pmatrix}1 & 0 & 0\\ 1 & 2 & 1\\ 2 & 3 & 2\end{pmatrix}$$ 4+1+1

(ii) Find the eigen values of the matrix $\begin{pmatrix}1 & 3\\ 4 & 5\end{pmatrix}$ and find the eigen vectors for one eigen value of A. 4

(b) (i) Let $V=\{(x, y, z): x, y, z \in \mathbb{R}\}$, where $\mathbb{R}$ is a field of real numbers.

Show that

$W=\{(x, y, z) \in \mathbb{R}^3 : x+y+z=0\}$ is a subspace of V over R. Find the dimension of W. 5

(ii) Determine the linear mapping $T: R^3 - R^2$ which maps the basis vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) of $\mathbb{R}^3$ to the vectors (1, 1), (2, 3), (3, 2) respectively. 5
