2017

MATHEMATICS

[Generic Elective]

(CBCS)

[First Semester]

PAPER – GE1T

Full Marks : 60

Time : 3 hours

The figures in the right-hand margin indicate marks

UNIT – I

(*Calculus* - I)

1. Answer any *three* questions : 2×3

(*a*) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$. 2

(*b*) Find the envelope of the straight line $y = mx + \frac{a}{m}$, m being the variable parameter $(m \neq 0)$. 2

(c) Find the asymptotes of the curve $y = xe^{\frac{1}{xr}}$. 2

(d) Find the point(s) of inflexion on the curve $x = (y-1)(y-2)(y-3)$. 2

(e) State Leibnitz's rule for successive differentiation. 2

2. Answer any *one* question : 10×1

(a) (i) If $y = \sin(m\sin^{-1}x)$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)y_{n+1}x - (n^2 - m^2)y_n = 0$$

and hence prove that $y_n(0) = 0$, for even n.

(ii) Find the value of p and q such that

$$\lim_{x \to 0} \frac{x(1 - p\cos x) + q\sin x}{x^3} = \frac{1}{3}$$ 10

(b) (i) Trace the curve $r^2 = a^2\cos 2\theta$. 5

(ii) Find the envelope of circles whose centres lie on the rectangular hyperbola

$xy = c^2$ and which passes through its centre. 5

UNIT – II

(*Calculus* - II)

3. Answer any *two* questions : 2×2

(*a*) Show that the area of the circle $r = 2a\sin\theta$ is πa^2. 2

(*b*) If $I_n = \int_0^{\pi/2} x^n \sin x\, dx$, n being positive integer > 1, then show that

$$I_n + n(n-1)\, I_{n-2} = n \cdot \left(\frac{\pi}{2}\right)^{n-1}$$ 1 + 1

(*c*) Find the length of the circumference of a circle of radius a. 2

4. Answer any *two* questions : 5×2

(*a*) Find the volume of the solid generated by revolving the cardioid $r = a(1 - \cos\theta)$ about the initial line.

(*b*) If $I_n = \int_0^{\pi/2} \cos^{n-1} x \sin nx \, dx$, show that

$$2(n-1) I_n = 1 + (n-2) I_{n-1}$$

(*c*) Show that the area bounded by the parabolas

$$y^2 = 4ax \text{ and } x^2 = 4ay \text{ is } \frac{16}{3} a^2 .$$

UNIT – III

(*Geometry*)

5. Answer any *three* questions : 3×2

(*a*) Find the equation of the right circular cylinder whose axis is z-axis and radius equals to 1.

(*b*) Find the values of c for which the plane $x + y + z = c$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$$

(*c*) Find the polar equation of the straight line passing through the points $(1, \frac{\pi}{2})$ and $(2, \pi)$.

(*d*) Find the nature of the quadric surface given by the equation

$$2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z = 5$$

(*e*) Under what condition the surface $yz + zx + xy = a^2$ may produce a parabola as a plane section by the plane $lx + my + nz = p$?

6. Answer any *one* question : 5×1

(*a*) If a sphere touches the planes $2x + 3y - 6z + 14 = 0$ and $2x + 3y - 6z + 42 = 0$ and if its centre lies on the straight line $2x + z = 0$, $y = 0$, find the equation of the sphere.

(*b*) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes at A, B, C. Find the equation of the cone generated by the straight lines drawn from the centre O to meet the circle ABC.

7. Answer any *one* question : 10 × 1

(*a*) (*i*) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$, if

$$(l\cos\alpha-ep)^2=p^2-l^2\sin^2\alpha.$$

(*ii*) Find the equations of the generating lines of the hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$, which passes through the point (2, 3, –4).

(*b*) (*i*) A sphere of constant radius r passes through the origin and cuts the axes at A, B, C. Prove that the locus of the foot of the perpendicular from origin to the plane ABC is given be

$$(x^2+y^2+z^2)^2(x^{-2}+y^{-2}+z^{-2})=4r^2.$$

(ii) Find the equation of the cylinder whose generator are parallel to the straight line $\frac{x}{-1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=9, z=1$.

UNIT – IV

(*Differential Equation*)

8. Answer any *two* questions : 2×2

(*a*) By which condition the ODE
$M(x, y)\, dx + N(x, y)\, dy = 0$ will be exact ?
Is this condition necessary ?

(*b*) Determine the integrating factor of

$$(x^4 y^2 - y)\, dx + (x^2 y^4 - x)\, dy = 0$$

(*c*) The bacteria in a certain culture increase according to $dN/dt = 0{\cdot}25$ N. If originally $N = 200$, find N when $t = 8$.

9. Answer any *one* question : 5×1

(*a*) Solve

$$(x^2y^2 + xy + 1)\, ydx - (x^2y^2 - xy + 1)\, xdy = 0$$

(*b*) Find the singular solution of the differential equation

$$y = px + \sqrt{a^2 p^2 + b^2}, \ p = \frac{dy}{dx}.$$
