2017

MATHEMATICS

[Honours]

(CBCS)

[First Semester]

PAPER – C1T

Full Marks : 60

Time : 3 hours

The figures in the right hand margin indicate marks

UNIT – I

(*Calculus* – I)

1. Answer any *three* questions : 2 × 3

(*a*) What do you mean by asymptote ? Does asymptote exist for every curve ?

(*b*) Your answer find the value of

$$\lim_{x\to 1} x^{\frac{1}{x-1}}$$

(*Turn Over*)

(*c*) Define point of inflection of a curve.

(*d*) Find the envolpe of the straight line

$$\frac{x}{a}+\frac{y}{b}=1$$

where the parameters a and b are connected by the relation $ab = c^2$.

(*e*) Write the Leibnitz theorem successive derivatives up to 4^{th} order.

2. Answer any *one* question : 10 × 1

(*a*) (*i*) If s be the length of the arc $3ay^2 = x(x-a)^2$ measured from the origin to any point (x, y), show that

$$3s^2 = 4x^2 + 3y^2.$$ 5

(*ii*) Show that the curve

$$y = 3x^5 - 40x^3 + 3x - 20$$

is concave upwards for $-2 < x < 0$ and $2 < x < \infty$ but convex upwards for $-\infty < x < -2$ and $0 < x < 2$. Also show that $x = -2, 0, 2$ are its points of infexion.
2 + 2 + 1

(*b*) (*i*) Trace the curve :

$$x^2y^2 = a(y^2 - x^2)$$ 5

(*ii*) If α, β be the rools of the equation $ax^2 + bx + c = 0$ then show that

$$\lim_{x \to \alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2} = \frac{1}{2}a^2(\alpha-\beta)^2$$ 5

(*Calculus* – II)

3. Answer any *two* questions : 2 × 2

(*a*) If

$$I_n = \int_0^{\pi/4} \tan^n x dx,$$

n being a positive integer greater than 1, then prove that

$$I_n = \frac{1}{n-1} - I_{n-2}.$$

(*b*) The volume of the solid generated by the revolution of the curve $y = \frac{1}{x}$, bounded by

$y = 0$, $x = 2$, $x = b$ $(0 < b < 2)$ about x-axis is 3. Find the value of b.

(c) What is the formula for finding area of the curve $y = \psi(t)$, $x = \phi(t)$, where t is the parameter?

4. Answer any *two* questions : 5 × 2

(a) If

$$I_{m,n} = \int_0^{\pi/2} \cos m_x \sin nx \, dx,$$

then show that

$$I_{m,n} = \frac{1}{m+n} + \frac{1}{m+n} I_{m-1,n-1}.$$

Also, deduce that

$$I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \cdots + \frac{2^m}{m}\right].$$

(b) Find the area of the surface generated by revolving the curves $x = \cos t$, $y = 2 + \sin t$, $0 \le t \le 2\pi$ about x-axis.

(*c*) Find the arc length parameter along the curve

$$C : \vec{r}(t) = (1+2t)\hat{i} + (1+3t)\hat{j} + 6(1-t)\hat{k}$$

from the point, where $t = 0$.

UNIT – III

(*Geometry*)

5. Answer any *three* questions : 2 × 3

(*a*) Determine the type of the conic

$$8x^2 + 10xy + 3y^2 + 22x + 14y + 15 = 0$$

(*b*) Show the plane $z - 1 = 0$ which intersects the ellipsoid

$$\frac{x^2}{48} + \frac{y^2}{12} + \frac{z^2}{4} = 1$$

is an ellipse. Determine semi-axes.

(*c*) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 25, x + 2y - z + 2 = 0$ and the point (1, 1, 1).

(*d*) Find the equation of the cylinder where generators are parallel to the line $x = -\frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + y^2 = 1, z = 3$.

(*e*) Find the equation of a right circular cone whose axis is $\frac{x}{1} = \frac{y}{0} = \frac{z}{-2}$ and radius equal to 7.

6. Answer any *one* question : 5 × 1

(*a*) Find the polar equation of the tangent to the circle $r = 2d\cos\theta$ at the point whose vectorial angle is θ_1.

(*b*) Find the equations of the generating lines of the hyperboloid of one sheet

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$$

which passes through the point (2, 3, –4).

7. Answer any *one* question : 10 × 1

(*a*) (*i*) Find the locus of a luminous point, if the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ casts a circular shadow on the plane $z=0$ 5

(*ii*) Reduce the equation

$$4x^2-4xy+y^2-8x-6y+5=0$$

to canonial form. 5

(*b*) (*i*) Prove that the locus of the foot of the perpendecular from a focus of the conic $\frac{l}{r}=1-e\cos\theta$ on a tangent to it, is given by

$$r^2(1-e^2)-2ler\cos\theta-l^2=0$$ 5

(*ii*) Prove that the axes of sections of the conicoid $ax^2+by^2+cz^2=1$ which pass through the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ lie on the cone

$$\frac{(b-c)}{x}(mz-ny)+\frac{(c-a)}{y}(nx-lz)+\frac{(a-b)}{z}(ly-mx)=0.$$ 5

UNIT – IV

(*Differential Equation*)

8. Answer any *two* questions : 2 × 2

(*a*) Find the integrating factor of the following differential equation

$$\frac{dx}{dy}+\frac{xy}{1-y^2}-y\sqrt{x}=0$$

(*b*) Reduce the equation

$$\sin y\frac{dy}{dx}=\cos x(2\cos y-\sin^2 x)$$

into a linear equation.

(*c*) Show that the equation

$$M(x, y)dx+N(x, y)dy=0$$

will be exact if

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial y}.$$

9. Answer any *one* question : 5 × 1

(*a*) (*i*) Show that the substitution $z=ax+by+c$ changes

$$y' = f(ax + by + c)$$

into an equation with separable variables, and apply this method to solve the equation

$$y' = \sin^2(x - y + 1)$$

(*ii*) Reduce the equation

$$\left(2x^2 - 1\right)\left(\frac{dy}{dx}\right)^2 + \left(x^2 + y^2 + 2xy + 2\right)\frac{dy}{dx} + 2y^2 + 1 = 0$$

to Clairaut's form by the substitation $x + y = u$ and $xy - 1 = v$, hence solve the equation. 5

(*b*) (*i*) Show that if y_1 and y_2 be solutions of the equation

$$\frac{dy}{dx} + Py = Q$$

where P and Q are functions of x alone, and $y_2 = y_1 z$ then $z = 1 + ae^{-\int \frac{Q}{y_1} dx}$

(*ii*) Show that the solution of $\frac{dy}{dx} + py = Q$ can also be written in the form

$$y = \frac{Q}{P} - e^{\int P dx}\left[c + \int e^{\int P dx} d\left(\frac{Q}{P}\right)\right]$$ 5