2018

CBCS

3rd Semester

MATHEMATICS

PAPER—C7T

(Honours)

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Calculator is allowed in examination Hall]

Numerical Methods

UNIT—I

1. Answer any *two* questions : 2×2

(a) If a number 0.05418 is approximated to 0.05411, find the number of significant digits for such approximation.

(b) Define the terms :

(i) Truncation error

(ii) Round of error

(c) Let, $u = 4x^6 + 3x - 9$. Find the percentage error in computing u at x = 1.1, if the error in x is 0.05.

UNIT—II

2. Answer any *one* question : 1×2

(a) Write down the equation $x^3 + 2x - 10 = 0$ in the form $x = \varphi(x)$ such that the iterative scheme about $x = 2$ converges.

(b) What do you mean by the term as iterative method has the rate of convergence $p(\geq 1)$?

3. Answer any *one* question : 1×5

(a) Find the iterative formula for finding $\sqrt[k]{N}$, where N is a real number, using Newton-Raphson formula. Hence evaluate $\sqrt[3]{2}$ correct upto four significant figure. State the condition of convergence of this method.

(b) Describe the method of false position for finding a real root of an equation $f(x) = 0$ and obtain the corresponding interation formula. Discuss its advantages and disadvantages in comparison to Newton-Raphson Method.

UNIT—III

4. Answer any *one* question : 1×2

(a) State the conditions for convergence of Gauss-Scidel method for solving a system of linear equations. Are they necessary and sufficient ?

(b) Define ill-conditioned and well-conditioned system of Linear equation.

5. Answer any *one* question : 1×5

(a) Consider a system of equations

$$x + y - z = 2$$

$$2x + 3y + 5z = -3$$

$$3x + 2y - 3z = 6$$

Solve the system of equations by LU decompositon method.

(b) Describe Gauss elimination method with pivoting for solution of a system of linear equation. what is the total number of operations required for this method ?

UNIT—IV

6. Answer any *one* question : 1×10

(a) (i) Prove that $f(x+Kh)=\sum_{i=o}^{K}\left(K_{C_i}\right)\Delta^i f(x)$ 2

(ii) Find the missing term of the following table : 3

x	0	1	2	3	4	5
$f(x)$	0	–	8	15	–	35

(iii) Obtain the Error in the Lagrange Interpolating Polynomia.

Also show that the maximum error in linear interpolation is given by $\frac{(x_o - x_1)^2}{8}M$ where $M=\max\left|f''(\xi)\right|,\ x_0 \le \xi \le x_1.$ 3+2

(b) What is the nth order forward differences of a polynomial of degree n? If n is very small prove that $\Delta^{n+1} f(x) = h^{n+1} f^{n+1}(x)$.

Find the value of Sec 31.5° using the following table :

θ (in degree)	31°	32°	33°	34°
tanθ	0.6008	0.6249	0.6494	0.6747

UNIT—V

7. Answer any *one* question : 1×2

(a) Show that Simpson's ⅓ rule is exact for integrating a polynomial of degree 3.

(b) If $f(x)$ is a quadratic polynomial, deduce that

$$\int_0^3 f(x)dx \cong \frac{1}{12}[f(0) + 22f(2) + f(4)]$$

8. Answer any *one* question : 1×5

(a) Derive Simpson's one-third Rule from Newton cotes formula. 5

(b) Describe the method of least squares to fit a straight line $y = ax + b$.

In some determinations of the value v of carbon dioxide dissolved in a given volume of water at different temparature θ, the following pairs of values were obtained :

θ	0	5	10	15
v	1.80	1.45	1.18	1.00

Obtain by the method of least square a relation of the form $v = a + b\theta$ which best fit to this ob·

UNIT—VI

9. Answer any *one* question : 1×5

(a) Describe Euler's method for solving first order differential equation with initial condition. Compute $y(1.2)$ for the problem $\frac{dy}{dx} = 1 + xy$, $y(1) = 1$ by modified Euler's method taking $h = 0.1$.

(b) Find the values of $y(0.1)$ and $y(0.2)$ using Runge Kutta Method of 4th order taking $h = 0.1$. Given that $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.