2018

2nd Semester

MATHEMATICS

PAPER—GE2T

(Generic Elective)

Full Marks : 60

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Unit-I

(Classical Algebra)

[Marks : 22]

1. Answer any *one* question : 1×2

(a) Find the geomtric image of the complex number z satisfying $|z - i| \leq 3$.

(b) If $x + \frac{1}{x} = 2\cos\frac{\pi}{7}$, prove that $x^7 + x^{-7} = -2$.

(c) Use Descarte's rule of sign to show that the equation $x^8 + x^4 + 1 = 0$ has no real root.

2. Answer any *two* questions : 2×5

(a) If n be a positive integer, then prove that

$$(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}.$$ 5

(b) Solve the equation $3x^4 + 20x^3 - 70x^2 - 60x + 27 = 0$ given that the roots are in geometric progression. 5

(c) State and prove the Cauchy-Schwarz inequality. 1+4

3. Answer any *one* question : 1×10

(a) (i) If α, β, γ be the roots of the equation $x^3 - px^2 + qx - r = 0$, form an equation whose roots are $\beta\gamma + \frac{1}{\alpha}$, $\gamma\alpha + \frac{1}{\beta}$, $\alpha\beta + \frac{1}{\gamma}$.

(ii) Prove that $\sin(\log i^i) = -1$.

(iii) If x, y, z are positive real numbers such that $xy + yz = zx = 8$, then find the greatest value of xyz. 4+3+3

(b) (i) If $s_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$,

prove that $s_n > \frac{2n}{n+1}$ if $n > 1$.

(ii) Prove that the roots of the equation

$$\frac{1}{x+a_1} + \frac{1}{x+a_2} + \cdots + \frac{1}{x+a_n} = \frac{1}{x}$$

are all real, where $a_1, a_2, \ldots, a_n$ are all positive real numbers.

(iii) If a, b, c be positive real numbers, prove that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3 .$$ 3+5+2

Unit-II

(Sets and Integers)

[Marks : 15]

4. Answer any *five* questions : 5×2

(a) Prove that $1^n - 3^n - 6^n + 8^n$ is divisible by 10 $\forall\ n \in N$.

(b) When a function is invertible. Find the inverse of the function $f : R^- \to R^+$ defined by $f(x) = x^2$.

(c) Use mathematical induction to establish the following :

$$\sum_{i=1}^{n}(i+1)2^i = n.2^{n+1}.$$

(d) Prove that the intersection of two symmetric relations is a symmetric relation.

(e) Let $P = \{n \in Z : 0 \le n \le 5\}$, $Q = \{n \in Z : -5 \le n \le 0\}$ be two sets. Prove that the cardinality of two sets are equal.

(f) If two mappings $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined by $f(x) = x^2$ and $g(x) = x - 2$, respectively, then show that $f \circ g \neq g \circ f$.

(g) If a is prime to b, prove that a + b is prime to ab.

(h) Examine whether the mapping $f : z \rightarrow z$ defined by $f(x) = |x| \; \forall \; x \in z$ is injective.

5. Answer any *one* question : 1×5

(a) State Euclidean Algorithm for computation of gcd (a, b). Hence find gcd (1575, 231). 5

(b) (i) State the division algorithm on the set of integers.

(ii) Show that the product of any three consecutive integers is divisible by 6. 1+4

Unit-III

(System of Linear Equations)

[Marks : 9]

6. Answer any *two* questions : 2×2

(a) Find the condition(s) for which the system

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

has many solution and no solution.

(b) For what values of K the system of equations

$$2x + Ky = 0$$
$$5x + 2y = 0$$

has a non-trivial solution.

(c) Determine K so that the set s = {(K, 1, 1), (1, K, 1), (1, 1, K)} is linearly independent in R^3.

7. Answer any *one* question : 1×5

(a) Investigate for what values of λ and u the following equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = u$$

have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions. 2+2+1

(b) (i) For what values of K the planes $x + y + z = 2$, $3x + y - 2z = K$ and $2x + 4y + 7z = K + 2$ intersect in a line?

(ii) Find a row-reduced echelon matrix which is row equivalent to

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \end{pmatrix}.$$

3+2

Unit-IV

(Linear Transformations & Eigen Values)

[Marks : 14]

8. Answer any *two* questions : 2×2

(a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 1 & 1 \end{bmatrix}.$$

(b) If λ be an eigen value of an $n \times n$ matrix A, then show that λ is also an eigen value of its transpose matrix A^t.

(c) Let P_1 be the vector space of polynomials in t of degree 1 over the field of real numbers R. If $T : P_1 \to P_1$ is a linear transformation such that

$$T(1 + t) = t, \ T(1 - t) = 1, \text{ find } T(2 - 3t).$$

9. Answer any *one* question : 1×10

(a) (i) State Cayley-Hamilton theorem. Verity Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Hence find A^{-1} and A^{100}. 1+3+2+2

(ii) Find the eigen values of $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$. 2

(b) (i) Define rank and nullity of a linear transformation. Find the matrix of the linear transformation $T : R^3 \to R^3$ defined by

$T(a, b, c) = (a + b, a - b, 2c)$ with respect to the ordered basis $B = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$.

(ii) Let $V = \{(x, y, z) \mid x, y, z \in R\}$, where R is a field of real numbers.

Show that $W = \{(x, y, z) \mid x - 3y + 4z = 0\}$ is a sub-space of V over R. Find the dimension of W.

5+5
