2018

CBCS

1st Semester

MATHEMATICS

PAPER—C1T

(Honours)

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Calculus, Geometry and Differential Equation

Unit—I

1. Answer any *three* quesitons : 3×2

(a) Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upward.

(b) If n be any positive integer, find the value of

$$\lim_{x \to n} \frac{x-n}{\sin \pi x}.$$

(c) If $y = 2\cos x\,(\sin x - \cos x)$ then find the value of $(b_{20})_0$.

(d) Find the asymptotes, if any of the curve $y = \log \sec(x/a)$.

(e) Show that abscissa of the points of inflexion on the curve $y^2 = f(x)$ satisfying $[f'(x)]^2 = 2f(x)\,f''(x)$.

2. Answer any *one* question : 1×10

(a) (i) If $y = \sin\left(m \cos^{-1} \sqrt{x}\right)$ then prove that

$$\lim_{x \to 0} \frac{y_{n+1}}{y_n} = \frac{4n^2 - m^2}{4n+2}.$$ 4

(ii) Find all the asymptotes of the curve

$$x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0.$$ 3

(iii) If $f(x) = ax^3 + 3bx^2$. Find a and b so that $(1, -2)$ is a point of inflexion of f. 3

(b) (i) Trace the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. 4

(ii) Find the values of a and b so that 3

$$\lim_{x\to 0} \frac{a \sin 2x - b \sin x}{x^3} = 1$$

(iii) Obtain the envelope of the circle drawn upon the radii vectors of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as diameter. 3

Unit—II

3. Answer any *two* questions : 2×2

(a) Find the entire area enclosed by the curve $r = a \cos 2\theta$?

(b) Obtain reduction formula for $\int \operatorname{cosec}^n x \, dx$.

(c) Show that in the astroid $x^{2/3} + y^{2/3} = a^{2/3}$, $s \propto x^{2/3}$; s being measured from the point for which $x = 0$.

4. Answer any *two* questions : 2×5

(a) Prove that the surface of the solid obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ round its minor axis is $2\pi a^2\left[1+\frac{1-e^2}{2e}\log\left(\frac{1+e}{1-e}\right)\right]$ where $b^2 = a^2(1 - e^2)$. 5

(b) If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x \quad dx$, m, n being positive integers greater than 1, prove that

$$I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}$$

Hence find the value of $\int_0^1 x^6 \sqrt{1-x^2}\ dx$. 3+2

(c) Show that the arcs of the curves $x = f(t) - \varphi'(t)$, $y = \varphi(t) + f'(t)$ and $x = f'(t)\sin t - \varphi'(t)\cos t$, $y = f'(t)\cos t + \varphi'(t)\sin t$ corresponding to same interval of variation of t have equal lengths. 5

Unit—III

5. Answer any *three* questions : 3×2

(a) Find the angle of rotation about the origin which will transform the equation $x^2 - y^2 = 4$ into $x'y' + 2 = 0$.

(b) Prove that the equations $x = 1 + \lambda y = -1 + \frac{2z}{\lambda}$ represents a generator of $x^2 - 2yz = 1$. Find also other system of generators which lie on $x^2 - 2yz = 1$.

(c) Find the equation of the cylinder whose generating line is parallel to x-axis and guiding curve is

$$3x + 2y - 5 = 0,\ 5x^2 - 2y^2 + 7z^2 = 1.$$

(d) Find the point of intersection of the two forgents at α and β to the Conic $\frac{l}{r} = 1 + e\cos\theta$.

(e) Find the nature of the conicoid

$$3x^2 - 2y^2 - 12x - 12y - 6z = 0.$$

6. Answer any *one* question : 1×5

(a) Prove that the discriminant of the Conic $Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$ is invariant under rotation of axes. 5

(b) The section of a cone whose guiding curve is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that locus of the vertex is the surface $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$. 5

7. Answer any one questions : 1×10

(a) (i) Show that the Centre of the sphere which always touch the lines

$y = mx,\ z = c$ and $y = -mx,\ z = -c$

lie on the surface $mxy + cz(1+m^2) = 0$. 5

(ii) Find the equation of the right circular cylinder whose guiding carve is

$x^2 + y^2 + z^2 = 9,\ x - y + z = 3.$ 5

(b) (i) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$. 5

(ii) If the normal be drawn at one extrimity $\left(l, \frac{\pi}{2}\right)$ of the latus rectum PSP' on the conic $\frac{l}{r} = 1 + e\cos\theta$ where S is the pole, then show that the distance from focus S of the other point in which the normal meets the conic is $\frac{l\left(1+3e^2+e^4\right)}{1+e^2-e^4}$. 5

Unit—IV

8. Answer any *two* questions : 2×2

(a) For which value of m, $y = x^m$, is a solution of the equation $3x^2\frac{d^2y}{dx^2} + 11x\frac{dy}{dx} - 3y = 0$.

(b) Let the differential equation be $a\frac{dy}{dx} + by = ke^{-\lambda x}$ where a, b, k are positive constants and λ is non-negative constant. Find the solution of differential for $\lambda = 0$. Show that $y \to k/b$ as $x \to \infty (\lambda = 0)$.

(c) Find an integrating factor of the equation

$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0.$

9. Answer any *one* question : 1×5

(a) Reduce the equation $x^2p^2 + yp(2x + y) + y^2 = 0$ to Clairaut's form and obtain complete primitive. 5

(b) (i) In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that their number doubles in 4 hours, what should be their number at the end of 12 hours ?

(ii) Find the solution of $\frac{dy}{dx} - y\tan x = \cos x$ by substitution $y = y_1(x)\, v(x)$ where $y_1 = \sec x$

3+2

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