 C/18/B.Sc./2nd Sem/COSH/C4T

2018

2nd Semester

COMPUTER SCIENCE

PAPER—C4T

(Honours)

Full Marks : 60

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Discrete Structures)

1. Answer any *ten* questions. 10×2

(a) Explain 'tautology' and 'contradiction' with example.

(b) Define the following terms : Connected graph, Spanning tree.

(c) Find the closed form of the generating function of the sequence {1, –2, 3, –4, 5, –6, ...}.

(Turn Over)

(d) What do you mean by argument and valid argument in propositional logic ?

(e) Show that the maximum number of edges of a simple graph with n vertices is $\frac{n(n-1)}{2}$.

(f) Use the master method to give asymptotic tight bound of the following recurrence function

$f(n) = 2f(n/4) + \sqrt{n}$.

(g) Show that any graph contains even number of odd degree vertices.

(h) Explain uncountable infinite set with example.

(i) What is the coefficient of x^6y^3 in $(x + y)^9$?

(j) Give a connected graph which is Eulerian but not Hamiltonian.

(k) Explain pigeonhole principle with example.

(l) State the Modus Tollens inference rule for propositional logic.

(m) Find the asymptotic upper bound of $n^2 + 5n + 7$.

(n) Define binary relation on a non-empty set.

2. Answer any *four* questions. 4×5

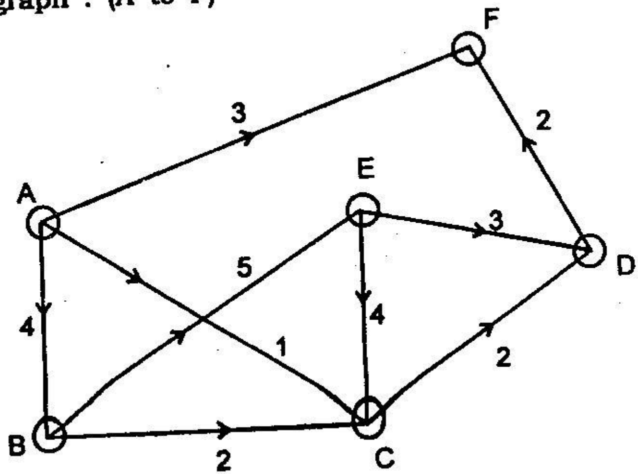
(a) Obtain the asymptotic tight bound of $\sum_{k=1}^{n} k^2 \log k$.

(b) Solve the following recurrence relation using generating function

$a_{n+2} - 2a_{n+1} + a_n = 2^n$, $a_0 = 2$, $a_1 = 1$.

(c) Show that every simple connected planar graph satisfies the following inequality $e \le 3n - 6$, where n be the number of vertices and e be the number of edges of the graph.

(d) Illustrate the steps for determine the minimal spanning tree of the following connected weighted graph : (A to F)



(Turn Over)

(e) Prove that the following proposition is Tautology (without truth table) :

$$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r.$$

(f) Using mathematical induction prove that

$$\frac{1^2}{1.3} + \frac{2^2}{3.5} + \ldots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}.$$

3. Answer any *two* questions. 2×10

(a) (i) Prove that the following set of premises is inconsistence $p \to q, p \to r, q \to \neg r, p$.

(ii) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $f(n) = f(n/2) + n^2$. Use the substitution method to verify your answer. 4+6

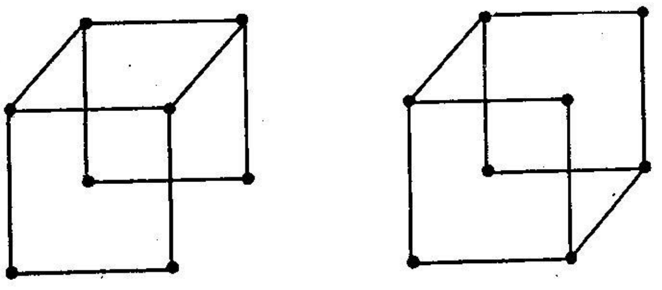
(b) (i) Find a recurrence relation and give initial conditions for a number of bit strings of length n that do not contain the pattern 11.

(ii) Estimate the growth of $\frac{n^2+1}{n+1}$.

(iii) Show that any cycle free graph with n vertices and (n-1) edges is a tree. 3+3+4

(c) (i) Show that the logical implications 'conditional and contrapositive' and 'inverse and converse' are logcially equivalence.

(ii) Define isomorphism between two graphs. Show that the following two graphs are not isomorphic. 5+5



(d) (i) How many positive integers between 100 and 999 are (A) not divisible by either 3 or 4? (B) divisible by 3 but not by 4?

(ii) Prove that $\sqrt{2}$ is irrational by contradiction method.

(iii) A function f on the set R of real numbers is defined as $(x) = \begin{cases} 2x+1, & 0 \le x \le 2 \\ x-2 & 2 \le x \le 5 \end{cases}$

Find

(A) The domains of f

(B) The image of f

(C) Whether the function is one-one [4+3+3]