বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

M.Sc. Examinations 2020
Semester IV
Subject: APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING
Paper: MTM 405 (Special Paper)
(Theory)

Full Marks: 20 **Time: 2hrs.**

Candidates are required to give their answers in their own words as far as practicable.

Paper / Unit : MTM-405A (Special Paper-OM: Dynamical Meteorology –II)

*Answer any **One** of the following questions*

1) (a) What are kinematic and dynamic boundary conditions for a front?
(b) What do you mean by grid points? How the variables such as pressure, temperature and velocities are represented on a two-dimensional grid cell.
(c) What do you mean by Hurricane and discuss the thermodynamics of Hurricane?
2) (a) Explain the pressure distribution near the fronts.
(b) Determine the potential temperature due to advection using finite difference method.
(c) How does relative vorticity ξ_r vary with distance x (east) in a planetary wave?
3) (a) Show that the isobars are always V-shaped at a front with lower pressures within the V and develop a formula for the angle between the two arms of the V made by the isobars at front.
(b) What do you mean by Global Circulation in the atmosphere?
(c) Define: potential vorticity and isentropic potential vorticity.
4) (a) Show that in a geostrophic wind field, an ideal front is stationary.
(b) Derive Meridional Temperature Gradient in the atmosphere?
(c) Find the meridional temperature gradient at 45° N latitude at sea level?
5) (a) What is the concept of frontogenesis and frontolysis?
(b) Explain Vertical Gradient of geostrophic Wind.

(c) Suppose temperature increases from 8℃ to 12℃ towards the east, across a distance of 100km. Find the vertical gradient of geostrophic wind. (f_c =10^{-4}/sec)

6) (a) Define frontal surface and front.

(b) Find the thickness of two isobaric surfaces.

(c) Suppose thickness of 100-70kpa layer is 2.9 km at one location and 3 km at a side 500km to the east. Find the components of thermal wind vector.

(d) Show that tangential velocity of hurricane must decrease with altitude and eventually change sign.

7) (a) Show that, slope of a frontal surface depends on temperatures and velocities in two air masses.

(b) Define: relative vorticity and absolute vorticity.

(c) An 11 km deep layer of air at 45° N latitude of density 0.5 kg/m^3 has no curvature but has a shear of -10 m/sec across distance 500 km and $\frac{\Delta\theta}{\Delta z}$ =3.3 kelvin/km. Find isentropic potential vorticity.

8) (a) Show that the kinematic boundary condition reduces to an equality of the wind components perpendicular to the front.

(b) Explain the jet stream in the atmosphere.

(c) What is the barotropic instability? Why Rossby wave is created?

9) (a) Explain perturbation technique.

(b) An 11 km deep layer of air at 45° N latitude has no curvature but a shear of -10 m/sec across a distance 500 km. Find the potential vorticity.

(c) Prove that, $\triangle P_T = a \ \triangle P_B$ – b $\triangle$T in a hurricane, where symbols have usual meaning.

10) (a) Derive the pressure tendency below the frontal surface.

(b) Prove that, $(\frac{2M_{tan}}{R} + f_c)\frac{\partial M_{tan}}{\partial z} = \frac{g}{T}\frac{\Delta T}{\Delta R}$ in a hurricane, where symbols have usual meaning.

11) (a) Derive the perturbation equations for a homogeneous incompressible fluid having free surface in the atmosphere. Hence deduce the wave travelling speed of pure gravity waves.

(b) What do you mean by storm surge?

12) (a) What is the concept of numerical weather prediction?

(b) What is the frontal surface? Derive the slope of frontal surface.

(c) In a strong Hurricane with eye pressure 90kpal causing ΔP_{max}=10kpal, what is the sea level height? (where symbols have their usual meaning)

Paper / Unit : MTM-405B (Special Paper-OR: Operational Research Modelling-II)

*Answer any **One** of the following questions*

1. (a) Suppose a system contains a primary element and a stand-by element. Let λ_p and λ_d represent the failure rates of primary element and stand-by element. Find the reliability of this system. Also, find the system reliability and MTBF when $\lambda_p = \lambda_d = \lambda$.
   (b) An industrial process is controlled by a computer and two similar components are operated in stand-by redundancy such that if a computer fails another is instantaneously brought into use in its place. The failure rate of each computer is given by $\lambda = 0.01$ failure/hour. Compare the improvement in reliability over a single computer when one and then two computers are used in a stand-by. The operating period is 100 hours and the switch is considered to be perfect.
2. Define joint and conditional entropies. Prove that
   $H(X,Y) = H(X/Y) + H(Y) = H(Y/X) + H(X)$, where $H(X) \geq H(X/Y)$.
3. Let a car be driven from a stationary position on a horizontal way to a stationary position in a garage moving a total distance '*a*'. The available controls for the driver are the accelerator and the break. Find the minimum time to bring the car in the stationary position at a distance '*a*' and the optimal control to be applied on the car.
4. (a) In a system, there are *n* number of components connected in series with reliability $R_i(t) = p, i = 1, 2, \dots, n$. Find the reliability of the system. If $R_i(t) = e^{-\lambda t}$, then find the reliability of the system.

   (b) The system connected in series consists of three independent parts *A, B* and *C* which have MTBFs of 100, 400 and 800 hours respectively. Find MTBF of the system and reliability of the system for 30 hours, how much MTBF of the parts A has to be increased to get and improvement of MTBF of the system by 30?

5. Apply Shannon's encoding procedure encode the message BADCADBABA using following information.

Alphabet	A	B	C	D
Probability	0.1	0.3	0.2	0.4

6. If *H* denotes the entropy function, then prove that
   $$H(p_1, p_2, \dots, p_{n-1}, q_1, q_2, \dots, q_m) = H(p_1, p_2, \dots, p_{n-1}) + p_n\, H\left(\frac{q_1}{q_n}, \frac{q_2}{q_n}, \dots, \frac{q_m}{q_n}\right)$$
   where $p_n = \sum_{i=1}^{m} q_i$.
   Verify the formula, defining additivity of entropies for events *A*, $ and *C* with probabilities $\frac{1}{5}, \frac{4}{15}, \frac{8}{15}$ respectively.