বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

M.Sc. Examinations 2020
Semester IV

Subject: APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

Paper: MTM 404 (Special Paper)

(Theory)

Full Marks: 40 **Time: 2hrs.**

Candidates are required to give their answers in their own words as far as practicable.

Paper/unit : MTM-404A (Special Paper-OM: Computational Oceanology)

*Answer any **One** of the following questions*

1. (a) Write the two-dimensional gravity wave equations.
   (b) Draw the grids for Grid-A (cell centered) and Grid-C (staggered grid).
   (c) Finally apply centred space differencing for space derivative on Grid-C and backward formula for time derivative to discretize the above equations.

2. (a) Write the non-dimensional continuity and Navier-Stokes equations for laminar two-dimensional incompressible fluid flow.
   (b) Draw the control volume for *v*-velocity and
   (c) Hence apply the finite volume method to the y-momentum equation.

3. (a) Draw the control volume for u-velocity and place the variables (velocity and pressure) on the respective faces for Quadratic Upwind Interpolation for Convective Kinematics (QUICK).
   (b) Then write the expressions for u-velocity at the east and west faces of the said control volume for both negative and positive fluxes.
   (c) Also with help of appropriate symbol, compose two expressions for negative and positive fluxes into one for both east and west faces separately.

4. (a) Write the two-dimensional system of shallow water equations in a single equation with appropriate matrices.

(b) Then apply the general finite volume method to the above equation.

5. (a) Write the expression for the First Order Upwind Scheme (FOU) and Second Order Upwind Scheme (SOU).
   (b) Hence apply the expression for the First Order Upwind Scheme (FOU) to the one-dimensional transport equation.

6. (a) What is Lax-Friedrichs Scheme?
   (b) In place of backward formula for time derivative, already used in Q.No.1(c), apply the Lax-Friedrichs Scheme to the discretize the shallow-water equations derived from Q.No.1(c), and write in a simplified form.

7. (a) Explain the group velocity.
   (b) Write an expression of group velocity for wave length (λ).
   (c) Derive an equation for surface tension (T) of capillary waves as $\frac{\partial^2 \phi}{\partial t^2} - g\frac{\partial \psi}{\partial x} + \frac{T}{\rho}\frac{\partial^3 \psi}{\partial x^3} = 0$, where symbols have their usual meaning.

8. Derive depth-average continuity and momentum equations for shallow water theory.

9. (a) Derive the wave equation for linear waves in the absence of rotation and hence, prove that the horizontal velocity is given by $u = -\frac{g}{2C_0}\{F(x + C_0 t) - F(x - C_0 t)\}$.
   (b) Considering step function, show that the total energy is conserved.

10. (a) Define Rossby radius of deformation.
    (b) Derive Klein-Gordon equation for long surface wave and hence, prove that geostrophic velocity in y-direction is given by $\bar{v} = \frac{gh}{2C_0}\exp(-|x|/a)$, where symbols have their usual meaning.

11. (a) Define circulation and vorticity of a rotating fluid.
    (b) Derive the vorticity equation for depth-averaged 2-D shallow water wave.
    (c) Explain, the terms relative vorticity, absolute vorticity and potential vorticity.

12. Write a short note on the following:
    a) Sverdrup wave
    b) Poincaré wave

Paper / Unit : MTM-404B (Non-Linear Optimization)

*Answer any **One** of the following questions*

1. Answer the questions.

(a) Define posynomial and polynomial in connection with geometric programming with an example.

(b) Let X^0 be an open set in R^n, let θ and g be defined on X^0. Find the conditions under which a solution $(\bar{x}, \bar{r_0}, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely.

(c) Define bi-matrix game with an example.

(d) State Dorn's duality theorem in connection with duality in quadratic programming.

(e) Write the basic difference(s) between Beale's and Wolfe's method for solving quadratic programming problem.

2. (a) Solve the quadratic programming problem using Wolfe's method

$$\text{Maximize } z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$$
$$\text{subject to } x_1 + 2x_2 \le 10,$$
$$x_1 + x_2 \le 9,$$
$$x_1,\ x_2 \ge 0.$$

(b) State and prove Weak duality theorem in connection with duality in non-linear programming.

3. (a) Minimize the following using geometric programming

$$f(x) = 16x_1x_2x_3 + 4x_1x_2^{-1} + 2x_2x_3^{-2} + 8x_1^{-3}x_2$$
$$x_1, x_2, x_3 > 0.$$

(b) State and prove Motzkin's theorem of alternative.

4. (a) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem:

(i) Complete optimal solution (ii) Pareto optimal solution

(ii) Local Pareto optimal solution (iv) Weak Pareto optimal solution

(b) Give the geometrical interpretations of differentiable convex function and concave function.

5 (a) State and prove Fritz-John saddle-point necessary optimality theorem.

(b) Use the chance constrained programming to find an equivalent deterministic problem to following stochastic programming problem, when c_j is a random variable:

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$
$$\text{Subject to } \sum_{j=1}^{n} a_{ij}\, x_j \le b_i$$

$$x_j \geq 0,\ i, j = 1, 2, \ldots\ldots, n$$

6 (a) Prove that a pair $\{y^*,\ z^*\}$ constitutes a mixed strategy Nash equilibrium solution to a bimatrix game (A, B) if and only if, there exists a pair $\{p^*,\ q^*\}$ such that $\{y^*,\ z^*,\ p^*, q^*\}$ is a solution of the following bilinear programming problem:

$$\begin{aligned} &\text{Minimize } [y'Az + y'Bz + p + q] \\ &\text{Subject to } \quad Az \geq -pl_m \\ &\qquad\qquad\quad B'y \geq -ql_n \\ &\qquad\qquad\quad y \geq 0, z \geq 0, y'l_m = 1, z'l_n = 1. \end{aligned}$$

(b) Define the following:

(i) Minimization problem;

(ii) Local minimization problem;

(iii) Kuhn-Tucker stationary point problem;

(iv) Fritz-John stationary point problem.

7. (a) Let X be an open set in R^n and θ and g be differential and convex on X and let $\bar{x}$ solve the minimization problem and let g satisfy the Kuhn-Tucker constraint qualification. Show that there exists a $\bar{u} \in R^m$ such that $(\bar{x}, \bar{u})$ solves the dual maximization problem and $\theta(\bar{x}) = \psi(\bar{x}, \bar{y})$.

(b) Prove that all strategically equivalent bimatrix game have the Nash equilibria.

8. (a) Let θ be a numerical differentiable function on an open convex set $\Gamma \complement R^n$. θ is convex if and only if $\theta(x^2) - \theta(x^1) \leqq \nabla\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$.

(b) Define the following terms:

(i) The (primal) quadratic minimization problem (QMP).

(ii) The quadratic dual (maximization) problem (QDP).

9. (a) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of FJSP?

(b) Define the following:

(i) Minimization problem;

(ii) Local minimization problem.

10. (a) How do you solve the following geometric programming problem?

Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function

$$f(x) = \sum_{j=1}^{n} U_j(x) = \sum_{j=1}^{N} \left(c_j \prod_{i=1}^{n} x_i^{a_{ij}}\right)$$

$c_j > 0,\ x_i > 0,\ a_{ij}$ are real numbers, $\forall\ i,\ j$

(b) Derive the Kuhn-Tucker conditions for quadratic programming problem.

11. (a) State and prove Slater's theorem of alternative.

(b) Use the chance constraints programming techniques to find an equivalent deterministic LPP to the following stochastic programming problem.

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to } P[\sum_{j=1}^{n} a_{ij} x_j \leq b_i] \geq p_i$$

$$x_j \geq 0, i, j = 1,2, \cdots n$$

where b_i are random variables and p_i are satisfied probabilities.

12. (a) Solve the following quadratic problems by using Beale's method:

$$\text{Maximize } Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$$

Subject to the constraints $x_1 + 2x_2 \leq 10$

$$x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0.$$

(b) Write short note on complementary slackness principle.