বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

M.Sc. Examinations 2020
Semester IV
Subject: APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING
Paper: MTM 403
(Theory)

Full Marks: 40 **Time: 2hrs.**

Candidates are required to give their answers in their own words as far as practicable.

Unit-01

Paper: MTM-403 (Magneto Hydro-Dynamics)

*Answer any **One** of the following questions*

1. (a) What is the difference between magneto-fluid dynamics (MFD) and magneto-hydro dynamics (MHD)?
(b) Define induced magnetic field.
(c) Explain any two applications of magneto-fluid dynamics.
(d) Define Lorentz force and hence derive the expression of Lorentz force for acting on the charge 'q' moving with the velocity 'v'.

2. (a) Write down the basic equations of magneto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows.
(b) Find the rate of change of magnetic energy in magneto-hydrodynamics.

3. (a) Define Hartman number and write its physical significance.
(b) State and prove Alfven's theorem.

4. (a) Define magnetic diffusivity.
(b) Give the mathematical formulation of MHD flow for Couette flow and derive its velocity and magnetic field expression.

5. A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate $Z = -L$ (lower) and a horizontal infinitely long non-conducting plate $Z = L$ (upper). Assume that a uniform magnetic field H_0 acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field.

6. Show that for B to be a force free magnetic field at all times it has to satisfy the integrability condition $B \times (\nabla\alpha.\nabla)B = 0$, in addition to satisfying the basic equation of force-free magnetic field (symbols have their usual meaning).

Unit-02

Paper: MTM-403 (Stochastic Process and Regression)

*Answer any **One** of the following questions*

7. Obtain the multiple regression equation of x_1 on $x_2,\ x_3, \dots, x_p$ in terms of the means, the standard deviations and the inter correlations of the variables.

8. State and prove Chapman-Kolmogorov equation for a homogeneous Markov chain $\{X_n\}$. Suppose a two state homogeneous Markov chain has the following transition probability matrix:

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \qquad 0 \le a, b \le 1, \qquad |1-a-b| < 1.$$

Prove that (by using Chapman-Kolmogorov equation) the n-step transition probability matrix P(n) is given

$$P(n) = \begin{bmatrix} \dfrac{b + a(1-a-b)^n}{a+b} & \dfrac{a - a(1-a-b)^n}{a+b} \\ \dfrac{b - b(1-a-b)^n}{a+b} & \dfrac{a + b(1-a-b)^n}{a+b} \end{bmatrix}.$$

9. Deduce the forward diffusion equation for the Wiener process. Also, write the backward diffusion equation from the deduced equation.

10. State birth and death process. Find the differential-difference equation for birth and death process.

11. Prove that

$$r_{1.23\dots p} = \left(1 - \frac{|R|}{R_{11}}\right)^{1/2}$$

where the symbols have their usual meanings.

12. Let $\{X_n, n \ge\ 0\}$ be a branching process. Show that if $m = E(X_1) = \sum_{k=0}^{\infty} k\, p_k$ and $\sigma^2 = Var(X_1)$ then $E(X_n) = m^n$ and

$$Var(x_n) = \begin{cases} \frac{m^{n-1}(m^n-1)}{m-1} \quad \sigma^2, \text{if } m \ne 1 \\ n\,\sigma^2, \qquad \text{if } m = 1 \end{cases}$$