Journal of Physical Sciences, Vol. 25, 2020, 21-30
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 16 October 2020

Mixed Fractional CEV Model with Stochastic Volatility and the Pricing of European Options

Yudong Sun[1] and Xie Wanshan[2]

[1]School of Business, Guizhou Minzu University, Guiyang GuiZhou, 550025 China
Email: yudongsun@yeah.net
[2]School of Data Science and Information Engineering, Guizhou Minzu University, Guiyang 550025, China
Email: 1719140947@qq.com

Received 28 April 2020; accepted 31 August 2020

ABSTRACT

In this work, we study the existence, uniqueness and continuity of solution to tock price equation of CEV model with stochastic volatility in fixed fractional Brown motion. Besides, we show a Monte Carlo simulation based on the discretization method to price the European option.

Keywords: Mixed fractional CEV model; Strong solution; Existence; Uniqueness; Continuity

Mathematical Subject Classification (2010): 35K99, 97M30

1. Introduction

Empirical evidences have shown that the volatility of market is not constant and its behavior is stochastic [1,2]. Many scholars paid attentions to the stochastic volatility (SV, for short) models which mainly include two situations. On the one hand, some studies use the functions of some stochastic process to describe the volatility [3]. On the other hand, some scholars introduce an additional Brown motion to character the stochastic parts of financial models. In this paper, we focus on the second case.

Hull and White in [4] introduced the SV models which were also developed by many scholars. Models in the category of "stochastic volatility" were first systematically studied by [5,6,7] with numerical methods. Specifically, Monte Carlo simulation was adopted by [5,6], while Wiggins proposed that the finite difference method be adopted in solving the corresponding PDEs for pricing financial derivatives, such as options[7].

The theoretical development of SV models was introduced in [8] where the authors studied the equation

$$\begin{cases} \mathrm{d}S(t) = rS(t)\mathrm{d}t + \sqrt{v(t)}S(t)\mathrm{d}B_1(t) + \sigma S(t)\mathrm{d}J(t) \\ \mathrm{d}v(t) = \kappa(\theta - v(t))\mathrm{d}t + \sigma_0(v(t))\mathrm{d}B_2(t) \end{cases} \tag{1}$$

whose stochastic parts added a Levy process $\{J(t), t \geq 0\}$. Here r, $\sigma, \kappa, \theta, \sigma$ and σ_0 are constants, $B_1(t)$ and $B_2(t)$ are standard Brown motions with assumption that $B_1(t)$, $B_2(t)$ and $J(t)$ are mutually independent. The existence and uniqueness of a strong solution to (1) were studied. Later, some L^p estimates were proved of (1) [9].

Unfortunately, all the SV model mentioned above are characted by Brown motion in which the increments follow the independent norm distribute. Many scholars argue that the returns of risky assets have long-range dependence properties which are expressed by increment of financial models. Regardless of the dependence in financial modeling, using Brown motion to express the stochastic parts may have some serious disadvantages [10].

Recently, scholars have paid their attentions to fractional Brown motion, and used it to character the stochastic parts of risky assets models, because the increments of fractional Brown motion have the self-similarity and long-range dependence properties. We refer the reader to [11] for the motivation and references concerning the study of fractional Brown motion.

In this paper, we use Mixed fractional Brown motion (mfbm) which is a linear combination of Brown motion and fractional Brown motion to driven the following stock price equation of CEV model

$$\mathrm{d}S(t) = rS(t)\mathrm{d}t + \sqrt{v(t)}S(t)^{\alpha}\mathrm{d}M_1^H(t), \tag{2}$$

where the variance process $\{v(t), t \geq 0\}$ driven by another mfbm satisfies

$$\mathrm{d}v(t) = \beta(v(t))\mathrm{d}t + \sigma(v(t))\mathrm{d}M_2^H(t), \tag{3}$$

$$\mathrm{d}M_1^H(t) \cdot \mathrm{d}M_2^H(t) = \rho(\mathrm{d}t^{2H} + \lambda^2\mathrm{d}t). \tag{4}$$

Here $M_1^H(t)$ and $M_2^H(t)$ are two mfbm processes whose concept and relative conclusions will be given later, r is the (constant) interest rate. The main goal of this work is to investigate the existence, uniqueness and continuity of solutions to the dynamic model (2)-(4). The existence and uniqueness are followed in Section 2. In Section 3, the continuity of solution to the dynamic model (2)-(4) is studied. European option is priced using discrete type of (2)-(4) and Monte-Carlo simulation.

2. The existence and uniqueness

Let λ and H be positive constants, $\lambda \geq 0$, $H \in (0,1)$. A mixed fractional Brownian motion with parameters λ and H is a linear combination of standard Brownian motion and fractional Brownian motion,

$$M_t^H = \lambda B(t) + B^H(t),$$

where $\{B(t), t \geq 0\}$ is a Brownian motion, $\{B^H(t), t \geq 0\}$ is an independent *fBm* of the Hurst parameter H [11].

We give the following lemmas with respect to *mfBm* which are used to prove our main results (for details, see [12]).

Lemma 2.1. A *mfBm* satisfy the following conditions

1. The paths of M^H are continuous and $M_0^H = 0$.
2. $E[M_t^H] = 0$ and $E[M_t^H] = \lambda^2 t + t^{2H}$, for any $t \geq 0$.
3. The increments of M^H are stationary.

Lemma 2.2. Suppose that $M_t^H = \lambda B_t + B_t^H$ is a mfBm process. Then, the path of the process is γ-Holder continuous such that $\gamma < 0.5 \wedge H$.

Lemma 2.3. A mfBm process with $H \in (0.75, 1)$ has long-range dependence.

Lemma 2.4. Let M_t^H be a mfBm process with Hurst parameter $H \in (0.75, 1)$ and $\lambda \in R$, then M_t^H and λB_t are locally equivalent.

According to Lemma 2.4, M_t^H is equivalent to λB_t. This process is suitable to display the random part of the financial model.

In this section, we prove the existence and uniqueness of solution for the mixed Heston model. To do this, we extend the idea of [13] for mixed stochastic differential equation.

Definition 2.1. For any $s < t$, suppose $C([s,t])$ denotes the Banach space of continuous functions equipped with the supremum norm we denote by $\| f \|_{s,t}$, $f \in C([s,t])$ with

$$\| f \|_{s,t,\infty} = \sup\{| f(r) |, s \le r \le t\} .$$

The space of Holder continuous functions of order $\beta > 0$ is denoted by $C^{\beta}([s,t])$ and its norm is

$$\| f \| s,t,\lambda = \sup\{\frac{| f(u) - f(\nu) |}{| u - \nu |^{\beta}}, s \le \nu < u < t\} .$$

Theorem 2.1. The volatility equation of the mixed CEV model has a unique positive solution v_t where $t \in [0,T)$ and $T = \inf\{t > 0 \mid X_t = 0\}$.

Proof: First, we confirm the existence of solution for relevant equation. In order to do, we define $Y_t^0 = v_0$ and $Y_t^{(k)} = Y_t^{(k)}(\omega)$ inductively as follows

$$Y_t^{(k+1)} = v_0 + \int_0^t \beta(Y_t^{(k)})\mathrm{d}s + \int_0^t \sigma(Y_t^{(k)})\mathrm{d}M_s^H . \tag{5}$$

Therefore

$$E[\left|Y_t^{(k+1)} - Y_t^{(k)}\right|^2] = E[\left|\int_0^t \beta(Y_t^{(k)}) - \beta(Y_t^{(k-1)})\mathrm{d}s + \int_0^t \sigma(Y_t^{(k)}) - \sigma(Y_t^{(k-1)})\mathrm{d}M_s^H\right|^2] .$$

We know that $(a+b)^n \le 2^{n-1}(a^n + b^n)$,so

$$E[\left|Y_t^{(k+1)} - Y_t^{(k)}\right|^2] \le 2E[\left|\int_0^t \beta(Y_t^{(k)}) - \beta(Y_t^{(k-1)})\mathrm{d}s\right|^2] + 2E[\left|\int_0^t \sigma(Y_t^{(k)}) - \sigma(Y_t^{(k-1)})\mathrm{d}M_s^H\right|^2] . \tag{6}$$

Using the Holder inequality, one derives

$$E[\left|\int_0^t \beta(Y_t^{(k)}) - \beta(Y_t^{(k-1)})\mathrm{d}s\right|^2] \le t\int_0^t E[| \beta(Y_t^{(k)}) - \beta(Y_t^{(k-1)}) |^2]\mathrm{d}s \le tM_1\int_0^t E[| Y_t^{(k)} - Y_t^{(k-1)} |^2]\mathrm{d}s . \tag{7}$$

Here M_1 is the Lipschitz coefficients related to the β. Next we pay attention to $E[\left|\int_0^t \sigma(Y_t^{(k)}) - \sigma(Y_t^{(k-1)})\mathrm{d}M_s^H\right|^2]$. Using the Ito lemma , we obtain

$$\begin{aligned} E[\left|\int_0^t \sigma(Y_t^{(k)}) - \sigma(Y_t^{(k-1)})\mathrm{d}M_s^H\right|^2] &\le (\lambda^2 + t^{2H-1})\int_0^t E[| \beta(Y_t^{(k)}) - \beta(Y_t^{(k-1)}) |^2]\mathrm{d}s \\ &\le (\lambda^2 + t^{2H-1})M_2\int_0^t E[| Y_t^{(k)} - Y_t^{(k-1)} |^2]\mathrm{d}s. \end{aligned} \tag{8}$$

The integral w.r.t. the Wiener process $\{B(t), t \geq 0\}$ is understood as the Ito integral, while that w.r.t. the process $\{B_H(t), t \geq 0\}$ as Wick integral. Now, putting together (6), (7), and (8), we have

$$E[\left|Y_t^{(k+1)} - Y_t^{(k)}\right|^2] \leq M \int_0^t E[|Y_t^{(k)} - Y_t^{(k-1)}|^2]\mathrm{d}s, \tag{9}$$

where $M = 2tM_1 + 2(\lambda^2 + t^{2H-1})M_2$.

Next we pay attention to $h_1 = E[\left|Y_t^{(1)} - v_0\right|^2]$. Taking $k = 0$ in (5), one obtains

$$h_1 = E[\left|\int_0^t \beta(v_0)\mathrm{d}s + \int_0^t \sigma(v_0)\mathrm{d}M_s^H\right|^2] = E[\left|\beta(v_0)t + \int_0^t \sigma(v_0)\mathrm{d}M_s^H\right|^2].$$

Now we use $(a+b)^n \leq 2^{n-1}(a^n + b^n)$ and Ito lemma to obtain

$$h_1 \leq 2t^2 E[\left|\beta(v_0)\right|^2] + 2(\lambda^2 t + t^{2H})E[\left|\sigma(v_0)\right|^2] \leq M_3 t,$$

where the constant M_3 depends on λ, C, T, $E[\left|\beta(Y_0)\right|^2]$ and $E[\left|\sigma(Y_0)\right|^2]$. Accordingly, by induction on k we obtain

$$h_{k+1} = E[\left|Y_t^{(k+1)} - Y_t^{(k)}\right|^2] \leq \frac{M_3 M^k t^{k+1}}{(k+1)!}.$$

Thus, the sequence $h_{k+1} = h_1 + \sum_{i=0}^{k}(h_{i+1} - h_i)$ is absolutely convergent with the L_2 norm. Hence, the existence is proved.

We now show that the solution of (3) is unique. Suppose $Y(t,\omega)$ and $Z(t,\omega)$ satisfy (3), $Y(0,\omega) = Y$ and $Z(0,\omega) = Z$. Therefore,

$$E[|Y(t,\omega) - Z(t,\omega)|^2] = E[|Y - Z + \int_0^t \beta(Y(s,\omega)) - \beta(Z(s,\omega))\mathrm{d}s + \int_0^t \sigma(Y(s,\omega)) - \sigma(Z(s,\omega))\mathrm{d}M_s^H|^2]$$

We may use Young's inequality to obtain

$$E[|Y(t,\omega) - Z(t,\omega)|^2]$$
$$\leq 3E[|Y - Z|^2] + 3E[\left(\int_0^t \beta(Y(s,\omega)) - \beta(Z(s,\omega))\mathrm{d}s\right)^2] + 3E[\left(\int_0^t \sigma(Y(s,\omega)) - \sigma(Z(s,\omega))\mathrm{d}M_s^H\right)^2]. \tag{10}$$

Following the similar proof of (7) and (8), we obtian

$$E[\left(\int_0^t \beta(Y(s,\omega)) - \beta(Z(s,\omega))\mathrm{d}s\right)^2] \leq tM_1 \int_0^t E[|Y(s,\omega) - Z(s,\omega)|^2]\mathrm{d}s, \tag{11}$$

$$E[\left(\int_0^t \sigma(Y(s,\omega)) - \sigma(Z(s,\omega))\mathrm{d}M_s^H\right)^2 \leq (\lambda^2 + t^{2H-1})M_2 \int_0^t E[|Y(s,\omega) - Z(s,\omega)|^2]\mathrm{d}s. \tag{12}$$

Substituting (11) and (12) into (10) and letting $M = 3tM_1 + 3(\lambda^2 + t^{2H-1})M_2$ yeild

$$E[|Y(t,\omega) - Z(t,\omega)|^2] \leq 3E[|Y - Z|^2] + M\int_0^t E[|Y(s,\omega) - Z(s,\omega)|^2]\mathrm{d}s.$$

Using Gronwall inequality, we have

$$E[|Y(t,\omega) - Z(t,\omega)|^2] \leq 3E[|Y - Z|^2]\exp\{At\}.$$

The uniqueness of solution can be proved using

$$Y = Y(0,\omega) = Z(0,\omega) = Z.$$

Consequently, the theorem is proved.□

Next, we will derive L_p estimate for the solution of the volatility equation under appropriate hypotheses on the functions of β and σ in which the restriction are weaker than that present in [10].

Assumption 1. There exists a positive function $M(t)\in L_2([0,T])$, such that for any $x\in R$, $\max\{|\beta(x)|,|\sigma(x)|\}\le M(t)(1+|x|)$.

Lemma 2.1. Assume that Assumption 1 holds, and Let $T>0$ be fixed. Then for any positive constant $C=C(x,T,p)$, we have

$$E_{S_0,i}[\sup_{t\in[0,T]}\left|X(t)\right|^p]\le C. \tag{13}$$

Proof: Let $\tau_N=\inf\{t\ge 0,|v(t)|>N\}$. Since

$$v(t\wedge\tau_N)=v_0+\int_0^{t\wedge\tau_N}\beta(v(s))\mathrm{d}s+\int_0^{t\wedge\tau_N}\sigma(v(s))\mathrm{d}M_s^H.$$

Using Young' s inequality, we have for any $p\ge 2$ that

$$\left|v(t)\right|^p\le 3^{p-1}(|v_0|^p+A_1+A_2), \tag{14}$$

where $A_1=\left|\int_0^{t\wedge\tau_N}\beta(v(s))\mathrm{d}s\right|^p$, $A_2=\left|\int_0^{t\wedge\tau_N}\sigma(v(s))\mathrm{d}M_s^H\right|^p$. Now, we compute A_1 and A_2. Using Holder inequality, and letting $|\mu_{\max}|=\max\{|\mu_1|,\ |\mu_2|\}$, we have

$$E[A_1]\le E[\sup_{t\in[0,T]}\left|\int_0^{t\wedge\tau_N}\mu(v(s))\mathrm{d}s\right|^p]\le E[\sup_{t\in[0,T]}\int_0^{t\wedge\tau_N}M(t)\left|1+|x|\right|^p\mathrm{d}s].$$

Note that for any $s\in[0,t\wedge\tau_N]$, $|v(s)|\le N$. Therefore,

$$E[A_1]\le E[\sup_{t\in[0,T]}\int_0^{t\wedge\tau_N}M(t)\left|1+N\right|^p\mathrm{d}s]. \tag{15}$$

Following similar proof which was performed for (14), we obtain

$$E_{S_0,i}[A_2]\le E[\sup_{t\in[0,T]}\int_0^{t\wedge\tau_N}M(t)^2\left|1+N\right|^{2p}\mathrm{d}s], \tag{16}$$

where $\sigma_{\max}=\max\{\sigma_1,\sigma_2\}$. Substituting (15) and (16) into (14), and letting

$$C_1=E[|v_0|^p]+E[\sup_{t\in[0,T]}\int_0^T M(t)\left|1+|x|\right|^p\mathrm{d}s]+E[\sup_{t\in[0,T]}\int_0^T M(t)^2\left|1+N\right|^{2p}\mathrm{d}s],$$

we obtain

$$\sup_{t\in[0,T]}E_{S_0,i}[\left|v(t\wedge\tau_N)\right|^p]\le 3^{p-1}C_1,\ p\ge 2. \tag{17}$$

Letting $N\to\infty$, by Fatou's lemma one finds that

$$\sup_{t\in[0,T]}E_{S_0,i}[\left|v(t)\right|^p]\le 3^{p-1}C_1,\ p\ge 2. \tag{18}$$

Second, we prove that (13) still holds for any $1\le p<2$.Using Cuachy inequality, obtains

$$E[\left|v(t)\right|^p]\le E[\left|v(t)\right|^{2p}]^{\frac{1}{2}}\le\left[\sup_{t\in[0,T]}E[\left|v(t)\right|^{2p}]\right]^{\frac{1}{2}}. \tag{19}$$

Note that $2p\ge 2$, and using (18) obtains

$E_{S_0,i}[|v(t)|^p] \le C(S_0, T, p)$.

Because $t \in [0,T]$ is arbitrary, (13) is proved when $1 \le p < 2$.

Finally, if $0 < p < 1$, note that

$$|v(t)|^p = |v(t)|^p I_{\{|v(t)|\ge 1\}} + |v(t)|^p I_{\{|v(t)|<1\}} \le |v(t)|^{p+1} I_{\{|v(t)|\ge 1\}} + |v(t)|^p I_{\{|v(t)|<1\}} .$$

Further we have

$$|v(t)|^p \le |v(t)|^{p+1} I_{\{|v(t)|\ge 1\}} + 1 \le |v(t)|^{p+1} + 1 .$$

Hence it follows from the case $0 < p < 1$

$$\sup_{t\in[0,T]} E[|v(t)|^p] \le C(v_0, T, p) + 1 .$$

This completes the proof of the lemma□

By following the proof of Theorem 2.1 and Lemma2.1 for Stock price equation can prove the following lemma.

Lemma 2.2. Stock price equation of CEV model has a unique solution In the case that $\beta(\cdot)$ and $\sigma(\cdot)$ satisfies Assumption 1, then

$$\sup_{t\in[0,T]} E[|S(t)|^p] \le C(v_0, S_0, T, p) .$$

3. Continuity

In this section, we are going to discuss the continuity to Stock price equation of CEV model.

Theorem 3.1. Stock price process of CEV model $\{S(t), t \ge 0\}$ is continuous.

Proof: Note that for any $0 \le s < t \le T$,

$$S(t) - S(s) = \int_s^t \mu S(s)\mathrm{d}s + \int_s^t v(s)S(s)^\alpha \mathrm{d}M_s^H .$$

Using Holder inequality

$$|S(t) - S(s)|^4 \le 2A_3 + 2^3 A_4 , \qquad (20)$$

where $A_3 = \left|\int_s^t \mu(\alpha(s))S(s)\mathrm{d}s\right|^4$, $A_4 = \left|\int_s^t \sigma(\alpha(s))S(s)\mathrm{d}w(s)\right|^4$. It follows by Cauchy inequality, $\left|\int_s^t S(s)\mathrm{d}s\right|^4 \le \left(\int_s^t |S(s)|^2 \mathrm{d}s\right)^2$. Therefore

$$E[A_3] \le |\mu|^4 E[\left|\int_s^t S(s)\mathrm{d}s\right|^4] \le |\mu|^4 E[(\int_s^t |S(s)|^2 \mathrm{d}s)^2] \le |\mu|^4 E[\int_s^t |S(s)|^2 \mathrm{d}s]^2 .$$

Using Cauchy inequality again, we obtain

$$E[A_3] \le |\mu|^4 \left(\int_s^t E[|S(s)|^2]\mathrm{d}s\right)^2 .$$

It follows by (19) that

$$E[A_3] \le |\mu|^4 C^2 |t-s|^2 . \qquad (21)$$

Now we pay attention to $E[A_4]$. Using Cauchy inequality, we obtain

$$E[A_4] \le E[\left|\int_s^t v(s)S(s)\alpha \mathrm{d}w(s)\right|^4] \le E[\left|\int_s^t S(s)\mathrm{d}w(s)\right|^2]^2 .$$

We may use Ito lemma and Holder inequality to arrive at

$$E[A_4] \le \left(\int_s^t E[v(s)^2 S(s)^{2\alpha}]\mathrm{d}s\right)^2 \le \left(\int_s^t \sqrt{E[v(s)^4]E[S(s)^{4\alpha}]}\mathrm{d}s\right)^2 .$$

It follows by (13) and (19) that

$$E[A_4] \le C^2 |t-s|^2 . \tag{22}$$

Substituting (21) and (22) into (20),yeilds

$$E[|S(t)-S(s)|^4] \le C^2 |t-s|^2 \quad . \tag{23}$$

Therefore, the theorem is proved.□

4. Option pricing

In this section, we consider the following European call option with

$$\text{terminal payoff } = \max\{S(T)-K\} .$$

Here K is the strike price and $S(T)$ is the terminal price of the underlying asset following extended Heston model

$$\mathrm{d}S(t) = rS(t)\mathrm{d}t + \sqrt{v(t)}S(t)\mathrm{d}B_1^H(t) , \tag{24}$$

$$\mathrm{d}v(t) = \kappa(\theta - v(t))\mathrm{d}t + \sigma_0\sqrt{v(t)}\mathrm{d}B_2^H(t) , \tag{25}$$

$$\mathrm{d}B_1^H(t)\cdot \mathrm{d}B_2^H(t) = \rho \mathrm{d}t^{2H} , \tag{26}$$

which is special case of model (1.1)-(1.3) if

$$\lambda = 0 , \beta(v(t)) = k(\theta - v(t)) , \quad \sigma(v(t)) = \sigma_0\sqrt{v(t)} .$$

From the risk-neutral valuation principle, the price of European call option at time t can be written as

$$c(t, S(t)) = \exp\{-r(T-t)\}E_{S(t)}[\max\{S(T)-K\}] ,$$

Now we are going to describe the time discretization of the SDE (24)-(25). First the time interval $[0,T]$ is divided into N time steps, with $\Delta t = T/N$ and $t_n = n\Delta t$, $n = 0,1,2,\cdots,N$. Let $\{S_n\}$ and $\{v_n\}$ be approximation of $\{S(t)\}$ and $\{v(t)\}$ at time level t_n respectively. The implementation of discretization to (24) and (25) produces

$$S_{n+1} = S_n + rS_n\Delta t + \sqrt{v_n}S_n\Delta B_n^{1,H} , S(0) = S_0 , \tag{27}$$

$$v_{n+1} = v_n + \kappa(\theta - v_n)\Delta t_n + \sigma_0\sqrt{v_n}\Delta B_n^{2,H} , v(0) = v_0 , \tag{28}$$

where $\Delta t_k = t_{k+1} - t_k$, $\Delta B_n^{1,H} = B_1^H(t_{n+1}) - B_1^H(t_n)$ and $\Delta B_n^{2,H} = B_2^H(t_{n+1}) - B_2^H(t_n)$.

Since the closed-form solution for the extended Heston model has not been found yet, we consider the numerical computations in this section. Thus, for European call

$$\hat{c}_M(t, S(t)) = \exp\{-r(T-t)\}\frac{1}{M}\sum_{k=1}^{M}\max\{S_N^{(k)} - K, 0\} , \tag{29}$$

where $S_N^{(k)}$ is the k th simulation of S_N .

Example 1. If $\kappa = 0 , \sigma_0 = 0$, the value of European call has the closed form

$$c(t, S(t)) = S(t)\Phi(d_1) - K\exp\{-r(T-t)\}\Phi(d_2) , \tag{30}$$

where

$$d_1 = \frac{\ln(S(T)) - \ln(K) + r(T-t) + \frac{1}{2}v_0(T^{2H} - t^{2H})}{\sqrt{v_0(T^{2H} - t^{2H})}}, \quad d_2 = d_1 - \sqrt{v_0(T^{2H} - t^{2H})}.$$

Here we compare the value of European call obtained using Scheme (27)-(29) with (30). Consider an European call with parameters $T = 1$, $t = 0.5$, $r = 0.05$, $S(t) = 110$, $K = 100$, $v_0 = 0.3$, $\lambda = 0$ and $H = 0.5$. Let $M \in \{100, 200, 300, \cdots, 10000\}$, the curve of the European call is plotted in Fig1 with $N = 20$. From the Fig1, we see that $\hat{c}_M(t, S(t))$ converges to $c(t, S(t))$ as $M \to \infty$. Let $S \in \{90, 91, \cdots, 150\}$, Fig2 shows accurate approximations for large numbers M (Here we set $M = 20000$) and the relationship between asset price S and the values of European call.

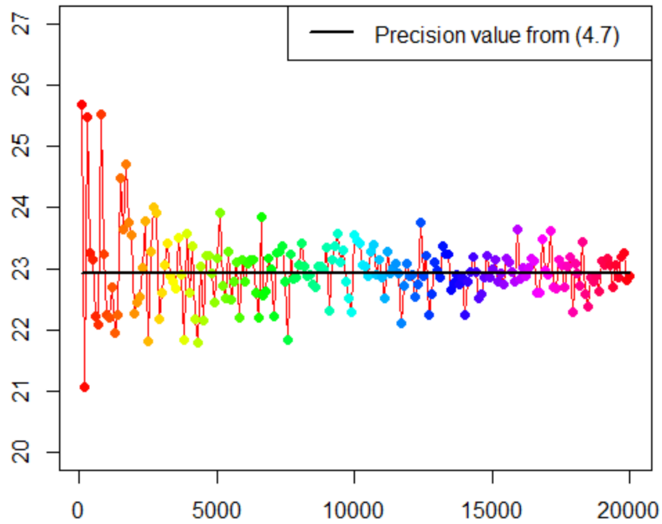

Figure 1: European call for different value of M

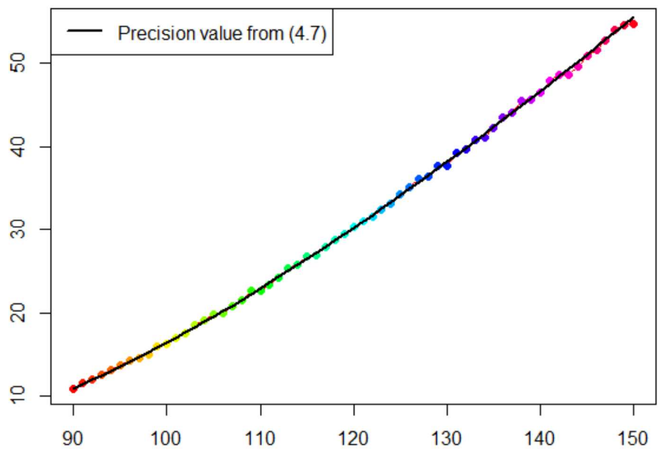

Figure 2: European call for different value of S

The next example shows the effect on values of European call is obvious by the volatility equation.

Example 2. Here the parameters for European call are $T=1$, $t=0.5$, $r=0.05$, $S(t)=110$, $K=100$, $v_0=0.3$, $\lambda=0$, $H=0.5$, $M=20000$ and $N=20$. Let $\theta\in\{0.05,0.1,0.15,\cdots,0.50\}$, Fig3 reports the relationship between θ and the values of European call with $\kappa=1,\sigma_0=0.1$. Further, the value of European call computed by (29) for different value of σ_0 is plotted in Fig 4.The relationship between the values of European call and σ_0 showed an U - shaped curve.

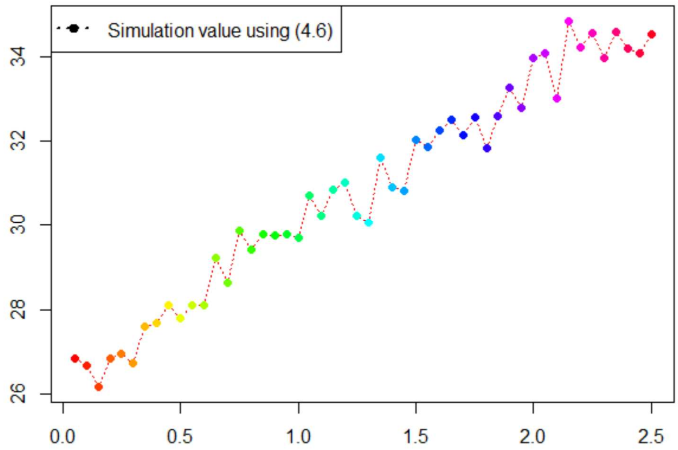

Figure 3: European call for different value of θ

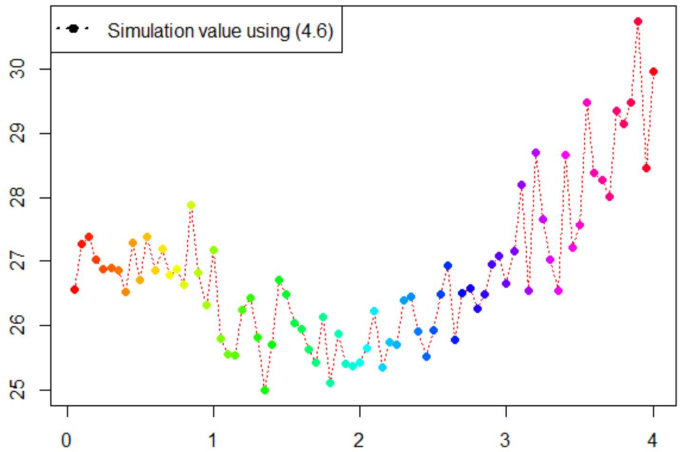

Figure 4: European call for different value of σ_0

REFERENCES

1. C.C.Lo, D.Nguyen and K.Skindilias, A unified tree approach for options pricing

under stochastic volatility models, Finance Research Letters, 20 (2017) 260-268.
2. G. Wang, X. Wang and K. Zhou, Pricing vulnerable options with stochastic volatility, Physica A, 485 (2017) 91-103.
3. R.Griego and A.Swishchuk, Black-Scholes formula for a market in Markov environment, Theory of Probability and Mathematical Statistics, 62 (2000) 9-18.
4. J. Hull and A. White, The pricing of options on assets with stochastic volatilities, The Journal of Finance, 42 (1987) 281-300.
5. K.Shafi, N.Latif, S.A.Shad, et al. Estimating option greeks under the stochastic volatility using simulation, Physica A, 503 (2018) 1288–1296.
6. D.K.Sakaria and J.E.Griffin, On efficient Bayesian inference for models with stochastic volatility, Econometrics and Statistics, 3 (2017) 23–33.
7. X.He and S.Zhu, An alternative form used to calibrate the Heston option pricing model, Computers and Mathematics with Applications, 71 (2016) 1831-1842.
8. M.Barczy, M.B.Alaya and A.Kebaier, et al., Asymptotic behavior of maximum likelihood estimators for a jump-type Heston model, Journal of Statistical Planning and Inference, 198 (2019) 139-164.
9. J.Kim, B.Kim, K.Moon, et al. Valuation of power options under Heston's stochastic volatility model, Journal of Economic Dynamics & Control, 36 (2012) 1796-1813.
10. F. Mehrdoust, A.R.Najafi, S.Fallah, et al. Mixed fractional Heston model and the pricing of American options, Journal of Computational and Applied Mathematics, 330 (2018) 141–154.
11. P. Cheridito. Arbitrage in fractional Brownian motion models, Finance Stochastic, 7 (4) (2003) 533-553.
12. P. Cheridito, Mixed fractional Brownian motion, Bernoulli,7 (6) (2001) 913–934.
13. F. Biagini, Y. Hu, B. Øksendal, et al., Stochastic Calculus for Fractional Brownian Motion and Applications, Springer, Berlin, 2008.