Journal of Physical Sciences, Vol. 25, 2020, 31-38
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 16 October 2020

Faculty Evaluation using Fuzzy Numbers

Md. Ashraful Alam, Sahidul Islam and M.K.Pandit

Department of Mathematics, Jahangirnagar University, Savar, Dhaka, Bangladesh
E-mail: ashraf_math20@juniv.edu

Received 8 June 2020; accepted 5 September 2020

ABSTRACT

This research work provides a method based on fuzzy numbers to evaluate a faculty member of a university associated with degrees of confidence of the evaluator (student). The satisfaction levels for each question of faculty evaluation form are represented by fuzzy numbers associated with degrees of confidence between zero and one. The arithmetic operations between the α–cuts of fuzzy numbers are used to evaluate the total points of each faculty.

Keywords: Fuzzy Numbers, α–cuts of Fuzzy Numbers, Satisfaction levels, Degrees of Confidence

Mathematical Subject Classification (2010): 94D05

1. Introduction

The quality of education is the lifeline of a university. The emphasis of improving the quality of education is mainly to improve the quality of teaching. Universities evaluate teachers to facilitate decisions about teacher's status and to help teachers to improve their performance. As institutional constraints and calls for increased accountability continue into the decade of the 1990s in universities, faculty evaluation programs need reexamining to see how they fit with institutional purposes of evaluation. An assessment of practices of evaluation also should help determine a program's effectiveness in promoting faculty development and productivity. To provide adequate and unbiased evaluation programs, administrators must involve faculty members in the process of determining the evaluation's purpose, as well as its scope, sources of data, participants, and assessment of effectiveness. A strong teacher evaluation system is central to improving teacher quality. It provides the means to recognize and reward great teachers so we can learn from and replicate their success. It also helps to identify those who need help so they can get the extra training they need to be effective. So the question is—how to find a scientific evaluation model to solve the problem. In order to decrease the subjectivity and randomness of the evaluation, this research work, presents an evaluation model for the faculty of university utilizing the fuzzy theory (Fuzzy Numbers).

2. Related work

In recent years, some methods have been presented for dealing with students' evaluation [5-12], [14-19]. In [5], Biswas presented a fuzzy evaluation method (fem) and a generalized fuzzy evaluation method (gfem) for applying fuzzy sets in student's answer scripts

evaluation. In [6], Chang and Sun presented a method for dealing with fuzzy assessment of learning performance of junior high school students. In [7], Chen *et al.* presented two methods for applying fuzzy sets in student's answer scripts evaluation. In [8], Cheng and Yang presented a method using fuzzy sets in education grading systems. In [9], Chiang and Lin presented a method for applying the fuzzy set theory for teaching assessment. In [10], Echauz and Vachtsevanos presented a fuzzy grading system. In [11], Frair presented a method for student peer evaluations based on the analytic hierarchy process (AHP) method. In [12], Kaburlasos *et al.* presented a software tool for computer-based testing and evaluation used in the Greek higher education system. In [14], Law built a structure model of a fuzzy education grading system and proposed an algorithm with it. He also presented a method to build membership functions of linguistic values with different weights. In [15], Ma *et al.* presented a fuzzy set approach for the assessment of student-centered learning. In [16], McMartin *et al.* used scenario assignments as assessment tools for undergraduate engineering education. In [17], Nolan presented an expert fuzzy classification system for supporting the grading of student writing samples. In [18], Pears *et al.* presented a method for student evaluation in an international collaborative project courses. In [19], Wu presented a method based on the fuzzy set theory and item response theory to evaluate learning performance. In [20], A.R. Khan et al discussed on the application of expert system with fuzzy logic in teachers 'performance evaluation. In this work, we present a methods for faculty evaluation using fuzzy numbers associated with degrees of confidence, where the satisfaction levels given by the evaluator awarded to the questions of the faculty evaluation form are represented by triangular fuzzy numbers associated with degrees of confidence between zero and one. The arithmetic operations between the α −cuts of fuzzy numbers are used to evaluate the total point of each faculty, where $\alpha \in [0, 1]$. The proposed methods can evaluate a faculty of university in a more flexible and more intelligent manner than the existing methods.

3. Literature review

In this section, we briefly review some basic definitions of fuzzy numbers and the arithmetic operations between α–cuts operations of fuzzy numbers from [13]. A fuzzy number is a fuzzy set in the universe of discourse *X* that is both convex and normal. A fuzzy number *A* of the universe of discourse *X* can be characterized by a triangular membership function parameterized by a triplet (a, b, c) as shown in Figure 1.

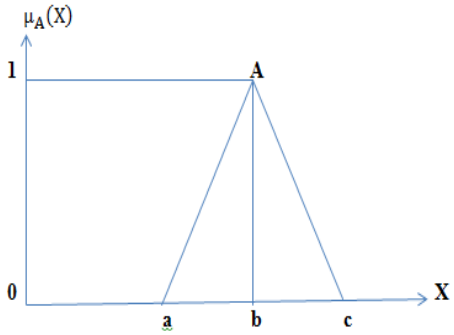

Figure 1: A fuzzy number A

The α–cut A_α of the fuzzy number *A* in the universe of discourse *X* is defined by

$$A_\alpha = \{x_i : \mu_A(x_i) \geq \alpha \; and \; x_i \in X\} = \left[{\alpha_1}^{(\alpha)}, {\alpha_2}^{(\alpha)}\right], \ldots \ldots \ldots \text{(i)}$$

where $\alpha \in [0,1], 0 \leq {\alpha_1}^{(\alpha)} \leq {\alpha_2}^{(\alpha)} \leq Max(X)$and Max($X$) denotes the maximum value in the universe of discourse X.

Let $\left[{\alpha_1}^{(\alpha)}, {\alpha_2}^{(\alpha)}\right]$ and $\left[{b_1}^{(\alpha)}, {b_2}^{(\alpha)}\right]$ be the α–cuts of the fuzzy numbers A and B, respectively, where $\alpha \in [0,1], 0 \leq {\alpha_1}^{(\alpha)} \leq {\alpha_2}^{(\alpha)} \leq Max(X), 0 \leq {b_1}^{(\alpha)} \leq {b_2}^{(\alpha)} \leq Max(X)$and Max($X$) denotes the maximum value in the universe of discourse X.

The arithmetic operations between the α–cuts $\left[{\alpha_1}^{(\alpha)}, {\alpha_2}^{(\alpha)}\right]$ and $\left[{b_1}^{(\alpha)}, {b_2}^{(\alpha)}\right]$ of the fuzzy numbers A and B, respectively, are defined as follows:
The addition between α–cuts$\left[{\alpha_1}^{(\alpha)}, {\alpha_2}^{(\alpha)}\right]$ and $\left[{b_1}^{(\alpha)}, {b_2}^{(\alpha)}\right]$:

$$\left[{\alpha_1}^{(\alpha)}, \; {\alpha_2}^{(\alpha)}\right] + \left[{b_1}^{(\alpha)}, \; {b_2}^{(\alpha)}\right] = \left[{\alpha_1}^{(\alpha)} + {b_1}^{(\alpha)}, {a_2}^{(\alpha)} + {b_2}^{(\alpha)}\right].$$

The multiplication between α–cuts$\left[{\alpha_1}^{(\alpha)}, {\alpha_2}^{(\alpha)}\right]$ and $\left[{b_1}^{(\alpha)}, {b_2}^{(\alpha)}\right]$:

$$\left[{\alpha_1}^{(\alpha)}, \; {\alpha_2}^{(\alpha)}\right] \times \left[{b_1}^{(\alpha)}, \; {b_2}^{(\alpha)}\right] = \left[{\alpha_1}^{(\alpha)} \times {b_1}^{(\alpha)}, \; {a_2}^{(\alpha)} \times {b_2}^{(\alpha)}\right].$$

4. Methodology and result analysis

In this section, we present a method for faculty evaluation using fuzzy numbers associated with degrees of confidence between zero and one, where six satisfaction levels are used to evaluate a faculty regarding a set of ten questions provided by the concern university, i.e., *Excellent* (E), *Very Good* (VG), *Good* (G), *Fair* (F), *Poor* (P) and *Extremely Poor* (EP). These six satisfaction levels are represented by triangular fuzzy numbers parameterized by the triples shown as follows:
Excellent = (100, 100, 100),
Very Good = (80, 90, 100),
Good = (70, 80, 90),
Fair = (30, 50, 70),
Poor = (0, 10, 30),
Extremely Poor = (0, 0, 0).

Table I shows a fuzzy grade sheet with satisfaction levels associated with degrees of confidence of a student between zero and one, where $F_1, F_2, F_3, \ldots \ldots \ldots F_n$ are satisfaction levels represented by triangular fuzzy numbers corresponding to the questions *Q.1*, *Q.2*, … , and *Q.n*, respectively, and the six satisfaction levels are used, i.e., *Excellent* (E), *Very Good* (VG), *Good* (G), *Fair* (F), *Poor* (P) and *Extremely Poor* (EP); α, β, …, and δ are the degrees of confidence of the satisfaction levels $F_1, F_2, F_3, \ldots \ldots \ldots F_n$ respectively, where α∈[0, 1], β∈[0, 1], …, and δ ∈ [0, 1]. It is obvious that the satisfaction level awarded to each Question *Q.i* shown in Table I is associated with a degree of confidence between zero and one, where $1 \leq i \leq n$. The larger the value, the higher the confidence of the student to give the satisfaction level regarding the answer to the question of faculty evaluation form.

Table 1: A faculty evaluation form with satisfaction levels associated with degrees of confidence

Question No.	**Satisfaction Levels**	**Degrees of Confidence of Satisfaction Levels**
Q.1	F_1	α
Q.2	F_2	β
⋮	⋮	⋮
Q.n	F_n	δ
Total Point =		
The Degree of Confidence of the Total Point =		

Consider the situation that the total point of a faculty evaluation form is 100. Assume that the faculty evaluation form contains *n* questions i.e.,
TOTAL POINTS = 100,
Q.1 carries$s_1$points,
Q.2 carries $s_2$points,

⋮
Q.n carries s_n points,
where $\sum_{i=1}^{n} s_i = 100, 0 \le s_i \le 100, 1 \le i \le n$

Assume that a student will evaluate a faculty as shown in Table I, where the satisfaction levels F_1, F_2, F_3, ... $F_n$are described by six satisfaction levels represented by triangular fuzzy numbers, α denotes the degree of confidence of the satisfaction level F_1awarded to the question *Q.1*, β denotes the degree of confidence of the satisfaction level F_2 awarded to the question *Q.2*, …, and δ denotes the degree of confidence of the satisfaction level $F_n$awarded to the question *Q.n*, where α ∈ [0, 1], β ∈ [0,1], …, and δ ∈ [0, 1]. Assume that an optimism index λ [8] determined by the student is used to indicate the degree of optimism of the student for evaluating a faculty, where λ∈[0, 1]. If 0 ≤λ< 0.5, then the evaluator (student) is a pessimistic evaluator. If λ= 0.5, then the evaluator (student) is a normal evaluator. If 0.5 <λ≤ 1.0, then the evaluator(student) is an optimistic evaluator. The method for faculty evaluation is now presented as follows:

Step 1: Calculate the α–cut $(F_1)\alpha$ of the fuzzy number F_1, the β–cut $(F_2)\beta$ of the fuzzy number F_2…, and the δ–cut (F_n) δ of the fuzzy number F_n, respectively, where $(F_1)\alpha = [a_1, a_2]$, $(F_2)\beta = [b_1, b_2]$, …, $(F_n)\delta = [z_1, z_2]$, α∈[0, 1], β∈[0,1], …, and δ∈[0, 1].

Step 2: Calculate the interval-valued total point $[m_1, m_2]$ which is given by students for a faculty, where

$$[m_1, m_2] = [\frac{s_1}{s_1 + s_2 + \cdots \ \ldots\ldots\ldots + s_n} \times (F_1)\alpha + \frac{s_2}{s_1 + s_2 + \cdots \ \ldots\ldots\ldots + s_n} \times (F_2)\alpha + \cdots \ldots\ldots\ldots + \frac{s_n}{s_1 + s_2 + \cdots \ \ldots\ldots\ldots + s_n} \times (F_n)\alpha]$$

$$= [\frac{s_1}{s_1 + s_2 + \cdots \ldots\ldots\ldots + s_n} \times [a_1, a_2] + \frac{s_2}{s_1 + s_2 + \cdots \ldots\ldots\ldots + s_n} \times [b_1, b_2] + \cdots \ldots\ldots\ldots + \frac{s_n}{s_1 + s_2 + \cdots \ldots\ldots\ldots + s_n} \times [z_1, z_2]\ ]$$

Step 3: The total point of a faculty is evaluated as follows:
$(1 - \lambda) \times m_1 + \lambda \times m_2$, where λ denotes the value of the optimism index which is determined by the students and $\lambda \in [0, 1]$. The degree of confidence of the total point awarded to the faculty is equal to Min (α,β,γ, …, δ), where Min $(\alpha,\beta,\gamma,\ldots,\delta) \in [0, 1]$. Put this total point and the degree of confidence in the appropriate box at the bottom of the faculty evaluation form.

In the following, we use an example to illustrate the faculty evaluation process.

Example 1. Consider the situation that total point of a faculty evaluation form is 100. Assume that in total there are ten questions to be responded by the students which are:
Q.1: Teaching procedure
Q.2: Classroom management
Q.3: Knowledge of subject matter
Q.4: Personal Characteristics
Q.5: Responsibility & Punctuality
Q.6: Explain difficult thing clearly
Q.7: Makes lessons interesting
Q.8: Returns homework in a timely manner
Q.9: Grades fairly
Q.10: Encourages students to speak up and be active in the class

TOTAL POINTS = 100,
Q.1 carries 10 marks,
Q.2 carries 10 marks,
Q.3 carries 10 marks,
Q.4 carries 5 marks,
Q.5 carries 10 marks,
Q.6 carries 10 marks,
Q.7 carries 15 marks,
Q.8 carries 5 marks,
Q.9 carries 10 marks,
Q.10 carries 15 marks.

Assume that an evaluator (student) fill up a faculty evaluation form as shown in Table II and assume that the optimism index λ of the evaluator is 0.70 (i.e., λ =0.70).

Table 2: A faculty evaluation form of example 1

Question No.	**Satisfaction Levels**	**Degrees of Confidence of Satisfaction Levels**
Q.1 (Teaching procedure)	Fair	0.75
Q.2 (Classroom management)	Good	0.9
Q.3 (Knowledge of subject matter)	Very Good	0.70
Q.4 (Personal Characteristics)	Poor	0.95
Q.5 (Responsibility & Punctuality)	Excellent	0.75
Q.6 (Explain difficult thing clearly)	Very Good	0.95
Q.7 (Makes lessons interesting)	Good	0.8
Q.8 (Returns homework in a timely manner)	Fair	0.9
Q.9 (Grades fairly)	Very Good	0.85
Q.10 (Encourages students to speak up and be active in the class)	Good	0.95
Total Point		
The Degree of Confidence of the Total Point		

[Step 1]
The 0.75-cut of the satisfaction level, Fair is [45, 55], the 0.9-cut (Good)is [79, 81], the 0.70-cut (*Very Good*) is [87, 93], the 0.95-cut (Poor) is [9.5, 11], the 0.75-cut (Excellent) is [100, 100], the 0.95-cut (*Very Good*) is [89.5, 90.5], the 0.8-cut (Good) is [78, 82], the 0.9-cut (Fair) is [48, 52], the 0.85-cut (*Very Good*) is [88.5, 91.5], and the 0.95 cut (*Good*) is [79.5, 80.5].

[Step 2] Based on formula, we can calculate the interval-valued total mark $[m_1, m_2]$of the faculty evaluation form, where

$$[m_1, m_2] = \frac{10}{100} \times (\text{Fair})_{0.75} + \frac{10}{100} \times (\text{Good})_{0.9} + \frac{10}{100} \times (\text{Very Good})_{0.70} + \frac{5}{100} \times (\text{Poor})_{0.95} + \frac{10}{100} \times (\text{Excellent})_{0.75} + \frac{10}{100} \times (\text{Very Good})_{0.95} + \frac{15}{100} \times (\text{Good})_{0.80} + \frac{5}{100} \times (\text{Fair})_{0.90} + \frac{10}{100} \times (\text{Very Good})_{0.85} + \frac{15}{100} \times (\text{Good})_{0.95}$$

= 0.1 × [45, 55] + 0.1 × [79, 81] + 0.1 × [87, 93] + 0.05× [9.5, 11]+0.1×[100, 100]+0.1×[89.5, 90.5]+0.15×[78, 82]+0.05×[48, 52]+0.1×[88.5, 91.5]+0.15×[79.5, 80.5]
= [4.5, 5.5] + [7.9, 8.1] + [8.7, 9.3] + [0.475, 0.55] + [10, 10] + [8.95, 9.05] + [11.7, 12.3] + [2.4, 2.6] + [8.85, 9.15] + [11.925, 12.075]
= [75.4, 78.625].

[Step 3] Since the value of the optimism index λ determined by the evaluator is 0.70 (i.e., λ= 0.70), based on formula, the total point of the faculty can be evaluated as follows:
(1 – 0.70) × 75.4 + 0.70 × 78.625
= 0.30 × 75.4 + 0.70 × 78.625
= 22.62 + 55.04 = 77.66

The degree of confidence of the total point is equal to Min (0.75, 0.9, 0.7, 0.95, 0.75, 0.95, 0.8, 0.9, 0.85, 0.95) = 0.70.

5. Conclusion

We have presented a method to evaluate faculty of university using fuzzy numbers associated with degrees of confidence, where the satisfaction levels given by the evaluator (students) awarded to the questions of the faculty evaluation form are represented by triangular fuzzy numbers associated with degrees of confidence between zero and one. The arithmetic operations between the α–cuts of fuzzy numbers are used to evaluate the total points of each faculty, where $\alpha \in [0, 1]$. The degree of confidence of the total points of each faculty also can be calculated by the method. The proposed methods can evaluate a faculty of university in a more flexible and more intelligent manner than the existing methods.

Acknowledgement. We would like to thank the anonymous referees for providing very helpful comments and suggestions. We are also immensely grateful to the Editor of this Journal for providing insight and expertise that greatly assisted the research.

REFERENCES

1. M.A.Alam, S.K.Saha and N.K.Mitra, Fuzzy cognitive map to analyze the impact of modern technology on Bangladeshi Handloom weavers, International Journal of Computer Applications, 174(5) (2017) 24-27.
2. M.A.Alam, M.A.Kahn, M.N.Khan and M.B.Hossain, Fuzzy relational equation based on triangular norms to detect the problems of Bangladeshi weavers, Journal of Physical Sciences, 21 (2016) 37-45.
3. M.A.Alam, Fuzzy cognitive maps approach to identify risk factors of diabetes, Journal of Physical Sciences, 22 (2017)13-21.
4. M.A.Alam and M.B.Hossain, Fuzzy cognitive maps to study the fact behind homelessness in Bangladesh, Journal of Physical Sciences, 23 (2018) 57-63.
5. R. Biswas, An application of fuzzy sets in student's evaluation, Fuzzy Sets and Systems, 74(2) (1995) 187-194.
6. D. F. Chang and C. M. Sun, Fuzzy assessment of learning performance of junior high school students, Proceedings of the 1993 First National Symposium on Fuzzy Theory and Applications, Hsinchu, Taiwan, Republic of China, (1993) 10-15.
7. S. M. Chen and C. H. Lee, New methods for students' evaluating using fuzzy sets, Fuzzy Sets and Systems, 104(2) (1999) 209-218.
8. C. H. Cheng and K. L. Yang, Using fuzzy sets in education grading system, Journal of Chinese Fuzzy Systems Association, 4(2) (1998) 81-89.
9. T .T. Chiang and C. M. Lin, Application of fuzzy theory to teaching assessment, Proceedings of the 1994 Second National Conference on Fuzzy Theory and Applications, Taipei, Taiwan, Republic of China, (1994) 92-97.
10. J. R. Echauz and G. J. Vachtsevanos, Fuzzy grading system, IEEE Transactions on Education, 38(2) (1995) 158-165.
11. L. Frair, Student peer evaluations using the analytic hierarchy process method, Proceedings of 1995 Frontiers in Education Conference, 2 (1995) 1-5.

12. V. G. Kaburlasos, C. C. Marinagi, and V. T. Tsoukalas, PARES: A software tool for computer-based testing and evaluation used in the Greek higher education system, Proceedings of the 2004 IEEE International Conference on Advanced Learning Technologies, (2004) 771-773.
13. A. Kaufmann and M. M. Gupta, Fuzzy Mathematical Models in Engineering and Management Science, Amsterdam: North-Holland, (1988).
14. C. K. Law, Using fuzzy numbers in education grading system, Fuzzy Sets and Systems, 83(3) (1996) 311-323.
15. J. Ma and D. Zhou, Fuzzy set approach to the assessment of student-centered learning, IEEE Transactions on Education, 43(2) (2000) 237-241.
16. F. McMartin, A. Mckenna, and K. Youssefi, Scenario assignments as assessment tools for undergraduate engineering education, IEEE Transactions on Education, 43(2) (2000) 111-119.
17. J. R. Nolan, An expert fuzzy classification system for supporting thegrading of student writing samples, Expert Systems with Applications, 15(1) (1998) 59-68.
18. A. Pears, M. Daniels, A. Berglund, and C. Erickson, Student evaluation in an international collaborative project course, Proceedings of the 2001 Symposium on Applications and the Internet Workshops, (2001) 74-79.
19. M. H. Wu, A Research on Applying Fuzzy Set Theory and Item Response Theory to Evaluate Learning Performance, Master Thesis, Department of Information Management, Chaoyang University of Technology, Wufeng, Taichung County, Taiwan, Republic of China, (2003).
20. A.R.Kahn, H.U. Amin and Z.U.Rehman, Application of expert system with fuzzy logic in teacher's performance evaluation, International Journal of Advanced Computer Science and Applications, 2(2) (2011).