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Employee's Bonus Distribution System: A Fuzzy Set Approach

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ABSTRACT

This work proposes two methods based on fuzzy set for distributing rewards (bonus) to the employee of a company considering their performances. Fuzzy reward distribution sheet for different criteria is developed here. For each criteria the degrees of confidence between zero and one can be found by a precise formula.

Keywords: Fuzzy set, fuzzy reward distribution sheet, weighted fuzzy reward distribution sheet, degrees of confidence

Mathematical Subject Classification (2010): 94D05

1. Introduction

Employee assessment in terms of their performance is an unavoidable element of organizational life [13, 17]. There are many decisions in modern organizations that depend on employee assessment, and they are widely used in most organizations [11-12, 14-15]. Employee assessment allows organizations to inform their employees about their rates of growth, their competencies, and their potentials. It enables employees to be intentional in creating their individual developmental goals to help in their personal growth. If used well, Employee assessment is an influential tool that organizations have to organize and coordinate the power of every employee of the organization towards the achievement of its strategic goals [16]. It can focus each employee's mind on the organization's mission, vision, and core values. However, if employee assessment is not done well, In [16] Grote suggests the process can become the object of jokes and the target of ridicule.

To overcome these limitations, we propose two scientific methods based on fuzzy sets to distribute rewards to the employee. Here, we set five performance levels: Excellent (E), Very Good (VG), Good (G), Fair (F) and Poor (P). The number of level can be modified by the authority of any organization according to their convenience. In our proposed method, an employee will get reward consequent upon their percentage of performance. Another intension of this research is to examine the parameter affecting employee performance satisfaction in the public sector. Some other uses of fuzzy sets are cited in [18-22].

2. Existing work

In recent years, some methods have been presented for dealing with the assessment procedure of employee of different organizations and student's evaluation [11-17], [5-10].

In [5], Biswas presented a fuzzy evaluation method (fem) and a generalized fuzzy evaluation method (gfem) for applying fuzzy sets in student's answer scripts evaluation. In [6], Chang and Sun presented a method for dealing with fuzzy assessment of learning performance of junior high school students. In [7], Chen *et al.* presented two methods for applying fuzzy sets in student's answer scripts evaluation. In [8], Cheng and Yang presented a method using fuzzy sets in education grading systems. In [9], Chiang and Lin presented a method for applying the fuzzy set theory for teaching assessment. In [10], Echauz and Vachtsevanos presented a fuzzy grading system. In [11], Denisi talked about a cognitive approach to performance appraisal. In [12], Burkhaulter and Buford Jr., discussed on Performance appraisal: Concepts and techniques for post secondary education. In [13], Longeneckerand Fink dealt with creating effective performance appraisals. In [14], Wanguari, presented a review, integration, and a critique of cross-disciplinary research on performance appraisals, evaluations, and feedback. In [15], Davis worked on approaches to performance appraisal in student affairs. In [16], Grote discussed about the performance appraisal question and answer book, actually a survival guide for managers. In [17], Brown focused on performance appraisal as a tool for staff development.

In this research work, we propose two methods to assess employee of different organizations by giving them (employee) bonus or reward according to their performance that will be measured by the tools of fuzzy set.

3. Method for allocating employee's bonus using fuzzy set

In this section, we present a new method for distributing rewards to the employee based on their monthly performance. Assume a market based company decides to give reward to it's employee. The company sets five performance levels to distribute the rewards to the employee.

Let X be a set of performance level, X={Excellent (E), Very Good (VG), Good (G), Fair (F), Poor (P)} and let T be a mapping function which maps a performance level to the maximum degree of confidence of the corresponding performance level, where $T: X \rightarrow [0,1]$.

Table 1: Performance level, percentage of performance and their corresponding degree of confidence

Performance levels	Percentage of performance	Degree of confidence
Excellent (E)	95% - 100%	1.00
Very Good (VG)	81% - 94%	0.9
Good (G)	61% - 80%	0.8
Fair (F)	41% - 60%	0.7
Poor (P)	1% - 40%	0

From Table 1, we can see that

T(Excellent)=1.00 i.e.,T(E)=1.00
T(Very Good)=0.9 i.e., T(VG)=0.9
T(Good)=0.8 i.e., T(G)=0.8
T(Fair)=0.7 i.e.,T(F)=0.7
T(Poor)=0 i.e., T(P)=0 (1)

Again, the company offer rewards to the employees based on their performance regarding some criteria, say, there are n criteria.

Table 2: A fuzzy reward distribution sheet

Criteria	Performance levels					Degree of confidence
	E	VG	G	F	P	
C_1						
C_2						
C_3						
C_4						
. .	. .	. .	. .	. .	. .	. .
C_n						
						Total reward=

Table II describes about a fuzzy reward distribution sheet which is a matrix type structure containing n rows and 7 columns, where n is the number of criteria set by the company to observe employee's performance. At the bottom of the sheet there is a box for total reward. The first column reveals the criteria ; in any row the columns from second to sixth shows the fuzzy reward corresponding to each criteria in the first column, where the fuzzy reward is represented as a fuzzy set in the universe of discourse X, X={Excellent (E), Very Good (VG), Good (G), Fair (F), Poor (P)}. The last column shows the degree of confidence.

For example, assume that a company is using a fuzzy reward distribution sheet to give reward to an employee for the first criteria. From Table III we can see that the performance level regarding the first criteria of an employee is represented by a fuzzy set $F(C_1)$ of the universe of discourse X, where X={Excellent (E), Very Good (VG), Good (G), Fair (F), Poor (P)} and $F(C_1)$= {(E,0), (VG, 0.93), (G, 0), (F, 0), (P, 0)}. It can be written as $F(C_1)$= {(VG, 0.93)}.

It indicates that the performance level of an employee regarding to the first criteria is 93% very good.

Table 3: An example of a fuzzy reward distribution sheet

Criteria	Performance levels					Degree of confidence
	E	VG	G	F	P	
C_1	0	0.93	0	0	0	
. .	. .	. .	. .	. .	. .	. .

The method for giving the reward to an employee is now presented as follows:

Step 1: Assume that a company is using a fuzzy reward distribution sheet to give reward y_i to an employee for the i^{th} criteriaC_j, which is shown in Table 4, where $1 \leq i \leq 5$ and

$1 \leq j \leq n$. From formula (1) , we get T(E)=1.00, T(VG)=0.9, T(G)=0.8, T(F)=0.7 and T(P)=0. In this case, the degree of performance $D(C_j)$ of the criteria C_j can be evaluated by the function D.

$$D(C_j) = \frac{y_1 * T(\mathrm{E}) + \mathrm{y}_2 * \mathrm{T(VG)} + \mathrm{y}_3 * \mathrm{T(G)} + \mathrm{y}_4 * \mathrm{T(F)} + \mathrm{y}_5 * \mathrm{T(P)}}{y_1 + y_2 + y_3 + y_4 + y_5} \qquad (2)$$

where $D(C_j) \in [0,1]$.

Table 4: Fuzzy reward distribution sheet for criteria C_j

Criteria	Performance levels					Degree of performance
	E	VG	G	F	P	
.	.	.	.	.	.	.
.	.	.	.	.	.	.
C_j	y_1	y_2	y_3	y_4	y_5	$D(C_j)$
.	.	.	.	.	.	.
.	.	.	.	.	.	.

For example let us consider the example shown in Table 3. From formula (1) , we get T(E)=1.00, T(VG)=0.9, T(G)=0.8, T(F)=0.7 and T(P)=0. By applying formula (2) the degree of performance for criteria C_1 can be evaluated as

$$D(C_1) = \frac{0 * 1 + 0.93 * 0.9 + 0 * 0.8 + 0 * 0.7 + 0 * 0}{0 + 0.93 + 0 + 0 + 0} = 0.9$$

Step 2: Consider a total amount of reward offered by the company to an employee for his performance regarding n criteria. Assume that the total amount of reward offered by the company to an employee for his performance regarding n criteria be R Tk. i.e.,
Total reward= R Tk.
Assume that,
C_1 carries r_1 Tk.
C_2 carries r_2 Tk.
.
.
C_n carries r_n Tk.
where $\sum_{j=1}^{n} r_j = R$, $0 \leq r_j \leq R$ and $1 \leq j \leq n$. Assume that the degree of performance of the criteria C_1, C_2, ……, C_n are $D(C_1)$, $D(C_2)$, ……, $D(C_n)$ respectively. Then the total amount of reward attained by an employee based on his performance regarding n criteria be as follows:

$$r_1 * D(C_1) + r_2 * D(C_2) + \ \ldots\ldots\ldots + r_n * D(C_n) \qquad (3)$$

Example 1. Assume that a market based company declares 50,000 Tk. as monthly reward based on an employee's monthly performance. Assume that the company selects it's employee based on the following five criteria:
C_1: Punctuality
C_2: Sincerity
C_3: Efficiency
C_4: Monthly sell of the product

C_5: Supervision of the market
Suppose each criteria carry equal reward which is equal to 10,000 Tk..

Table 5: Fuzzy reward distribution sheet for different criteria

Criteria	Performance levels					Degree of performance
	E	VG	G	F	P	
C_1	0	0.93	0	0	0	$D(C_1) = 0.9$
C_2	0.97	0	0	0	0	$D(C_2) = 1$
C_3	0	0	0.75	0	0	$D(C_3) = 0.8$
C_4	1	0	0	0	0	$D(C_4) = 1$
C_5	0	0	0	0	0.40	$D(C_5) = 0$

Step. 1: Based on formula (1) and by applying formula (2), we get

$$D(C_1) = \frac{0*1+0.93*0.9+0*0.8+0*0.7+0*0}{0+0.93+0+0+0} = 0.9$$

$$D(C_2) = \frac{0.97*1+0*0.9+0*0.8+0*0.7+0*0}{0.97+0+0+0+0} = 1$$

$$D(C_3) = \frac{0*1+0*0.9+0.75*0.8+0*0.7+0*0}{0+0+0.75+0+0} = 0.8$$

$$D(C_4) = \frac{1*1+0*0.9+0*0.8+0*0.7+0*0}{1+0+0+0+0} = 1$$

$$D(C_5) = \frac{0*1+0*0.9+0*0.8+0*0.7+0.40*0}{0+0+0+0+0.40} = 0$$

Step. 2: By applying formula (3), the total amount of reward (r) attained by an employee based on his performance regarding five criteria as follows:

$$r = 10000*0.9+10000*1+10000*0.8+10000*1+10000*0$$

$$r = 37000$$

Hence, total amount of reward attained by an employee in a month is 37,000 Tk.

4. A weighted method for allocating employee's bonus using fuzzy set

In this section, we present a new weighted method for distributing rewards to the employee based on their on their monthly performance. Assume a market based company decides to give reward to it's employee. The company sets five performance levels to distribute the rewards to the employee.

Let X be a set of performance level, X={Excellent (E), Very Good (VG), Good (G), Fair (F), Poor (P)} and let T be a mapping function which maps a performance level to the maximum degree of performance of the corresponding performance level, where $T: X \rightarrow [0,1]$.

To observe an employee's performance more closely, the company decides to divide a month into four weeks W_1, W_2, W_3, W_4.

Table 6: Weighted fuzzy reward distribution sheet

Week	Criteria	Performance Level					Degree of Performance for Criteria	Degree of Performance for weeks
		E	VG	G	F	P		
W_1	C_1						$D(C_{11})$	$P(W_1)$
	C_2						$D(C_{12})$	
	C_3						$D(C_{13})$	
	. .							
	C_n						$D(C_{1n})$	
W_2	C_1						$D(C_{21})$	$P(W_2)$
	C_2						$D(C_{22})$	
	C_3						$D(C_{23})$	
	. .							
	C_n						$D(C_{2n})$	
W_3	C_1						$D(C_{31})$	$P(W_3)$
	C_2						$D(C_{32})$	
	C_3						$D(C_{33})$	
	. .							
	C_n						$D(C_{3n})$	
W_4	C_1						$D(C_{41})$	$P(W_4)$
	C_2						$D(C_{42})$	
	C_3						$D(C_{43})$	
	. .							
	C_n						$D(C_{4n})$	
	Total reward= $r_1 * P(W_1) + r_2 * P(W_2) + r_3 * P(W_3) + r_4 * P(W_4)$							

Step.1: Assume total amount of reward offered by a company be R Tk..
Amount of reward for the first week, $W_1 = r_1$Tk.
Amount of reward for the second week, $W_2 = r_2$Tk.
Amount of reward for the third week, $W_3 = r_3$Tk.
Amount of reward for the fourth week, $W_4 = r_4$Tk.
Here, $\sum_{i=1}^{4} r_i = R$ and $0 \leq r_i \leq R$.
There are n criteria set by the company to evaluate an employee's performance to give him performance reward. The weight of the criteria C_1 , C_2 , C_3 , ……., C_n are $w_1, w_2, w_3, \ldots\ldots\ldots w_n$ respectively, where $w_j \in [0,1]$ and $1 \leq j \leq n$. The degree of performance for criteria C_1, C_2,C_3, ……., C_n are $D(C_{i1}), D(C_{i2}), D(C_{i3}), \ldots\ldots, D(C_{in})$ respectively and $0 \leq D(C_{ij}) \leq 1$ where $1 \leq i \leq 4$ and $1 \leq j \leq n$.
Step. 2: The degree of performance $P(W_i)$for weeks W_i of an employee's performance

can be evaluated as follows:

$$P(W_i) = \frac{w_1 * D(C_{i1}) + w_2 * D(C_{i2}) + w_3 * D(C_{i3}) + \cdots + w_n * D(C_{in})}{w_1 + w_2 + w_3 + \cdots + w_n} \quad (4)$$

where $P(W_i) \in [0,1]$

The total reward attained by an employee can be evaluated as follows: $r_1 * P(W_1) + r_2 * P(W_2) + r_3 * P(W_3) + r_4 * P(W_4)$

Example 2. Assume that a market based company declares 50,000 Tk. as monthly reward based on an employee's monthly performance. Assume that the company selects it's employee based on the following five criteria:

C_1: Punctuality
C_2: Sincerity
C_3: Efficiency
C_4: Monthly sell of the product
C_5: Supervision of the market

Suppose that the weight of each criteria be equal, i.e., $w_1 = w_2 = w_3 = w_4 = w_5 = 0.2$. Since there are four weeks, the amount of reward allocated for each week be $r_1 = r_2 = r_3 = r_4 = 10000$Tk.

Table 7: Weighted fuzzy reward distribution sheet for particular data

Week	Criteria	Performance Level					Degree of performance for criteria	Degree of performance for weeks
		E	VG	G	F	P		
W_1	C_1	0	0.93	0	0	0	$D(C_{11})$	$P(W_1)$
	C_2	0.97	0	0	0	0	$D(C_{12})$	
	C_3	0	0	0.75	0	0	$D(C_{13})$	
	C_4	1	0	0	0	0	$D(C_{14})$	
	C_5	0	0	0	0	0.40	$D(C_{15})$	
W_2	C_1	1	0	0	0	0	$D(C_{21})$	$P(W_2)$
	C_2	0	0	0	0	0.35	$D(C_{22})$	
	C_3	0	0	0	0.50	0	$D(C_{23})$	
	C_4	0	0.90	0	0	0	$D(C_{24})$	
	C_5	0	0	0.80	0	0	$D(C_{25})$	
W_3	C_1	0	0	0	0	0.30	$D(C_{31})$	$P(W_3)$
	C_2	0.96	0	0	0	0	$D(C_{32})$	
	C_3	0	0.89	0	0	0	$D(C_{33})$	
	C_4	0	0	0.76	0	0	$D(C_{34})$	
	C_5	0	0	0	0.57	0	$D(C_{35})$	
W_4	C_1	1	0	0	0	0	$D(C_{41})$	$P(W_4)$
	C_2	0.98	0	0	0	0	$D(C_{42})$	
	C_3	0	0.91	0	0	0	$D(C_{43})$	
	C_4	0	0.82	0	0	0	$D(C_{44})$	
	C_5	0.97	0	0	0	0	$D(C_{45})$	

Degree of performance for first week:

$$D(C_{11}) = \frac{0.93 * 0.9}{0.93} = 0.9$$
$$D(C_{12}) = \frac{0.97 * 1}{0.97} = 1$$
$$D(C_{13}) = \frac{0.75 * 0.8}{0.75} = 0.8$$
$$D(C_{14}) = \frac{1 * 1}{1} = 1$$
$$D(C_{15}) = \frac{0.40 * 0}{0.40} = 0$$
$$P(W_1) = \frac{0.2(0.9 + 1 + 0.8 + 1 + 0)}{0.2 + 0.2 + 0.2 + 0.2 + 0.2} = 0.75$$

Degree of performance for second week:

$$D(C_{21}) = \frac{1 * 1}{1} = 1$$
$$D(C_{22}) = \frac{0.35 * 0}{0.35} = 0$$
$$D(C_{23}) = \frac{0.5 * 0.7}{0.5} = 0.7$$
$$D(C_{24}) = \frac{0.9 * 0.9}{0.9} = 0.9$$
$$D(C_{25}) = \frac{0.80 * 0.80}{0.80} = 0.8$$
$$P(W_2) = \frac{0.2(1 + 0 + 0.7 + 0.9 + 0.8)}{0.2 + 0.2 + 0.2 + 0.2 + 0.2} = 0.68$$

Degree of performance for third week:

$$D(C_{31}) = \frac{0.3 * 0}{0.3} = 0$$
$$D(C_{32}) = \frac{0.96 * 1}{0.96} = 1$$
$$D(C_{33}) = \frac{0.89 * 0.9}{0.89} = 0.9$$
$$D(C_{34}) = \frac{0.76 * 0.8}{0.76} = 0.8$$
$$D(C_{35}) = \frac{0.57 * 0.70}{0.57} = 0.7$$
$$P(W_3) = \frac{0.2(0 + 1 + 0.9 + 0.8 + 0.7)}{0.2 + 0.2 + 0.2 + 0.2 + 0.2} = 0.68$$

Degree of performance for fourth week:

$$D(C_{41}) = \frac{1 * 1}{1} = 1$$
$$D(C_{42}) = \frac{0.98 * 1}{0.98} = 1$$

$$D(C_{43}) = \frac{0.91 * 0.9}{0.91} = 0.9$$
$$D(C_{44}) = \frac{0.82 * 0.9}{0.82} = 0.9$$
$$D(C_{45}) = \frac{0.97 * 1}{0.97} = 1$$
$$P(W_4) = \frac{0.2(1 + 1 + 0.9 + 0.9 + 1)}{0.2 + 0.2 + 0.2 + 0.2 + 0.2} = 0.96$$

The total reward attained by an employee
=10000*0.75+10000*0.68+10000*0.68+10000*0.96
=30700 Tk.

5. Conclusion

The employee assessment procedures introduced here, give a numerical figure as indicator of an employee's performance. So, judgment will be fair and there is no chance for anyone to be deprived as well as for someone to get undue facilities. Strength and weakness of an employee for different criteria, can be identified easily by the proposed methods. So, it will be helpful for the authority to manage and control the organizations by taking necessary action regarding these.

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