Chapter 9

A Multi-Objective Solid Transportation Problem with Discount and Two-level Fuzzy Programming Technique

9.1 Introduction

In the third world countries like India, China, Nepal, etc, where there is a different competition amongst the companies to win over the maximum possible market, it is common practice of the transportation managers is that, the companies offer discount to the users on the unit transportation cost for the orders of more quantities. Such all units discount policy already have been introduced in different area of Operations Research Management very few researchs Ojha et al used this conception on TP or STP. In today's life, any decisions maker is not concious about a single criteria. The decision maker related to transportation system also thinks about multicriteria systems consideration of total cost is a long established criteria for STP or TP. Jain and Saksena [61] assume a transportation problem with time minimizing criteria. Chakraborty and Chakraborty [18] solved a time-cost transportation problem numerically.

It is often difficult to estimate the accurate values of transportation cost, delivery time, quantity of goods delivered, demands, availabilities, the capacity of different modes of transport between origins to destinations, etc. Depending upon different aspects, these fluctuate due to uncertainty in judgement, lack of evidence, insufficient information, etc. i.e., it is not possible to get relevant precise data, which are assumed by several researchers cf. Shell [136],Giri. et al. [44] proposed a STP with fuzzy random costs and constraints. So, a transportation model becomes more realistic if these parameters are assumed to be flexible / imprecise in nature i.e., uncertain in non-stochastic sense and may be represented by fuzzy numbers. Bit et al. [14], Jimenez and Verdegay [64], Li and Lai [79], Gen et al. [41], Kundu et al. [74], Meiyi et al. [95], Waiel [143] and others presented the fuzzy compromise programming approach to multi-objective TPs and fuzzy TPs. Based on Chanas and Kuchta [19], Das et al. [28] converted the interval number TPs into deterministic multi-objective problems and proposed a method using extension principle to solve a fuzzy stochastic two dimensional transportation problem.

In this chapter, a multi-criterium solid transportation problem with a discount on costs has been formulated as a linear programming problem. Here, the discount on transportation costs depending upon the transported amount are taken in the form of AUD. Obviously, when the cost coefficients or the supply and demand quantities are fuzzy in nature, the total transportation cost will also be fuzzy as well. In this model, we develop a solution procedure that is able to calculate the fuzzy objective value of the fuzzy transportation problem. The idea is to apply the Zadeh's extension principle to solve the proposed fuzzy solid transportation problem. In this problem, two objective functions corresponding to two criteria such as time and cost of transported amount from source to destination via conveyance have been optimized by $LINGO - 9.0$. This multi-objective solid transportation problem has been solved using Weighted Average Method. Finally, the proposed model has been illustrated using a numerical example.

9.2 Notations and Assumptions

9.2.1 Notations

In this solid transportation problem, instead of common notations the following additional notations have been used.

(i) T : Total time taken for the item to be transportated from $i-th$ origin to $j-th$ destination by $k-th$ conveyance.

9.2.2 Assumptions

In this solid transportation problem, the following assumptions are made.

(i) Here the problem is fully constracted with fuzzy parameters i.e, (a) the cost coefficient ($\widetilde{C}_{ijk}$), (b) the availability the suply of source ($\widetilde{a}_i$) (c) the demand of the destination ($\widetilde{b}_j$) (d) the capacity of the convayance ($\widetilde{e}_k$) all are impresize in nature.

(ii) The model is formulated under the consideration of multi objective problems in term of total cost and total time.

(iii) The unit transportation cost has an oppourtunity of all unit discount (AUD) facility.

(iv) A bi-level fuzzy programming technique has been considered to solve the problem.

9.3 Mathematical Formulation of Fuzzy Solid Transportation Problem

A classical solid transportation problem may be considered with M supply nodes (origin), N demand nodes (destination) and K types of conveyance, where all STP parameters are fuzzy in nature and followed traditional STP is in the following form

$$\begin{aligned}
& x_{ijk} \geq 0, \qquad \forall i, j, k. \\
& \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_i, \qquad i = 1, 2, ...M \\
& \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq b_j, \qquad j = 1, 2......N \\
& \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_k, \qquad k = 1, 2, ...K \\
& \sum_{i=1}^{M} a_i = \sum_{j=1}^{N} b_j = \sum_{k=1}^{K} e_j \\
& \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} C_{ijk}\ x_{ijk} \text{ is minimum}
\end{aligned} \tag{9.1}$$

In this model, we consider the resources a_i, b_j and e_k as a fuzzy quantity i.e., $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$. The unit transportation costs are not only fuzzy amount but it is also decided through a discount (AUD) policy, i.e.,

$$\tilde{C}_{ijk} = \begin{cases} \tilde{C}^1_{ijk} & if \;\; 0 < x_{ijk} < R_1 \\ \tilde{C}^2_{ijk} & if \;\; R_1 < x_{ijk} < R_2 \\ .... & .... \\ \tilde{C}^l_{ijk} & if \;\; R_{l-1} \leq x_{ijk} < \infty \end{cases} \tag{9.2}$$

where $\tilde{C}^1_{ijk} > \tilde{C}^2_{ijk} > .... > \tilde{C}^l_{ijk}$.

Here, the decision maker also introduces the fact of “time” in addition with “cost”. Hence, the problem is to determine x_{ijk} and t_{ijk} which is written as

$$\text{Minimize } \widetilde{Z}_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \tilde{C}_{ijk}\ x_{ijk} \tag{9.3}$$

$$\text{Minimize } Z_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} t_{ijk} y_{ijk}(x_{ijk}) \tag{9.4}$$

subject to constraints

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i, \qquad i = 1, 2, ...M$$
$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \tilde{b}_j, \qquad j = 1, 2......N \tag{9.5}$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \tilde{e}_k, \qquad k = 1, 2, ...K$$
$$\sum_{i=1}^{M} \tilde{a}_i = \sum_{j=1}^{N} \tilde{b}_j = \sum_{k=1}^{K} \tilde{e}_k$$
$$\text{and } x_{ijk} \geqq 0, \qquad \forall i, j, k.$$

where

$$y_{ijk} = \begin{cases} 1 & if \;\; x_{ijk} > 0 \\ 0 & if \;\; x_{ijk} = 0 \end{cases} \tag{9.6}$$

and the costs $\tilde{C}_{ijk}$ follows the policy (9.2)

9.4 Two Level Fuzzy Mathematical Programming

To find the solution space of (9.5), (9.6) for the optimization of (9.3), (9.4), a solution procedure "two level fuzzy mathematical programming technique" has been modified. It is supposed that the membership functions of unit shipping cost $\widetilde{C}_{ijk}$, supply $\widetilde{a}_i$, demand $\widetilde{b}_j$ and conveyance $\widetilde{e}_k$ are denoted by $\mu_{\widetilde{C}_{ijk}}, \mu_{\widetilde{a}_i}, \mu_{\widetilde{b}_j}, \mu_{\widetilde{e}_k}$ respectively. Now these fuzzy numbers can be written as

$$\widetilde{C}_{ijk} = \{(C_{ijk}, \mu_{\widetilde{C}_{ijk}}(C_{ijk})) \mid C_{ijk} \in S(\widetilde{C}_{ijk})\}$$
$$\widetilde{a}_i = \{(a_i, \mu_{\widetilde{a}_i}(a_i)) \mid a_i \in S(\widetilde{a}_i)\}$$
$$\widetilde{b}_j = \{(b_j, \mu_{\widetilde{b}_j}(b_j)) \mid b_j \in S(\widetilde{b}_j)\}$$
$$\widetilde{e}_k = \{(e_k, \mu_{\widetilde{e}_k}(e_k)) \mid e_k \in S(\widetilde{e}_k)\}$$

where $S(\widetilde{C}_{ijk})$, $S(\widetilde{a}_i)$, $S(\widetilde{b}_j)$ and $S(\widetilde{e}_k)$ are the supports of $\widetilde{C}_{ijk}$, $\widetilde{a}_i$, $\widetilde{b}_j$ and $\widetilde{e}_k$.

Now, the respective α-cuts of the fuzzy numbers $\tilde{C}_{ijk}$, $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$ are defined as:

$$C^{\alpha}_{ijk} = [C^{\alpha}_{ijkL}, C^{\alpha}_{ijkU}] = [min\{C_{ijk} \in S(\tilde{C}_{ijk})\}, max\{C_{ijk} \in S(\tilde{C}_{ijk})\}]$$
$$\text{where } \mu_{\tilde{C}_{ijk}}(C_{ijk}) \geq \alpha$$
$$a^{\alpha}_{i} = [a^{\alpha}_{iL}, a^{\alpha}_{iU}] = [min\{a_i \in S(\tilde{a}_i)\}, max\{a_i \in S(\tilde{a}_i)\}] \text{ where } \mu_{\tilde{a}_i}(a_i) \geq \alpha$$
$$b^{\alpha}_{j} = [b^{\alpha}_{jL}, b^{\alpha}_{jU}] = [min\{b_j \in S(\tilde{b}_j)\}, max\{b_j \in S(\tilde{b}_j)\}] \text{ where } \mu_{\tilde{b}_j}(b_j) \geq \alpha$$
$$e^{\alpha}_{k} = [e^{\alpha}_{kL}, e^{\alpha}_{kU}] = [min\{e_k \in S(\tilde{e}_k)\}, max\{e_k \in S(\tilde{e}_k)\}] \text{ where } \mu_{\tilde{e}_k}(e_k) \geq \alpha$$

These intervals indicate where the unit shipping cost, supply, demand and conveyance lie with possibility level α. Now we are interested to find the membership function and the α-cut of the total transportation cost $\widetilde{Z}_1$. Based on the Zadeh's extension principle, the membership function $\mu_{\widetilde{Z}_1}$ can be defined as follows:

$$\mu_{\widetilde{Z}_1}(z) = \sup\left\{ \min \{\mu_{\tilde{C}_{ijk}}(C_{ijk}), \mu_{\tilde{a}_i}(a_i), \mu_{\tilde{b}_j}(b_j), \mu_{\tilde{e}_k}(e_k) \forall i,j,k\} \mid z = \widetilde{Z}_1(\widetilde{C}, \widetilde{a}, \widetilde{b}, \widetilde{e}) \right\}$$

where $\widetilde{Z_1}(\widetilde{C}, \widetilde{a}, \widetilde{b}, \widetilde{e})$ is defined in Model (9.3).

Since the α-cut $Z{1}^{\alpha}_{L}$ is the minimum of $\widetilde{Z_1}(\widetilde{C}, \widetilde{a}, \widetilde{b}, \widetilde{e})$ and $Z{1}^{\alpha}_{U}$ is the maximum of $\widetilde{Z_1}(\widetilde{C}, \widetilde{a}, \widetilde{b}, \widetilde{e})$; they can be expressed as:

$$Z{1}^{\alpha}_{L} = \min\left\{ \widetilde{Z_1}(\widetilde{C}, \widetilde{a}, \widetilde{b}, \widetilde{e}) : C_{ijk} \in [C^{\alpha}_{ijkL}, C^{\alpha}_{ijkU}], a_i \in [a^{\alpha}_{iL}, a^{\alpha}_{iU}], b_j \in [b^{\alpha}_{jL}, b^{\alpha}_{jU}], e_k \in [e^{\alpha}_{kL}, e^{\alpha}_{kU}] \right\}$$

$$Z{1}^{\alpha}_{U} = \max\left\{ \widetilde{Z_1}(\widetilde{C}, \widetilde{a}, \widetilde{b}, \widetilde{e}) : C_{ijk} \in [C^{\alpha}_{ijkL}, C^{\alpha}_{ijkU}], a_i \in [a^{\alpha}_{iL}, a^{\alpha}_{iU}], b_j \in [b^{\alpha}_{jL}, b^{\alpha}_{jU}], e_k \in [e^{\alpha}_{kL}, e^{\alpha}_{kU}] \right\}$$

These two expressions reformulates the problem into the following pair of two-level mathematical model:

Step-1:

$$Z_{1L}^{\alpha} = \begin{array}{c} \text{Min} \\ C_{ijkL}^{\alpha} \leq \widetilde{C}_{ijk} \leq C_{ijkU}^{\alpha} \\ a_{iL}^{\alpha} \leq \widetilde{a}_i \leq a_U^{\alpha} \\ b_{jL}^{\alpha} \leq \widetilde{b}_j \leq (b_j)_U^{\alpha} \\ e_{kL}^{\alpha} \leq \widetilde{e}_k \leq e_{kU}^{\alpha} \\ \forall i,j,k \end{array} \left\{ \begin{array}{l} \text{Min } \displaystyle\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{i=1}^{K} \widetilde{C}_{ijk}x_{ijk} \\ \text{Min } \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} t_{ijk}y_{ijk} \\ \text{subject to constraint} \\ \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \widetilde{a}_i, \qquad i = 1,2,......M \\ \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \widetilde{b}_j, \qquad j = 1,2....N \\ \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \widetilde{e}_k, \qquad k = 1,2,......K \\ y_{ijk} = \begin{cases} 1 & if \;\; x_{ijk} > 0 \\ 0 & if \;\; x_{ijk} = 0 \end{cases} \\ x_{ijk} \quad \geq 0 \qquad \forall i,j,k \end{array} \right.$$

$$Z_{1U}^{\alpha} = \begin{array}{c} \text{Max} \\ C_{ijkL}^{\alpha} \leq \widetilde{C}_{ijk} \leq C_{ijkU}^{\alpha} \\ a_{iL}^{\alpha} \leq \widetilde{a}_i \leq a_{iU}^{\alpha} \\ b_{jL}^{\alpha} \leq \widetilde{b}_j \leq b_{jU}^{\alpha} \\ e_{kL}^{\alpha} \leq \widetilde{e}_k \leq e_{kU}^{\alpha} \\ \forall i,j,k \end{array} \left\{ \begin{array}{l} \text{Min } \displaystyle\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{i=1}^{K} \widetilde{C}_{ijk}x_{ijk} \\ \text{Min } \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} t_{ijk}y_{ijk} \\ \text{subject to constraint} \\ \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \widetilde{a}_i, \qquad i = 1,2,......M \\ \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \widetilde{b}_j, \qquad j = 1,2....N \\ \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \widetilde{e}_k, \qquad k = 1,2,......K \\ y_{ijk} = \begin{cases} 1 & if \;\; x_{ijk} > 0 \\ 0 & if \;\; x_{ijk} = 0 \end{cases} \\ x_{ijk} \quad \geq 0 \qquad \forall i,j,k \end{array} \right.$$

Now, the necessary and sufficient conditions for the above Model to have feasible solutions are $\sum_{i=1}^{M} \widetilde{a}_i \geq \sum_{j=1}^{N} \widetilde{b}_j$ and $\sum_{j=1}^{N} \widetilde{b}_j = \sum_{k=1}^{K} \widetilde{e}_k$. Adding these feasible conditions, the above problem becomes

Step-2:

$$
Z1^{\alpha}_{L} = \begin{array}{c} \text{Min} \\ C^{\alpha}_{ijkL} \leq \widetilde{C}_{ijk} \leq C^{\alpha}_{ijkU} \\ a^{\alpha}_{iL} \leq \widetilde{a}_i \leq a^{\alpha}_{iU} \\ b^{\alpha}_{jL} \leq \widetilde{b}_j \leq b^{\alpha}_{jU} \\ e^{\alpha}_{kL} \leq \widetilde{e}_k \leq e^{\alpha}_{kU} \\ \sum_{i=1}^{M} \widetilde{a}_i \geq \sum_{j=1}^{N} \widetilde{b}_j \\ \sum_{j=1}^{N} \widetilde{b}_j = \sum_{k=1}^{K} \widetilde{e}_k \\ \forall i,j,k \end{array}
\left\{ \begin{array}{l} \text{Min } \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{i=1}^{K} \widetilde{C}_{ijk} x_{ijk} \\ \text{Min } \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} t_{ijk} y_{ijk} \\ \text{subject to constraint} \\ \sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk} \leq \widetilde{a}_i, \qquad i = 1,2,......M \\ \sum_{i=1}^{M} \sum_{k=1}^{K} x_{ijk} \geq \widetilde{b}_j, \qquad j = 1,2....N \\ \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ijk} \leq \widetilde{e}_k, \qquad k = 1,2,......K \\ y_{ijk} = \begin{cases} 1 & if \;\; x_{ijk} > 0 \\ 0 & if \;\; x_{ijk} = 0 \end{cases} \\ x_{ijk} \geq 0 \qquad \forall i,j,k \end{array} \right.
$$

$$
Z1^{\alpha}_{U} = \begin{array}{c} \text{Max} \\ C^{\alpha}_{ijkL} \leq \widetilde{C}_{ijk} \leq C^{\alpha}_{ijkU} \\ a^{\alpha}_{iL} \leq \widetilde{a}_i \leq a^{\alpha}_{U} \\ b^{\alpha}_{jL} \leq \widetilde{b}_j \leq b^{\alpha}_{jU} \\ e^{\alpha}_{kL} \leq \widetilde{e}_k \leq e^{\alpha}_{kU} \\ \sum_{i=1}^{M} \widetilde{a}_i \geq \sum_{j=1}^{N} \widetilde{b}_j \\ \sum_{j=1}^{N} \widetilde{b}_j = \sum_{k=1}^{K} \widetilde{e}_k \\ \forall i,j,k \end{array}
\left\{ \begin{array}{l} \text{Min } \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{i=1}^{K} \widetilde{C}_{ijk} x_{ijk} \\ \text{Min } \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} t_{ijk} y_{ijk} \\ \text{subject to constraint} \\ \sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk} \leq \widetilde{a}_i, \qquad i = 1,2,......M \\ \sum_{i=1}^{M} \sum_{k=1}^{K} x_{ijk} \geq \widetilde{b}_j, \qquad j = 1,2....N \\ \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ijk} \leq \widetilde{e}_k, \qquad k = 1,2,......K \\ y_{ijk} = \begin{cases} 1 & if \;\; x_{ijk} > 0 \\ 0 & if \;\; x_{ijk} = 0 \end{cases} \\ x_{ijk} \geq 0 \qquad \forall i,j,k \end{array} \right.
$$

Now, we simplify the first portion of the above Model. This model will be infeasible for any α level if $\sum_{i=1}^{M} a^{\alpha}_{iU} \leq \sum_{j=1}^{N} b^{\alpha}_{jL}$ and $\sum_{j=1}^{N} b^{\alpha}_{jL} \geq \sum_{k=1}^{K} e^{\alpha}_{kU}$.

In other words, a fuzzy transportation problem is feasible if the upper bound of the total fuzzy supply is greater than or equal to the lower bound of the total fuzzy demand.

Step-3:

To derive the lower bound of the objective value in first Model, we can directly set $\widetilde{C}_{ijk}$ to its lower bound C^{α}_{ijkL} $\forall i, j, k$, to find the minimum objective value. Hence, the lower level problem can be reformulated in conventional way as follows:

$$Z_{1L}^{\alpha} = \text{Min} \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{i=1}^{K} C^{\alpha}_{ijkL} x_{ijk} \tag{9.7}$$

$$Z_2 = \text{Min} \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} t_{ijk} y_{ijk} \tag{9.8}$$

subject to the constraints

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_i \quad \text{and} \quad a^{\alpha}_{iL} \leq a_i \leq a^{\alpha}_{iU}, \qquad i = 1, 2, ...M$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq b_j \quad \text{and} \quad b^{\alpha}_{jL} \leq b_j \leq b^{\alpha}_{jU}, \qquad j = 1, 2....N$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_k \quad \text{and} \quad e^{\alpha}_{kL} \leq e_k \leq e^{\alpha}_{kU}, \qquad k = 1, 2, ...K$$

$$\sum_{i=1}^{M} a^{\alpha}_{iU} \leq \sum_{j=1}^{N} b^{\alpha}_{jL}$$

$$\sum_{j=1}^{N} b^{\alpha}_{jL} \geq \sum_{k=1}^{K} e^{\alpha}_{kU} \qquad \forall i, j, k$$

$$\text{where } y_{ijk} = \begin{cases} 1 & if \;\; x_{ijk} > 0 \\ 0 & if \;\; x_{ijk} = 0 \end{cases} \tag{9.9}$$

Similarly, the upper level problem conventionally can be written as

$$Z_{1U}^{\alpha} = \text{Min} \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{i=1}^{K} C_{ijkU}^{\alpha} x_{ijk} \tag{9.10}$$

$$Z_2 = \text{Min} \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} t_{ijk} y_{ijk} \tag{9.11}$$

subject to the constraints

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_i \quad \text{and} \quad a_{iL}^{\alpha} \leq a_i \leq a_{iU}^{\alpha}, \qquad i = 1, 2, ...M$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq b_j \quad \text{and} \quad b_{jL}^{\alpha} \leq b_j \leq b_{jU}^{\alpha}, \qquad j = 1, 2....N$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_k \quad \text{and} \quad e_{kL}^{\alpha} \leq e_k \leq e_{kU}^{\alpha}, \qquad k = 1, 2, ...K$$

$$\sum_{i=1}^{M} a_{iU}^{\alpha} \leq \sum_{j=1}^{N} b_{jL}^{\alpha}$$

$$\sum_{j=1}^{N} b_{jL}^{\alpha} \geq \sum_{k=1}^{K} e_{kU}^{\alpha} \qquad \forall i, j, k$$

$$\text{where } y_{ijk} = \begin{cases} 1 & if \;\; x_{ijk} > 0 \\ 0 & if \;\; x_{ijk} = 0 \end{cases} \tag{9.12}$$

First and second problems are assured to be feasible, if the lower bound of the total fuzzy demand is smaller than the upper bound of the total fuzzy supply and the upper bound of the total fuzzy demand is equal to the lower bound of total number of fuzzy conveyance, i.e. $\sum_{i=1}^{M} a_i \leq \sum_{j=1}^{N} b_j$ and $\sum_{j=1}^{N} b_j = \sum_{k=1}^{K} e_k$. If this condition is not fulfilled then the problems will be infeasible.

9.5 Numerical Illustration

A STP with two suppliers, two destinations and three conveyances in fuzzy environment is considered. The imprecise unit shipping costs with maximum discount (minimum possible unit costs) are given below:

$$\widetilde{C}_{111} = \begin{cases} (5,6,7,8) & if x_{111} > 10 \\ (6,7,8,9) & if \ \ 5 < x_{111} \le 10 \\ (7,8,9,10) & if \ \ 0 \le x_{111} \le 5 \end{cases} \quad \widetilde{C}_{121} = \begin{cases} (3,7,9,11) & if x_{121} > 10 \\ (4,8,10,12) & if \ \ 5 < x_{121} \le 10 \\ (5,9,11,13) & if \ \ 0 \le x_{121} \le 5 \end{cases}$$

$$\widetilde{C}_{211} = \begin{cases} (3,6,9,11) & if \ \ x_{211} > 10 \\ (4,7,10,12) & if \ \ 5 < x_{211} \le 10 \\ (5,7,11,13) & if \ \ 0 \le x_{211} \le 5 \end{cases} \quad \widetilde{C}_{212} = \begin{cases} (7,8,9,10) & if \ \ x_{212} > 10 \\ (8,9,10,11) & if \ \ 5 < x_{212} \le 10 \\ (9,10,11,12) & if \ \ 0 \le x_{212} \le 5 \end{cases}$$

$$\widetilde{C}_{112} = \begin{cases} (2,3,5,9) & if \ \ x_{112} > 10 \\ (3,4,6,10) & if \ \ 5 < x_{112} \le 10 \\ (4,5,7,11) & if \ \ 0 \le x_{112} \le 5 \end{cases} \quad \widetilde{C}_{122} = \begin{cases} (4,6,8,12) & if \ \ x_{122} > 10 \\ (5,7,9,13) & if \ \ 5 < x_{122} \le 10 \\ (6,8,10,14) & if \ \ 0 \le x_{122} \le 5 \end{cases}$$

$$\widetilde{C}_{212} = \begin{cases} (3,7,9,12) & if \ \ x_{212} > 10 \\ (4,8,10,13) & if \ \ 5 < x_{212} \le 10 \\ (5,9,11,14) & if \ \ 0 \le x_{212} \le 5 \end{cases} \quad \widetilde{C}_{222} = \begin{cases} (5,8,10,13) & if \ \ x_{222} > 10 \\ (6,9,11,14) & if \ \ 5 < x_{222} \le 10 \\ (7,10,12,15) & if \ \ 0 \le x_{222} \le 5 \end{cases}$$

$$\widetilde{C}_{113} = \begin{cases} (11,12,13,14) & if \ \ x_{113} > 10 \\ (12,13,14,15) & if \ \ 5 < x_{113} \le 10 \\ (13,14,15,16) & if \ \ 0 \le x_{113} \le 5 \end{cases} \quad \widetilde{C}_{123} = \begin{cases} (5,9,12,14) & if \ \ x_{123} > 10 \\ (6,10.13,15) & if \ \ 5 < x_{123} \le 10 \\ (7,11.13,16) & if \ \ 0 \le x_{123} \le 5 \end{cases}$$

$$\widetilde{C}_{213} = \begin{cases} (6,8,13,15) & if \ \ x_{213} > 10 \\ (7,9,14,16) & if \ \ 5 < x_{213} \le 10 \\ (8,10,15,17) & if \ \ 0 \le x_{213} \le 5 \end{cases} \quad \widetilde{C}_{223} = \begin{cases} (3,5,9,13) & if \ \ x_{223} > 10 \\ (4,6,10,14) & if \ \ 5 < x_{223} \le 10 \\ (5,7,11,15) & if \ \ 0 \le x_{223} \le 5 \end{cases}$$

Again, the time for transporting the certain amount of quantity from i-th source to j -th destination by the k -th conveyance for only one trip is shown in the following time matrix.

Table-9.1: Input Data-Crisp Time matrix

	conveyance-1		conveyance-2		conveyance-3	
	D_1	D_2	D_1	D_2	D_1	D_2
O_1	**5**	**6**	**7**	**10**	**5**	**6**
O_2	**8**	**9**	**9**	**3**	**8**	**15**

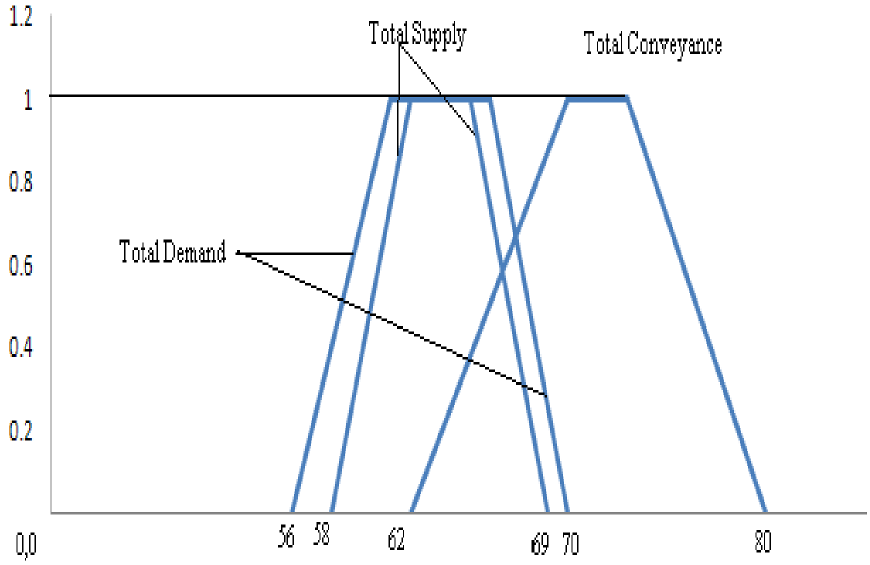

Figure 9.1: Total supply, total demand total conveyance

Now for this problem, the total supply, demand and conveyance are obtained using the addition rule of fuzzy numbers, which are as $\widetilde{A}_1 + \widetilde{A}_2 = (58, 62, 65, 69)$, $\widetilde{D}_1 + \widetilde{D}_2 = (56, 61, 66, 70)$ and $\widetilde{E}_1 + \widetilde{E}_2 + \widetilde{E}_3 = (62, 70, 73, 80)$ respectively and these are also trapezoidal fuzzy numbers which are graphically shown in *Figure* 9.1. The spread of the total fuzzy supply overlaps with the spread of the total fuzzy demand. In other words, the upper bound of the total fuzzy supply is greater than the lower bound of the total fuzzy demand, implying that the problem is feasible.

The lower α-cut of the unit shipping cost per unit item (with AUD policy) are given below.

$$C^{\alpha}_{111L} = \begin{cases} 5+\alpha & if x_{111} > 10 \\ 6+\alpha & if \;\; 5 < x_{111} \leq 10 \\ 7+\alpha & if \;\; 0 \leq x_{111} \leq 5 \end{cases} \quad C^{\alpha}_{121L} = \begin{cases} 3+4\alpha & if x_{121} > 10 \\ 4+4\alpha & if \;\; 5 < x_{121} \leq 10 \\ 5+4\alpha & if \;\; 0 \leq x_{121} \leq 5 \end{cases}$$

$$C^{\alpha}_{211L} = \begin{cases} 3+3\alpha & if \;\; x_{211} > 10 \\ 4+3\alpha & if \;\; 5 < x_{211} \leq 10 \\ 5+3\alpha & if \;\; 0 \leq x_{211} \leq 5 \end{cases} \quad C^{\alpha}_{221L} = \begin{cases} 7+\alpha & if \;\; x_{221} > 10 \\ 8+\alpha & if \;\; 5 < x_{221} \leq 10 \\ 9+\alpha & if \;\; 0 \leq x_{221} \leq 5 \end{cases}$$

$$C^{\alpha}_{112L} = \begin{cases} 2+\alpha & if \;\; x_{112} > 10 \\ 3+\alpha & if \;\; 5 < x_{112} \leq 10 \\ 4+\alpha & if \;\; 0 \leq x_{112} \leq 5 \end{cases} \quad C^{\alpha}_{122L} = \begin{cases} 4+2\alpha & if \;\; x_{122} > 10 \\ 5+2\alpha & if \;\; 5 < x_{122} \leq 10 \\ 6+2\alpha & if \;\; 0 \leq x_{122} \leq 5 \end{cases}$$

$$C^{\alpha}_{212L} = \begin{cases} 3+4\alpha & if \;\; x_{212} > 10 \\ 4+4\alpha & if \;\; 5 < x_{212} \leq 10 \\ 5+4\alpha & if \;\; 0 \leq x_{212} \leq 5 \end{cases} \quad C^{\alpha}_{222L} = \begin{cases} 5+3\alpha & if \;\; x_{222} > 10 \\ 6+3\alpha & if \;\; 5 < x_{222} \leq 10 \\ 7+3\alpha & if \;\; 0 \leq x_{222} \leq 5 \end{cases}$$

$$C^{\alpha}_{113L} = \begin{cases} 11+\alpha & if \;\; x_{113} > 10 \\ 12+\alpha & if \;\; 5 < x_{113} \leq 10 \\ 13+\alpha & if \;\; 0 \leq x_{113} \leq 5 \end{cases} \quad C^{\alpha}_{123L} = \begin{cases} 5+4\alpha & if \;\; x_{123} > 10 \\ 6+4\alpha & if \;\; 5 < x_{123} \leq 10 \\ 7+4\alpha & if \;\; 0 \leq x_{123} \leq 5 \end{cases}$$

$$C^{\alpha}_{213L} = \begin{cases} 6+2\alpha & if \;\; x_{213} > 10 \\ 7+2\alpha & if \;\; 5 < x_{213} \leq 10 \\ 8+2\alpha & if \;\; 0 \leq x_{213} \leq 5 \end{cases} \quad C^{\alpha}_{223L} = \begin{cases} 3+2\alpha & if \;\; x_{223} > 10 \\ 4+2\alpha & if \;\; 5 < x_{223} \leq 10 \\ 5+2\alpha & if \;\; 0 \leq x_{223} \leq 5 \end{cases}$$

Hence following the solution procedure discussed in section §4, the lower level crisp problem for the aforesaid fuzzy problem is :

$$\text{Minimize } Z^{\alpha}_{1L} = 5x_{111} + C^{\alpha}_{121L}x_{121} + C^{\alpha}_{211L}x_{211} + 7x_{221} + C^{\alpha}_{112L}x_{112} + C^{\alpha}_{122L}x_{122} + C^{\alpha}_{212L}x_{212}$$
$$+C^{\alpha}_{222L}x_{222} + 11x_{113} + C^{\alpha}_{123L}x_{123} + C^{\alpha}_{213L}x_{213} + C^{\alpha}_{223L}x_{223} \qquad (9.13)$$

$$\text{Minimize } Z_2 = t_{111} + t_{121} + t_{211} + t_{221} + t_{112} + t_{122} + t_{212} + t_{222} + t_{113} + t_{123}$$
$$+t_{213} + t_{223} \qquad (9.14)$$

subject to constraints

$$\begin{aligned}
&x_{111} + x_{112} + x_{113} + x_{121} + x_{122} + x_{123} \leq a_1 \\
&x_{211} + x_{212} + x_{213} + x_{221} + x_{222} + x_{223} \leq a_2 \\
&x_{111} + x_{112} + x_{113} + x_{211} + x_{212} + x_{213} \geq b_1 \\
&x_{121} + x_{122} + x_{123} + x_{221} + x_{222} + x_{223} \geq b_2 \\
&x_{111} + x_{121} + x_{211} + x_{221} \leq e_1 \\
&x_{112} + x_{122} + x_{212} + x_{222} \leq e_2 \\
&x_{113} + x_{123} + x_{213} + x_{223} \leq e_3 \\
&31 + 2\alpha \leq a_1 \leq 37 - 2\alpha \\
&27 + 2\alpha \leq a_2 \leq 32 - 2\alpha \\
&25 + 3\alpha \leq b_1 \leq 34 - 2\alpha \\
&31 + 2\alpha \leq b_2 \leq 36 - 2\alpha \\
&18 + 2\alpha \leq e_1 \leq 26 - 2\alpha \\
&23 + 3\alpha \leq e_2 \leq 28 - \alpha \\
&21 + 3\alpha \leq e_3 \leq 26 - \alpha \\
&a_1 + a_2 \geq b_1 + b_2 \\
&b_1 + b_2 = e_1 + e_2 + e_3 \\
&x_{ijk} \geq 0 \quad \forall \quad 0 \leq \alpha \leq 1 \\
&\forall \quad i = 1(1)2, j = 1(1)2, k = 1(1)3.
\end{aligned} \tag{9.15}$$

Similarly, when the decision maker chooses the upper level (pessimistic sense) of the imprecise parameters, the problem as reduced for the upper α-cut of the unit shipping cost as follows:

$$C^{\alpha}_{111U} = \begin{cases} 8-\alpha & if \;\; x_{111} > 10 \\ 9-\alpha & if \;\; 5 < x_{111} \leq 10 \\ 10-\alpha & if \;\; 0 \leq x_{111} \leq 5 \end{cases} \qquad C^{\alpha}_{121U} = \begin{cases} 11-2\alpha & if \;\; x_{121} > 10 \\ 12-2\alpha & if \;\; 5 < x_{121} \leq 10 \\ 13-2\alpha & if \;\; 0 \leq x_{121} \leq 5 \end{cases}$$

$$C^{\alpha}_{211U} = \begin{cases} 11-2\alpha & if \;\; x_{211} > 10 \\ 11.5-2\alpha & if \;\; 5 < x_{211} \leq 10 \\ 12-2\alpha & if \;\; 0 \leq x_{211} \leq 5 \end{cases} \qquad C^{\alpha}_{221U} = \begin{cases} 10-\alpha & if \;\; x_{221} > 10 \\ 11-\alpha & if \;\; 5 < x_{221} \leq 10 \\ 12-\alpha & if \;\; 0 \leq x_{221} \leq 5 \end{cases}$$

$$C^{\alpha}_{112U} = \begin{cases} 9-4\alpha & if \;\; x_{112} > 10 \\ 10-4\alpha & if \;\; 5 < x_{112} \leq 10 \\ 11-4\alpha & if \;\; 0 \leq x_{112} \leq 5 \end{cases} \qquad C^{\alpha}_{122U} = \begin{cases} 12-4\alpha & if \;\; x_{122} > 10 \\ 13-4\alpha & if \;\; 5 < x_{122} \leq 10 \\ 14-4\alpha & if \;\; 0 \leq x_{122} \leq 5 \end{cases}$$

$$C^{\alpha}_{212U} = \begin{cases} 12-3\alpha & if \;\; x_{212} > 10 \\ 13-3\alpha & if \;\; 5 < x_{212} \leq 10 \\ 14-3\alpha & if \;\; 0 \leq x_{212} \leq 5 \end{cases} \qquad C^{\alpha}_{222U} = \begin{cases} 13-3\alpha & if \;\; x_{222} > 10 \\ 14-3\alpha & if \;\; 5 < x_{222} \leq 10 \\ 15-3\alpha & if \;\; 0 \leq x_{222} \leq 5 \end{cases}$$

$$C^{\alpha}_{113U} = \begin{cases} 15-\alpha & if \;\; x_{113} > 10 \\ 16-\alpha & if \;\; 5 < x_{113} \leq 10 \\ 17-\alpha & if \;\; 0 \leq x_{113} \leq 5 \end{cases} \qquad C^{\alpha}_{123U} = \begin{cases} 14-2\alpha & if \;\; x_{123} > 10 \\ 15-2\alpha & if \;\; 5 < x_{123} \leq 10 \\ 17-2\alpha & if \;\; 0 \leq x_{123} \leq 5 \end{cases}$$

$$C^{\alpha}_{213U} = \begin{cases} 15-2\alpha & if \;\; x_{213} > 10 \\ 16-2\alpha & if \;\; 5 < x_{213} \leq 10 \\ 17-2\alpha & if \;\; 0 \leq x_{213} \leq 5 \end{cases} \qquad C^{\alpha}_{223U} = \begin{cases} 13-4\alpha & if \;\; x_{223} > 10 \\ 14-4\alpha & if \;\; 5 < x_{223} \leq 10 \\ 15-4\alpha & if \;\; 0 \leq x_{223} \leq 5 \end{cases}$$

The correspondence upper level problem is given by

$$\begin{aligned} \text{Minimize } Z^{\alpha}_{1U} = {} & 7x_{111} + C^{\alpha}_{121U}x_{121} + C^{\alpha}_{211U}x_{211} + 12x_{221} + C^{\alpha}_{112U}x_{112} + C^{\alpha}_{122U}x_{122} + C^{\alpha}_{212U}x_{212} \\ & + C^{\alpha}_{222U}x_{222} + 9x_{113} + C^{\alpha}_{123U}x_{123} + C^{\alpha}_{213U}x_{213} + C^{\alpha}_{223U}x_{223} \qquad (9.16) \end{aligned}$$

$$\begin{aligned} \text{Minimize } Z_2 = {} & t_{111} + t_{121} + t_{211} + t_{221} + t_{112} + t_{122} + t_{212} + t_{222} + t_{113} + t_{123} \\ & + t_{213} + t_{223} \qquad (9.17) \end{aligned}$$

subject to constraints

$$
\begin{aligned}
& x_{111} + x_{112} + x_{113} + x_{121} + x_{122} + x_{123} \leq a_1 \\
& x_{211} + x_{212} + x_{213} + x_{221} + x_{222} + x_{223} \leq a_2 \\
& x_{111} + x_{112} + x_{113} + x_{211} + x_{212} + x_{213} \geq b_1 \\
& x_{121} + x_{122} + x_{123} + x_{221} + x_{222} + x_{223} \geq b_2 \\
& x_{111} + x_{121} + x_{211} + x_{221} \leq e_1 \\
& x_{112} + x_{122} + x_{212} + x_{222} \leq e_2 \\
& x_{113} + x_{123} + x_{213} + x_{223} \leq e_3 \\
& 31 + 2\alpha \leq a_1 \leq 37 - 2\alpha \\
& 27 + 2\alpha \leq a_2 \leq 32 - 2\alpha \\
& 25 + 3\alpha \leq b_1 \leq 34 - 2\alpha \\
& 31 + 2\alpha \leq b_2 \leq 36 - 2\alpha \\
& 18 + 2\alpha \leq e_1 \leq 26 - 2\alpha \\
& 23 + 3\alpha \leq e_2 \leq 28 - \alpha \\
& 21 + 3\alpha \leq e_3 \leq 26 - \alpha \\
& a_1 + a_2 \geq b_1 + b_2 \\
& b_1 + b_2 = e_1 + e_2 + e_3 \\
& x_{ijk} \geq 0 \quad \forall \quad i = 1(1)2, j = 1(1)2, k = 1(1)3. \quad 0 \leq \alpha \leq 1
\end{aligned}
\tag{9.18}
$$

Since the above two crisp models (9.13) – (9.14) and (9.16) – (9.17) subject to (9.15) or (9.18) are multi-objective, hence these two models can be converted into the following single objective models (9.19) and (9.20) for the lower and upper level respectively as follows:

$$
\text{Minimize } Z_L = W_1 Z_{1}{}_{L}^{\alpha} + (1 - W_1) Z_2 \tag{9.19}
$$

with the constraints in (9.15) and

$$\text{Minimize } Z_U = W_2 Z1_U^{\alpha} + (1 - W_2) Z_2 \tag{9.20}$$

with the the constraints in (9.18).

Now, Lingo [9.0], a mathematical programming solver is used to optimize the above two models (9.19) and (9.20) taking $W_1 = W_2 = 0.8$ and $Table - 9.3$ shows the α-cuts of the objective function when transportation cost and time are simultaneously minimized at eleven distinct α values: $0, 0.1, 0.2, ...$ and 1.0.

Table-9.2: The α-cuts of the objective functions

α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$Z1_L^{\alpha}$ (Z_2)	147.5 (46)	161.6 (46)	175.8 (46)	190.3 (46)	205.1 (46)	240.3 (46)	249.2 (46)	258.6 (46)	268.61 (46)	278.4 (46)	289.0 (52)
$Z1_U^{\alpha}$ (Z_2)	570.0 (46)	566.1 (46)	565.7 (46)	561.4 (46)	550.0 (46)	543.0 (46)	537.7 (46)	535.5 (46)	530.0 (46)	519.6 (46)	510.0 (52)
Z_L	127.2	138.5	149.9	161.4	173.3	201.4	208.6	216.1	224.1	231.9	241.6
Z_U	465.2	462.2	461.2	458.3	449.1	443.6	439.4	437.6	432.5	424.9	418.4
Amount to be Transported in Z_L	56	56.5	57	57.5	58	58.5	59	59.5	60	60.5	61
Amount to be Transported in Z_U	56	56.5	57	57.5	58	58.5	59	59.5	60	60.5	61

which represents the possibility of the objective function in the associated range. Specifically, the '$\alpha = 1.0$' cut shows the total transportation cost that is most likely to be and the '$\alpha = 0$' cut shows the range that the total transportation cost could appear. In this example, while the total transportation cost is fuzzy, its most likely value falls between 289.60 and 510.00, and its value is impossible to fall outside the range of 147.50 and 570.00. Since the objective functions either transportation cost and time simultaneously or transportation cost individually are fuzzy in nature, so its membership functions are represented in *Figure* 9.2.

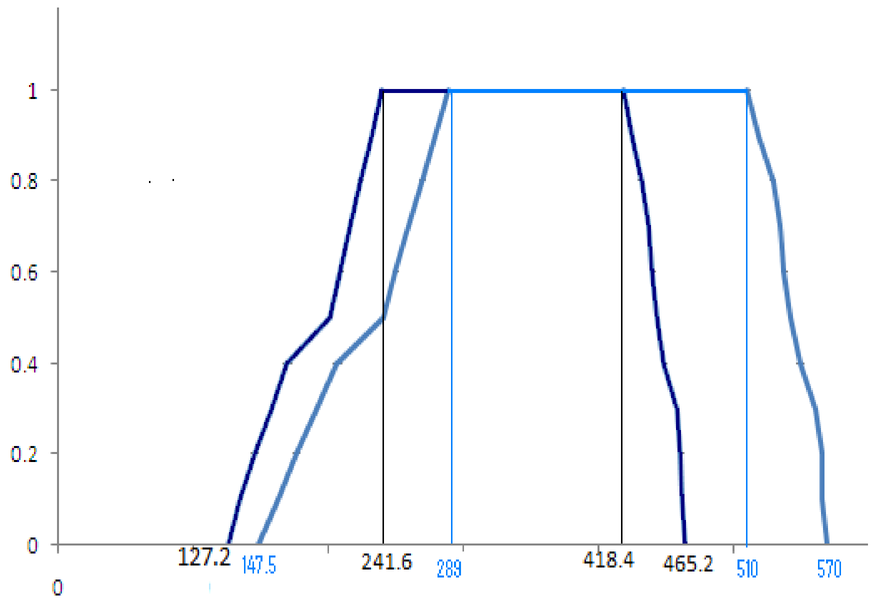

Figure 9.2: The membership function of the objective functions

9.6 Discussion

From $Table - 9.3$, it is observed that for the '$\alpha = 0$' cut, the lower bound of transportation cost $(\widetilde{Z_1})$ is 147.5 and total transported time is 46 units corresponding to optimum transported quantity and time from source to destination via conveyance such as follows $x_{111} = 0.14$, $x_{121} = 14$, $x_{211} = 0.86$, $x_{112} = 24$, $x_{223} = 17$, $x_{221} = x_{122} = x_{212} = x_{222} = x_{113} = x_{123} = x_{213} = 0$, $t_{111} = 5$, $t_{121} = 6$, $t_{211} = 8$, $t_{112} = 7$, $t_{223} = 15$, $t_{221} = t_{122} = t_{212} = t_{222} = t_{113} = t_{123} = t_{213} = 0$ with b_1=25,b_2=31; a_1=37,a_2=32 and e_1=15, e_2=24, $e_3 = 17$.

The upper bound of transportation cost $(\widetilde{Z_1})$ is 570 and total transported time 46 units corresponding to optimum transported quantity and time from source to destination via conveyance $x_{221} = 18$, $x_{112} = 11.5$, $x_{122} = 9.5$, $x_{113} = 13.5$, $x_{223} = 3.5$, $x_{111} = x_{121} = x_{211} = x_{212} = x_{222} = x_{123} = x_{213} = 0$ $t_{221} = 9$, $t_{112} = 7$, $t_{122} = 10$, $t_{113} = 5$, $t_{223} = 15$, $t_{111} = t_{121} = t_{211} = t_{212} = t_{222} = t_{123} = t_{213} = 0$ with $b_1 = 25$,$b_2 = 31$; a_1=37,a_2=32 and $e_1 = 18$, e_2=21, $e_3 = 17$.

At the other extreme end of $\alpha = 1$, the lower bound of $\widetilde{Z_1}$ is 241.60 and taken time 52 units corresponding to optimum transported quantity and time from source to destination via conveyance $x_{111} = 4$, $x_{121} = 3$, $x_{221} = 10$, $x_{112} = 24$, $x_{223} = 20$, $x_{211} = x_{122} = x_{212} = x_{222} = x_{113} = x_{123} = x_{213} = 0$ $t_{111} = 5$, $t_{121} = 6$, $t_{221} = 9$, $t_{112} = 7$, $t_{223} = 15$, $t_{211} = t_{122} = t_{212} = t_{222} = t_{113} = t_{123} = t_{213} = 0$ with b_1=28,b_2=33; a_1=35,a_2=30 and e_1=17, e_2=24, e_3=20.

The upper bound of $\widetilde{Z_1}$ is 418.40 and taken time 52 units corresponding to optimum transported quantity and time from source to destination via conveyance $x_{221} = 17$, $x_{112} = 22.5$, $x_{122} = 1.5$, $x_{113} = 5.5$, $x_{123} = 1.5$ $x_{223} = 3.5$, $x_{111} = x_{121} = x_{211} = x_{212} = x_{222} = x_{213} = 0$ $t_{221} = 9$, $t_{112} = 7$, $t_{122} = 10$, $t_{113} = 5$, $t_{123} = 6$ $t_{223} = 15$, $t_{111} = t_{121} = t_{211} = t_{212} = t_{222} = t_{213} = 0$ with b_1=28,b_2=33; a_1=35,a_2=30 and e_1=17, e_2=24, e_3=20.

9.7 Conclusion

In this chapter, a model is mainly investigated with uncertain cost of multi-criterium solid transportation problem based on two-level fuzzy programming technique. Since there are many uncertain parameters in the model, computing objective value and checking feasibility become complex in general. In order to solve the model conveniently, we have discussed the crisp equivalences for the model following Zadeh extension principal. Here, a realistic discounts policy (AUD) has been proposed to transportation cost depending on the amount of transportation units. The fixed time parameters also may be considered in different environment.