Chapter 7

A Solid Transportation Problem with Entropy Using Fuzzy Logic & Genetic Algorithm

7.1 Introduction

Due to several complexities, the nature of human thinking may be fluctuate, based on such fluctuation, Lofti Zadeh [152] has been introduced the fuzzy set theory or fuzzy logic in 1965. In the early seventies, John Holland developed a programming code namely, genetic algorithms by insisting the biological phenomena of reproduction, cross-over, mutation. These two fields are extansively been subjects of educational research in the real world. In the last few years, they have been experiencing extremely rapid growth in the industrial world, where they have been shown to be very effective in solving real-world problems. These two substantial fields are recognized as major factors of soft computing: a set of computing technologies already riding the waves of the next century to produce the human-centered intelligent systems of tomorrow in the existing literature (cf Rajak et al. [125], Melin et al. [97]Maria et al. [92]) . Fuzzy logic deals with the problems that have identified by imprecise nature of fuzziness or vagueness.

The decision maker with positive minded cannot satisfy in a single item management. They always expect for more than one item management system even in transportation system also. Such transported items are exchangeable in nature, i.e., lower availability or demand of one item motivate to increases the availability or demand of another item. This substitution technically managed by fuzzy logic/ inference. Pramanik et al. [118] have been worked about substitutable and damageable items in disaster response operations on fuzzy rough environment

Genetic algorithms are search and optimization evolutionary algorithms based on the principles of natural phenomenas. Genetic algorithm is generally composed of following processes, at first selection of individual is made for the production of next generation and second process is manipulation of the selected individual to form the next generation by crossover and mutation techniques Maria et al. [92], Ojha et al. [109], Maiti et al. [88].

In this chapter, two items solid transportation problem has been studied, where the items are substitutable in nature. Due to this substitutability, the demand and availability of one item depends on that of the other which can be calculated from the concept of fuzzy logic. Here holding cost has been considered in both of the origin and destination. The model is optimized using genetic algorithm for two different models, model with and with out entropy. Finally, the models are illustrated through numerical examples The basic differences of the proposed model from existing models have been given below:

Table-7.1: Comparison table anong the existing models with proposed model 7.3

References(s)	Type of Transportation	Entropy	Type of Item	Fuzzy Logic	Method
Bit et al.	2D	No	Single	No	Fuzzy Programming
Chena et al.	STP	No	Single	No	Goal Programming
Gen et al.	Two Stage TP	No	Single	No	Genetic Algorithm
Giri et al.	STP	No	Multi-item	No	GRG
Pramanik et al.	STP	NO	Substitutable and damageable item	No	Genetic Algorithm
Lotfi et al.	TP	Yes	Multi-item	No	Genetic Algorithm
This model	STP	Yes	substitutable items	Yes	Genetic Algorithm

7.2 Notations and Assumptions

7.2.1 Notations

Instead of common notations, here the following additional notations have been used.

(i) $\widetilde{a}_i^l$ = available amount of homogeneous product at i-th source for the $l-th$ item.

(ii) $\widetilde{b}_j^l$ = demand at j-th destination for the $l-th$ item.

(iii) C_{ijk}^l = unit transportation cost for transportation from i-th supply to j-th destination via k-th conveyance for the $l-th$ item.

(iv) d_{1i} = fuzzy infrastructural cost at i-th source to contain the available items.

(v) d_{2j} = fuzzy infrastructural cost at j-th destination to contain the received items.

(vi) x_{ijk}^l = amount of the product to be transported from i-th supply to j-th destination by k-th conveyance for the $l-th$ item.

7.2.2 Assumptions

To develop the proposed solid transportation model, the following assumptions have been made.

(i) The transported items are exchangeable in nature, i.e., lower availability or demand of one item increases the availability or demand of the another item. This substitution is maintained by fuzzy logic/ inference.

(ii) Each origin and destination has an infrastructural cost to contain the available and received quantity of item.

(iii) The entropy function is considered to ensure the transportation to all entities. Also infrastructural cost is taken in account.

(iv) The STP is balanced one , i.e., $\sum_{i=1}^{M} a_i = \sum_{j=1}^{N} b_j = \sum_{k=1}^{K} e_k$, more over, if the problem is unbalanced then it is converted into a balanced one as per existing rule.

7.3 Mathematical Formulation of Multi Item Solid Transportation Problem (MISTP)

In this fuzzy balanced transportation problem, if x^l_{ijk} units are transported from i-th origin to j-th destination via $k-th$ conveyance for $l-th$ item, then the optimization problem for solid transportation problems is written as

$$\text{Minimize } \widetilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{P} C^l_{ijk}\, x^l_{ijk} + \sum_{i=1}^{M} d_{1i}a_i + \sum_{j=1}^{N} d_{2j}b_j \tag{7.1}$$

subject to the constraints

$$\begin{aligned}
\sum_{j=1}^{N}\sum_{k=1}^{K} x^l_{ijk} &= \widetilde{a}^l_i \quad \forall \quad i \,\&\, l \\
\sum_{i=1}^{M}\sum_{k=1}^{K} x^l_{ijk} &= \widetilde{b}^l_j \quad \forall \quad j \,\&\, l \\
\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{l=1}^{P} x^l_{ijk} &= \widetilde{e}_k \quad \forall \quad k \\
\sum_{i=1}^{M}\sum_{l=1}^{P} \widetilde{a}^l_i &= \sum_{j=1}^{N}\sum_{l=1}^{P} \widetilde{b}^l_j = \sum_{k=1}^{K} \widetilde{e}_k \\
x_{ijk} &\geq 0 \quad \text{for all} \quad i, j, k.
\end{aligned} \tag{7.2}$$

In the objective function, the first term indicate the total transportation cost, second & third trems indicate the holding cost of the source & destination for the holding units. The first constraint represents the capacity of the sources, where as the second & third constraints of equation represent the demand of the each destination & capacity of the each conveyance respectively.

Now the membership functions of the fuzzy numbers for capacity a_i^l of the sources and the demand b_j^l of the destinations are defined by

$$\mu_{\widetilde{a_i^l low}}(x) = \begin{cases} 0 & \text{if } x \le a_0 \\ 1 & \text{if } a_0 \le x \le a_1 \\ \dfrac{a_2 - x}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ 0 & \text{if } x \ge a_2 \end{cases} \qquad \mu_{\widetilde{a_i^l medium}}(x) = \begin{cases} 0 & \text{if } x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{if } x \ge a_3 \end{cases}$$

$$\mu_{\widetilde{a_i^l high}}(x) = \begin{cases} 0 & \text{if } x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ 1 & \text{if } a_3 \le x \le a_4 \\ 0 & \text{if } x \ge a_4 \end{cases} \qquad \mu_{\widetilde{b_j^l low}}(x) = \begin{cases} 0 & \text{if } x \le b_0 \\ 1 & \text{if } b_0 \le x \le b_1 \\ \dfrac{b_2 - x}{b_2 - b_1} & \text{if } b_1 \le x \le b_2 \\ 0 & \text{if } x \ge b_2 \end{cases}$$

$$\mu_{\widetilde{b_j^l medium}}(x) = \begin{cases} 0 & \text{if } x \le b_1 \\ \dfrac{x - b_1}{b_2 - b_1} & \text{if } b_1 \le x \le b_2 \\ \dfrac{b_3 - x}{b_3 - b_2} & \text{if } b_2 \le x \le b_3 \\ 0 & \text{if } x \ge b_3 \end{cases} \qquad \mu_{\widetilde{b_j^l high}}(x) = \begin{cases} 0 & \text{if } x \le b_2 \\ \dfrac{x - b_2}{b_3 - b_2} & \text{if } b_2 \le x \le b_3 \\ 1 & \text{if } b_3 \le x \le b_4 \\ 0 & \text{if } x \ge b_4 \end{cases}$$

Using these membership functions, the following fuzzy inference rules for source $\widetilde{a}_i^l$ are given by

R_{11}: if the first item a_i^1 is high then scoend item a_i^2 is low.

R_{12}: if the first item a_i^1 is medium then scoend item a_i^2 is medium.

R_{13}: if the first item a_i^1 is low then scoend item a_i^2 is high.

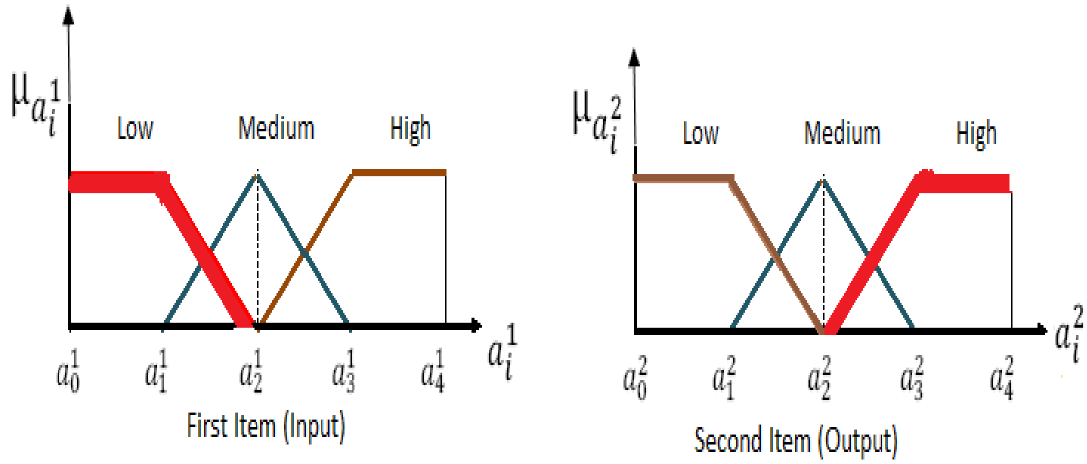

Figure 7.1: The first figure represents the membership function of the availability first item (taken as input) and corresponding membership function of that of second item (obtained by fuzzy logic) presents in second figure

Similarly, the fuzzy rules for demand b_j^l are proposed as follows:

R_{21}: if the demand of first item b_j^1 is high then the demand of scoend item b_j^2 is low.

R_{22}: if the demand of first item b_j^1 is medium then the demand of scoend item b_j^2 is medium.

R_{23}: if the demand of first item b_j^1 is low then the demand of scoend item b_j^2 is high.

7.3.1 MISTP with entropy

Let S be the transported amount i.e., $S = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L} x_{ijk}^l$.Consider a function F(X) which represents the number of possible assignment for the state $X = (x_{ijk}^l)$.

The (Shannon) entropy of a variable X is defined as

F(X)= the number of ways selecting x_{111}^1 from S, multiplied by the number of ways selecting x_{111}^2 from $S - x_{111}^1$, ... , multiplied by the number of ways selecting

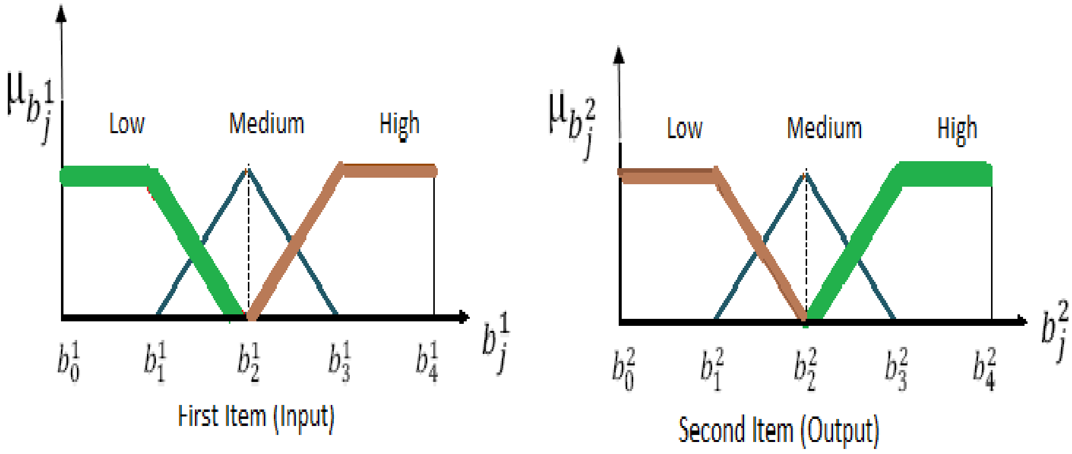

Figure 7.2: The first figure represents the membership function of the demand first item (taken as input) and corresponding membership function of that of second item (obtained by fuzzy logic) presents in second figure

x^l_{mnk} from $S - x^1_{111} - x^2_{111} - ..... - x^{l-1}_{mnk}$.

$$= \; {}^{S}C_{x^1_{111}} . {}^{(S-x^1_{111})}C_{x^2_{111}} . {}^{(S-x^1_{111}-x^2_{111})}C_{x^3_{111}} . \cdots\cdots \; {}^{(S-x^1_{111}-x^2_{111}-.....-x^{l-1}_{mnk})}C_{x^l_{mnK}}$$

$$= \frac{S!}{\prod_{i=1}^{M}\prod_{j=1}^{N}\prod_{k=1}^{K}\prod_{l=1}^{L} x^l_{ijk}!}$$

$$\begin{aligned} ln(F(X)) &= ln(S!) - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L} ln(x^l_{ijk}!) \\ &= ln(e^{-S}S^S) - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L} ln(e^{-x^l_{ijk}} {x^l_{ijk}}^{x^l_{ijk}}) \\ &\qquad\qquad \text{[By using Stirlings approximation formula]} \\ &= S.ln(S) - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L} x^l_{ijk} ln(x^l_{ijk}) \end{aligned}$$

Therefore $\frac{ln(F(X))}{S} = ln(S) - \frac{1}{S}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L} x^l_{ijk} ln(x^l_{ijk})$

Here the entropy function $E_n(x) = \frac{ln(F(X))}{S}$.

The function(Shannon)entropy can be expressed as

$$En(X) = -\sum_{x} f(x)$$

where

$$f(x) = \begin{cases} p(x) ln\, p(x) & \text{if } p(x) \neq 0 \\ 0 & \text{if } p(x) = 0 \end{cases}$$

p(x) being the probability that X is in the state x.

In transportation problem, normalizing the trip number x^l_{ijk} by dividing the total number of trips S $(=\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L} x^l_{ijk})$, a probability distribution, $p^l_{ijk} = x^l_{ijk}/S$ is formulated.

Therefore

$$\begin{aligned} En(X) &= -\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L}(x^l_{ijk}/S)ln(x^l_{ijk}/S) \qquad (7.3) \\ &= ln(S) - \frac{1}{S}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{L} x^l_{ijk} ln(x^l_{ijk}) \end{aligned}$$

In transportation problem, this entropy function acts as a measure of dispersal of trips among origins, destinations and conveyances. It becomes more useful, if we would like to have minimum transportation costs as well as maximum entropy amount.

Taking entropy function as an additional objective function, the Problem-I takes the following form:

Maximize $\widetilde{Z}(X)$ and Maximize En(X).

subject to the constraints (7.2).

7.4 Numerical Ilustration

Let us consider a solid transportation problem with two origins, two destinations and two conveyances for two items, where the availability and demand of one item (in proper units) are known and those for another item (in the same units) are obtained through fuzzy logic which are presented in $Table - 7.2$, $Table - 7.3$. The different range or limit of the fuzzy logic parameters are given by

Table-7.2: Input of source for item with its fuzzy logic value

	First Item ($\widetilde{a}_i^1$)	Secend Item ($\widetilde{a}_i^2$)
Low	$(0, 10, 20)$	$(10, 20, 30)$
Medium	$(10, 20, 30)$	$(20, 30, 40)$
High	$(20, 30, 40)$	$(30, 40, 50)$

Table-7.3: Input of demand for item with its fuzzy logic value

	First Item ($\widetilde{b}_j^1$)	Second Item ($\widetilde{b}_j^2$)
Low	$(0, 10, 20)$	$(10, 20, 30)$
Medium	$(10, 20, 30)$	$(20, 30, 40)$
High	$(20, 30, 40)$	$(30, 40, 50)$

Let the problem is optimized for the input data of first item for diffrent source/destination considered. Input of first item, the corresponding logical value of the second item are shown in $Table - 7.4$.

Table-7.4: Input of first item, the corresponding logical value of the second item

source/ destination	Input for l=1	parameter obtained from fuzzy logic l=2
a_1^l	18	37.050
a_2^l	28	15.459
b_1^l	27	17.230
b_2^l	19	32.120

Table-7.5: Other Inputs of the Model

Infrastructural Cost				Capacity of	
At Sources		At Destinations		Conveyance	
d_1^1	d_1^2	d_2^1	d_2^2	$\widetilde{e}_1$	$\widetilde{e}_2$
3	2	2	3	(40,45,50)	(50,55,60)

The corresponding unit transportation costs (in proper units) of the system are given by

Table-7.6: Unit Transportation Cost(C_{ijk}^l) of the Model

	Unit Transportation costs							
i	1				2			
j	1		2		1		2	
k	1	2	1	2	1	2	1	2
l=1	6	5	2	1	4	3	6	1
l=2	6	7	2	3	2	1	3	1

For the above considerable data, the model is defuzzified by taking the membership function of a fuzzy number and then it is optimized by using meta heuristic process MOGA. The obtained results are presented in $Table - 7.7$.

Table-7.7: Optimum results of Model, Using MOGA.

	Transported amounts									
i	1				2				Minimum	Maximum
j	1		2		1		2		Cost	Entropy
k	1	2	1	2	1	2	1	2	Z	Function(E_x)
l=1	2.099	1.262	9.593	3.822	4.78	6.502	7.805	5.489	6317.068	03.41
l=2	09.702	03.153	6.724	01.142	0.0	14.966	03.557	07.194		

The above optimal solutions form a set of pareto optimal solutions, since they are non-dominating to each other.The above results are shown that the allocation of transportation is normally occured in all the cells. This is due to the consideration of entropy functions. So another result is also obtained for without entropy, which are presented in $Table - 7.8$.

Table-7.8: Optimum results of Model, Using MOGA Without Entropy.

	Transported amounts								Minimum Cost
i	1				2				
j	1		2		1		2		
k	1	2	1	2	1	2	1	2	Z
l=1	01.788	05.839	11.668	10.617	14.180	02.342	3.617	06.604	6175.370
l=2	13.767	1.756	14.979	6.175	2.505	3.227	7.166	03.217	

From $Table-7.7$ and $Table-7.8$, it reveals that, when there are more objective functions, the system gives compromise solutions. It is also decided that the use of entropy produces maximum allocation to the transportation system.

7.5 Conclusion

This model represents a solid transportation problem (STP) for substitutable items with imprecise cost. Hence forth, the demand of one item may be fulfilled by the stock of another item. The substitutability of the items are measured by the tools of fuzzy logic. The fuzzy logic is scientifically controlled by meta heuristic genetic algorithm. The genetic algorithm also provides the non-dominated optimal solution to the decision maker. The model also consists of the infrastructural cost of the origin and destination. Such profit maximization STP is visible in any transportation management system where more than one item are transported. More over, most of the optimal solution of STP are feasible with respect to the availability of the origin, demand of the destination and capacity of the conveyances only, they do not agree with maximum number of allocation. In this regard, an imprecise multi objective STP is formulated here in terms of maximization of entropy function.