Chapter 6

Genetic algorithm approach of a solid transportation problem based on fuzzy logic

6.1 Introduction

In reality, the amount of transportation from an origin inversely depends on the level of unit cost of transportation. When the unit cost of transportation in a certain path is low, then one attempts to transport maximum amount of the item though that path i.e, the amount of transportation is high. Again on the contrary, if the unit cost of transportation is high, then less resource is transported. For medium unit cost of transportation, amount of transportation is also medium. If the two parameters such as quality (percentage of breakability) transportation amount and distance of destination have been considered for transportation in a transportation system, then transported cost varies inversely with the item of transportation and the distance of destination. This means that if quality of transportated item low then the transportation cost will be maximum and if the distance from source to destination is low then the transportation cost will be maximum. When these conditions are automatically hold with fuzzy parameters under IF-THEN rules,

these rules are called fuzzy references.
In the 1960s. the idea of fuzzy logic was advanced by Dr. Lotfi Zadeh [152]. He was working on computer understanding problem of natural language (like most other activities in life and indeed the universe) which is not comfortably converted into the absolute terms of 0 and 1. It may help to see fuzzy logic as the way reasoning really works and binary or Boolean logic is simply a special case of it.These two substantial fields are recognized as major factors of soft computing: a set of computing technologies already riding the waves of the next century to produce the human-centered intelligent systems of tomorrow in the existing literature (cf Rajak et al. [125], Ross et al. [124], Maria et al. [92]) . Haley [52] developed the solution procedure of a solid transportation problem and made a comparison between the STP and the classical transportation problem. Gupta et al. solved a fuzzy STP following two level fuzzy programming technique. Thereafter, a series of literature [9], [146] and reports [37] became available in which the fixed charge has been considered in deterministic nature. [20], [43], [15], [16], [51], [73] and [111] discussed the solution algorithm for solving the transportation problem in uncertain environment. In 1970, Genetic algorithms are search and optimization evolutionary algorithms based on the principles of natural phenomenas, which were first introduced by John Holland [58]. Genetic algorithms also implement the optimization strategies by simulating evolution of species through natural selections, cross over and mutation. Through a genetic evolution, a fitter chromosome has the tendency to yield good-quality offspring, which means a better solution to the problem.

In this chapter, a solid transportation problem is considered where the unit cost is depended into parts: one for type of item like, liquid item, glass goods, solid items, etc, another part of cost depends on the distance from source to destination. Here, both the unit costs are logically executed from three different measures of membership: low, medium and high. The proposed model is optimized through genetic algorithm based on fuzzy logic. Finally, a numerical example has been illustrated to discuss the model for both TP and STP.

6.2 Notations and Assumptions

6.2.1 Notations

The proposed model is formed with some notations. These notations are the following additional notations to be used. Other used notations in this model followed by the common notations.

(i) $\widetilde{C}_{ijk}$ = fuzzy unit transportation cost of the solid transportation to be depended on distance for transportation from i-th supply to j-th destination via k-th conveyance.

(ii) $\widetilde{\acute{C}}_{ijk}$ = fuzzy unit transportation cost of the solid transportation to be depended on quality for transportation from i-th supply to j-th destination via k-th conveyance.

(iii) $\widetilde{d}_{ijk}$ = fuzzy transportation distance from i-th supply to j-th destination via k-th conveyance.

(iv) $\widetilde{q}_{ijk}$ = fuzzy quality material to be transported from i-th supply to j-th destination via k-th conveyance.

6.2.2 Assumptions

To develop the proposed solid transportation model, the following assumptions have been made.

(i) In the traditional transportational problem, it is seen that unit transportation cost is fixed for any quantity of item which is transported from a source to a destination. But, in practical business system, it is not always true. In most of the cases of transportation problems in which the unit transportation cost depends on distance. When distance is increased then the unit transportation cost is decreased. Again, it is seen that due to the various factors the unit transportation costs may also vary i.e., realistically it may be uncertain. In this model, it has been considered as fuzzy numbers. Therefore, the fuzzy unit transformation costs due to distance have been taken in the following form

$$\widetilde{C}_{ijk}\widetilde{d}^{\alpha}_{ijk} \quad \text{where, } \alpha \geq 0$$

When $\alpha = 0$, then the unit transportation cost does not vary with the distance.

(ii) Here the quality of material means the percentage of breakability of that material. The quality of the material becomes high then the percentage of breakability of this material as well as risk will be high and at that time the unit transportation cost becomes high.

(iii) The total unit transportation cost of the materials from i-th source to j-th destination by k-th convenance, is sum of the unit transportation cost due to the distance and the unit transportation cost due to the quality.

6.3 Mathematical Formulation of the Solid Transportation Problem

If x_{ijk} units are transported from $i-th$ origin to $j-th$ destination via $k-th$ conveyance, then the optimization problem for solid transportation problems is given by

$$\text{Minimize} \quad Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(\widetilde{C}_{ijk} + \widetilde{C}_{ijk}\widetilde{d}^{\alpha}_{ijk})\ x_{ijk} \tag{6.1}$$

subject to the constraints

$$\begin{aligned} \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} &= a_i \quad \forall \quad i \\ \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} &= b_j \quad \forall \quad j \\ \sum_{i=1}^{M}\sum_{n=1}^{N} x_{ijk} &= e_k \quad \forall \quad k \\ \sum_{i=1}^{M} a_i &= \sum_{j=1}^{N} b_j = \sum_{k=1}^{K} e_k \\ x_{ijk} \geq 0 & \quad \text{for all} \quad i, j, k. \end{aligned} \tag{6.2}$$

The unit transportation costs $\tilde{\acute{C}}_{ijk}$ and $\widetilde{C}_{ijk}$ are depending on the quality or type of the goods and distance between source and destination respectively, whose values are generated from the following fuzzy logic inference. The first constraint in equation (6.2) represents the availability of the sources, where as the second & third constraints in equation (6.2) represent the demand of the each destination & capacity of the each conveyance respectively.
Now, the membership functions of the fuzzy numbers for unit costs $\widetilde{C}_{ijk}$(depends on distance) are $\mu_{\widetilde{Clow}}(x)$, $\mu_{\widetilde{Cmedium}}(x)$,$\mu_{\widetilde{Chigh}}(x)$ and unit costs $\widetilde{C}'_{ijk}$(depends on quality) are $\mu_{\widetilde{Clow}}(x)$, $\mu_{\widetilde{Cmedium}}(x)$,$\mu_{\widetilde{Chigh}}(x)$ defined by

$$\mu_{\widetilde{Clow}}(x) = \begin{cases} 0 & \text{if } x \le d_0 \\ 1 & \text{if } d_0 \le x \le d_1 \\ \dfrac{d_2 - x}{d_2 - d_1} & \text{if } d_1 \le x \le d_2 \\ 0 & \text{if } x \ge d_2 \end{cases} \qquad \mu_{\widetilde{Cmedium}}(x) = \begin{cases} 0 & \text{if } x \le d_1 \\ \dfrac{x - d_1}{d_2 - d_1} & \text{if } d_1 \le x \le d_2 \\ \dfrac{d_3 - x}{d_3 - d_2} & \text{if } d_2 \le x \le d_3 \\ 0 & \text{if } x \ge d_3 \end{cases}$$

$$\mu_{\widetilde{Chigh}}(x) = \begin{cases} 0 & \text{if } x \le d_2 \\ \dfrac{x - d_2}{d_3 - d_2} & \text{if } d_2 \le x \le d_3 \\ 1 & \text{if } d_3 \le x \le d_4 \\ 0 & \text{if } x \ge d_4 \end{cases} \qquad \mu_{\widetilde{C'low}}(x) = \begin{cases} 0 & \text{if } x \le q_0 \\ 1 & \text{if } q_0 \le x \le q_1 \\ \dfrac{q_2 - x}{q_2 - q_1} & \text{if } q_1 \le x \le q_2 \\ 0 & \text{if } x \ge q_2 \end{cases}$$

$$\mu_{\widetilde{C'medium}}(x) = \begin{cases} 0 & \text{if } x \le q_1 \\ \dfrac{x - q_1}{q_2 - q_1} & \text{if } q_1 \le x \le q_2 \\ \dfrac{q_3 - x}{q_3 - q_2} & \text{if } q_2 \le x \le q_3 \\ 0 & \text{if } x \ge q_3 \end{cases} \qquad \mu_{\widetilde{C'high}}(x) = \begin{cases} 0 & \text{if } x \le q_2 \\ \dfrac{x - q_2}{q_3 - q_2} & \text{if } q_2 \le x \le q_3 \\ 1 & \text{if } q_3 \le x \le q_4 \\ 0 & \text{if } x \ge q_4 \end{cases}$$

Using these membership functions, the following fuzzy rules are proposed for quality related cost $\tilde{\acute{C}}_{ijk}$.

R_{11}: if quality is high then transportation cost is high.

R_{12}: if quality is mid then transportation cost is mid.

R_{13}: if quality is low then transportation cost is low.

Similarly, the fuzzy rules for distance related cost $\widetilde{C}_{ijk}$ are proposed as.

R_{21}: if distance is high then transportation cost is low.

R_{22}: if distance is mid then transportation cost is mid.

R_{23}: if distance is low then transportation cost is high.

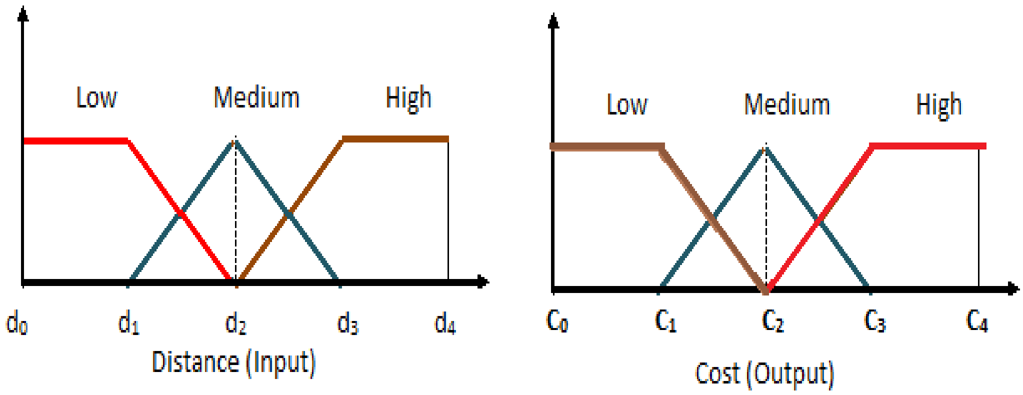

Figure 6.1: Membership function of Distance and corresponding unit transportation cost

Algorithm for proposed problem:

The fuzzy inference module for calculation of unit transportation cost(for distance and quality both) is given below-

Step-1: Take transportation distance and quality of items as input and unit transportation cost as output.

Step-2: Calculate the membership values to the fuzzy sets Low, Medium and High for finding cost.

Step-3: Evaluate the rules and find the rule strengths of each rule.

Step-4: Calculates the membership functions of the fuzzy transportation cost Low, Medium, High which are represented by the rules with non-zero strength

Step-5: Apply fuzzy union operator to find the fuzzy output.

6.4 Numerical Illustration

Example-6.1: Let us consider a solid transportation problem with fuzzy unit cost. Also, let the no of source $M = 2$, no of destination $N = 2$ and the no of conveyance $K = 2$. Also the imprecise input data for unit costs, qualities and distances are taken as

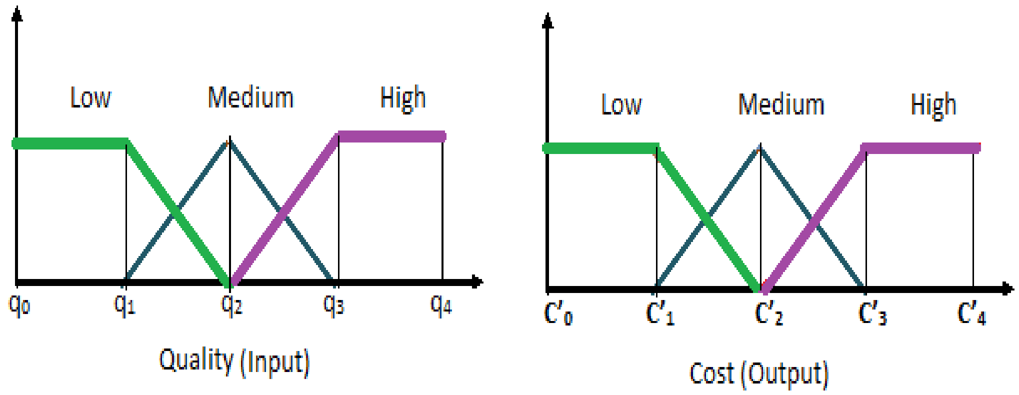

Figure 6.2: Membership function of quality and corresponding unit transportated amount

triangular fuzzy number.

Table-6.1:Input of distance with its fuzzy logic value

	Distance $(\widetilde{d}_{ijk})$	Distance related cost $\widetilde{C}_{ijk}$ (found on fuzzy logic)
Low	$(0, 40, 80)$	$(2.0, 4.5, 7.0)$
Medium	$(40, 80, 120)$	$(4.5, 7.0, 9.5)$
High	$(80, 120, 160)$	$(7.0, 9.5, 12.0)$

Table-6.2:Input of quality with its fuzzy logic value

	Quality $(\widetilde{q}_{ijk})$	Quality related cost $\widetilde{\acute{C}}_{ijk}$ (found on fuzzy logic)
Low	$(0, 25, 50)$	$(5.0, 7.5, 10.0)$
Medium	$(25, 50, 75)$	$(7.5, 10.0, 12.5)$
High	$(50, 75, 100)$	$(10.0, 12.5, 15)$

The availability of the sources, demand of the destinations and capacity of the conveyances are as follows: $a_1 = 40$, $a_2 = 60$, $b_1 = 50$, $b_2 = 50$, $e_1 = 45,$, $e_2 = 55$. and $\alpha = 0.5$

Let the problem is optimized for the input data of distance between the source and destination and the percentage of breakability i.e quality of the item are considered as in the following Table-6.3

Table-6.3:Input of distance and quality

	$k=1$		$k=2$	
	$i=1$	$i=2$	$i=1$	$i=2$
$j=1$	120, 85	106, 90	55, 39	10, 13
$j=2$	30, 48	78, 29	96, 73	45, 55

The corresponding logical value of unit transportation cost due to distances and percentage of breakability i.e quality of the item (in respective units) are shown in $Table-6.4$.

Table-6.4: Output of logical value for C_{ijk} & C'_{ijk}

	$k=1$		$k=2$	
	$i=1$	$i=2$	$i=1$	$i=2$
$j=1$	3.94, 13.05	5.00, 13.05	8.90, 8.62	10.00, 6.94
$j=2$	10.00, 9.71	7.18, 7.45	6.17, 12.79	9.65, 10.67

Using the fuzzy logic algorithm and then by using GA, the optimum value (in respective units) of transportation arc shown thc $Table-6.5$.

Table-6.5: Output of transportation amount(x_{ijk})

	$k=1$		$k=2$	
	$i=1$	$i=2$	$i=1$	$i=2$
$j=1$	9.10	15.00	18.40	7.50
$j=2$	7.90	13.00	4.60	24.50

The optimum solution presented in $Table-6.5$ is non degenerate, since the number of allocation cell is more than $(M+N+K-2)$. And hence the corresponding transportation cost is 6719.82 unit.

Example-6.2: The problem is similar as Example-1 only consider $k = 1$, i.e. number of conveyance is one. Then corresponding optimum amounts of transportation (in respective units) are shown in the *Figure* 6.3.

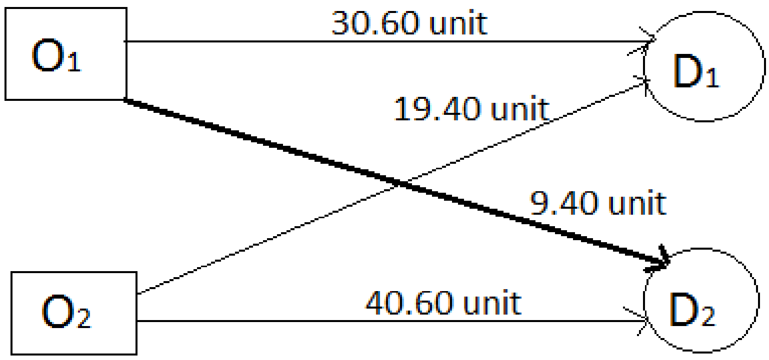

Figure 6.3: Output of transportation amount for example-6.2

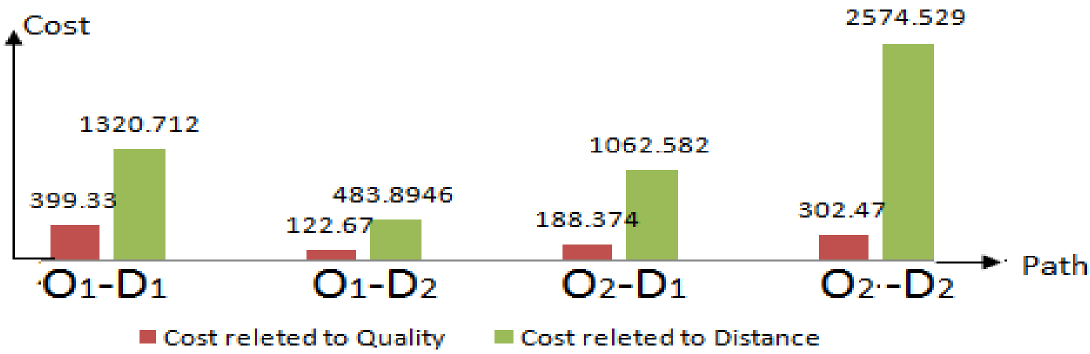

Figure 6.4: Output the cost related to quality of item and the distances for example-6.2

From *Figure* 6.3 and *Figure* 6.4, it is also seen that the unit costs (both distance related and quality related) increases with the increasing nature of transportation amount.

6.5 Conclusion

In reality, unit transportation cost depends on various parameters, like type of the materials whether it is liquid or solid or breakable, etc, distance of transportation and many more things. Beside that, the unit rate also may be vary whether the distance is low or high. In this model, an imprecise solid transportation problem is considered where unit transportation cost has two parts: unit transportation cost for the type of item and unit transportation cost related to distance. Both the unit cost are executed through fuzzy inference logic. The proposed model is optimized through genetic algorithm based on fuzzy logic for a transportation model in two dimension also. Finally, a numerical example has been illustrated to discuss the model.