Chapter 4

Two staged solid transportation problem of breakable items with safety cost

4.1 Introduction

The transportation problem is a special type of linear programming problem which arises in many practical situations. The transportation model has a wide implementation, not only in transportation systems, but also in other systems such as production planning, networks, etc. The solid transportation problem (STP) was first stated by Shell [136]. Then, Haley [54] developed the solution procedure of a solid transportation problem and made a comparison between STP and the classical transportation problem. Its bounds are given on three parameters, namely, supply of the origins, demand of the destinations and capacity of the conveyances. Basu et al. [11], Golmohamadi et al. [45] and Gupta et al. [51] developed the solution techniques to solve STP with a fixed cost

Transportation of breakable items is a growing interest of today's researchers. Most of the researchers considered this situation in the constraints of destination's demand (cf. Ojha et al. [110] [111] [107]). Few researchers consider transportation of the non-breakable items only, which does not certify the transportation of breakable items. To overcome this situation, here, for the first time, a safety cost is introduced for the breakable items, which reduced the amount of breakability as well as felt an effect to the receives of fresh amounts.

The main objective of the two stage transportation problem is to find a shipment plan to the destinations from the origins in two stages such that the total shipping costs is minimum. Geokrion and Graves [42] were the first researchers studied on two-stage transportation problem. After that so many researchers, Pirkul and Jayaraman [121], Pandian et al. [114] Amiri [7], Antony et al. [8], Gen et al. [40], Mahapatra et al. [90], Pandian and Natarajan [114] study two-stage TP. A parametric approach has been developed to solve a two stage fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers by Nagoor et al. [106]. Using geometric programming approach, Ritha and Merline Vinotha [128] proposed a method for finding a multi objective two stage fuzzy transportation problem

The concept of fuzzy set theory, first to be introduced by Zadeh [152], is used for solving different types of linear programming problems (cf. [20] [19] [43] [65] [134]).

Reality is less or more uncertain in nature. To a decision maker who is connected with transportation or transshipment problem, it is quite difficult to determine the amount of quantity to be transported when the different parameters are uncertain in nature. In this chapter, a two stage solid transportation problem (TSSTP) has been considered where almost all known parameters are imprecise (fuzzy) in nature. The items may be broken or destroy with a constant rate of breakability which depend on the mode of conveyance. As the item may be broken, so to reduce its amount, we consider a safety cost on that path of the transportation. Proposed TSSTP for the breakable item, all related constraints are taken into account. Finally, a numerical example is allowed to verify such TSSTP and its simple form TSTP to stabilized the model. The basic differences of the proposed model from other existing models have been given below:

Table-4.1: Comparison table anong the existing models with proposed model 4.3

References(s)	Stage	TP / STP	Safety Cost	Nature of breakability	Environment
Antony et al. (2012)	Two	TP	No	No	Crisp
Chanas et al. (1996)	One	TP	No	No	Fuzzy
Giri et al. (2015)	One	STP	No	No	Fuzzy
Baidya et al. (2013)	One	STP	Yes	No	Fuzzy
Ojha et al. (2010)	One	STP	No	Yes	Crisp
Chena et al. (2017)	One	STP	No	No	Fuzzy
This model	Two	STP	Yes	Yes	Fuzzy

4.2 Notations and Assumptions

4.2.1 Notations

In this STP, instead of common notations the following additional notations have been used

(i) R = number of destinations of the solid transportation problem.

(ii) P = number of conveyances in stage-1, i.e., different modes of the solid transportation problem from source to distribution center.

(iii) Q = no of conveyances in stage-2, i.e., different modes of the solid transportation problem from distribution center to destination.

(iv) $\widetilde{t}_u$ = fuzzy amount of product which can be carried by u-th conveyance from source to distribution center.

(v) $\widetilde{r}_v$ = fuzzy amount of product which can be carried by v-th conveyance from distribution center to distribution.

(vi) $\widetilde{C}_{iju}$ = fuzzy unit transportation cost of the solid transportation problem to be transported from i-th supply to j-th distribution center by u-th conveyance.

(vii) $\widetilde{d}_{jkv}$ = fuzzy unit transportation cost of the solid transportation problem to be transported from j-th distribution center to k-th destination via v-th conveyance.

(viii) x_{iju} = amount of the product to be transported from i-th supply to j-th distribution center by u-th conveyance.

(ix) y_{jkv} = amount of the product to be transported from j-th distribution center to k-th destination by v-th conveyance.

(x) λ_u, μ_v = rate of breakability in u-th and v-th conveyances respectively.

(xi) $\widetilde{s}_{1u}$ = fuzzy safety cost for uth conveyance.

(xii) $\widetilde{s}_{2v}$ = fuzzy safety cost for vth conveyance.

(xiii) S=total breakable item.

4.2.2 Assumptions

To develop the proposed solid transportation model, the following assumptions have been made.

(i) In the traditional transportational problem, it is seen that quantity is transported from a source to a destination directly. But, in practical business system, it is not always possible. There may exist some transportation problems in which the materials to be required can not be transported directly from a source to a destination by one conveyance only. In such situation, the materials are transported to a station(distribution center) near by the destination and then from distribution center, it is transported to the exact destination.

(ii) In the proposed solid transportation problem, it is assumed that the total capacities of distribution centers are unlimited.

(iii) The transported item are breakable in nature. The rate of breakability depends on the type of conveyances.

(iv) A safety cost has been taken into consideration at the time of transportation which depends on the condition of the path through which the conveyance is moved. Its value is fixed by the conveyance itself. In this case, we assume that f_{iju}, the function of x_{iju} takes the values 0 and 1 to describe the transportation activity from source i to distribution center j through transport mode u and it is defined as

$$f_{iju} = f(x_{iju}) = \begin{cases} 1, & if \;\; 0 < x_{iju} \\ 0, & \text{otherwise} \end{cases} \tag{4.1}$$

Similarly, for the transportation from jth distribution center to kth destination through transport mode v, the function g_{jkv} is defined as

$$g_{jkv} = g(y_{jkv}) = \begin{cases} 1, & if \;\; 0 < y_{jkv} \\ 0, & \text{otherwise} \end{cases} \tag{4.2}$$

4.3 Mathematical Formulation of TSSTP

Let us consider the problem where it needs to transport a homogeneous product from M sources to R destinations. There exist some unlimited capacities of N distribution centers between source & final destination. The availability of sources like warehouses, production facilities or supply quantities are $\widetilde{a_i}$. The destination of consumption facilities, warehouses, or demand points, is characterized by required levels of demand $\widetilde{b_k}$. $\widetilde{r_u}$ $(u = 1, 2, ...P)$ and $\widetilde{t_v}$ $(v = 1, 2, ...Q)$ represent the amount of product which can be carried by two conveyances. Then on the basis of the proposed assumptions, the problem is to determine the amounts of quantities x_{iju} and y_{jkv} which minimize the total transportation cost.

The solid transportation problem can be formulated as a minimizing problem with some

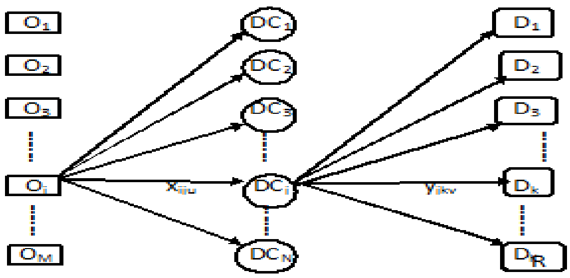

Figure 4.1: Design of the proposed problem

linear constraints in the following way.

$$Min\ \widetilde{Z}(x_{iju}, y_{jkv}) = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{u=1}^{P}\widetilde{C}_{iju}\ x_{iju}\ + \sum_{j=1}^{N}\sum_{k=1}^{R}\sum_{v=1}^{Q}\widetilde{d}_{jkv}\ y_{jkv}\ + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{u=1}^{P}\widetilde{s}_{1u}\ f_{iju}$$
$$+\sum_{j=1}^{N}\sum_{k=1}^{R}\sum_{v=1}^{Q}\widetilde{s}_{2v}\ g_{jkv} \tag{4.3}$$

subject to constraints

$$\begin{aligned}
&\sum_{j=1}^{N}\sum_{u=1}^{P} x_{iju} \le \widetilde{a}_i && i = 1, 2, 3, ..., M\\
&\sum_{i=1}^{M}\sum_{j=1}^{N} x_{iju} \le \widetilde{t}_u && u = 1, 2, 3, ..., P\\
&\sum_{j=1}^{N}\sum_{v=1}^{Q} y_{jkv} \ge \widetilde{b}_k && k = 1, 2, 3, ..., R\\
&\sum_{j=1}^{N}\sum_{k=1}^{R} y_{jkv} \le \widetilde{r}_v && v = 1, 2, 3, ..., Q\\
&\sum_{i=1}^{M}\sum_{u=1}^{P}(1-\lambda_u)x_{iju} \ge \sum_{k=1}^{R}\sum_{v=1}^{Q}(1-\mu_v)y_{jkv} && j = 1, 2, 3, ..., N\\
&\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{u=1}^{P}\lambda_u\, x_{iju}\ + \sum_{j=1}^{N}\sum_{k=1}^{R}\sum_{v=1}^{Q}\lambda_v\, y_{jkv} \le S &&\\
&x_{iju} \ge 0, y_{jkv} \ge 0 && \forall\ i,\ j,\ k,\ u, v
\end{aligned} \tag{4.4}$$

4.3.1 Defuzzification Procedures

The fuzzy numbers $\widetilde{C}_{iju}$, $\widetilde{d}_{jkv}$, $\widetilde{s}_{1u}$, $\widetilde{s}_{2v}$ and fuzzy quantities $\widetilde{a}_i$, $\widetilde{b}_k$, $\widetilde{t}_u$, $\widetilde{r}_v$ are converted to their nearest interval approximation following §2.1.8 Then the objective function $\widetilde{Z}(x_{iju}, y_{jkv})$ is reduced to Z_L^α and Z_U^α and corresponding restricted parameters become $a_{iL}^\alpha, a_{iU}^\alpha, b_{kL}^\alpha, b_{kU}^\alpha, t_{uL}^\alpha, t_{uU}^\alpha, r_{vL}^\alpha, r_{vU}^\alpha$. Such α-cut yields multiobjectives $Z_L^\alpha(x_{iju}, y_{jkv})$ and $Z_U^\alpha(x_{iju}, y_{jkv})$. From these two objectives, the weighted sum function $Z_C^\alpha(x_{iju}, y_{jkv}) = \frac{Z_L^\alpha(x_{iju}, y_{jkv}) + Z_U^\alpha(x_{iju}, y_{jkv})}{2}$ is considered for minimization and hence the above solid transportation problem becomes in the following form:

$$
\begin{aligned}
Min\ Z^\alpha(x_{iju}, y_{jkv}) = & \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{u=1}^{P} C_{iju}^\alpha\ x_{iju} + \sum_{j=1}^{N}\sum_{k=1}^{R}\sum_{v=1}^{Q} d_{jkv}^\alpha\ y_{jkv} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{u=1}^{P} s_{1u}^\alpha\ f_{iju} \\
& + \sum_{j=1}^{N}\sum_{k=1}^{R}\sum_{v=1}^{Q} s_{2v}^\alpha\ g_{jkv}
\end{aligned} \tag{4.5}
$$

subject to constraints

$$
\begin{aligned}
& a_{iL}^\alpha \le \sum_{j=1}^{N}\sum_{u=1}^{P} x_{iju} \le a_{iU}^\alpha \qquad i = 1, 2, 3, ..., M \\
& b_{kL}^\alpha \le \sum_{j=1}^{N}\sum_{v=1}^{Q} y_{jkv} \le b_{kU}^\alpha \qquad k = 1, 2, 3, ..., R \\
& t_{uL}^\alpha \le \sum_{i=1}^{M}\sum_{j=1}^{N} x_{iju} \le t_{uU}^\alpha \qquad u = 1, 2, 3, ..., P \\
& r_{vL}^\alpha \le \sum_{j=1}^{N}\sum_{k=1}^{R} y_{jkv} \le r_{vU}^\alpha \qquad v = 1, 2, 3, ..., Q \\
& \sum_{i=1}^{M}\sum_{u=1}^{P}(1-\lambda_u)x_{iju} \ge \sum_{k=1}^{R}\sum_{v=1}^{Q}(1-\lambda_v)y_{jkv} \qquad j = 1, 2, 3, ..., N \\
& \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{u=1}^{P}\lambda_u\, x_{iju} + \sum_{j=1}^{N}\sum_{k=1}^{R}\sum_{v=1}^{Q}\lambda_v\, y_{jkv} \le S \\
& x_{iju} \ge 0, y_{jkv} \ge 0 \qquad \forall\ i,\ j,\ k,\ u, v
\end{aligned} \tag{4.6}
$$

4.4 Algorithm of Proposed Model

To get optimal solution of the proposed model with equations (4.5) – (4.6), the following algorithm has been developed.

Step-1: For the given fuzzy numbers $\widetilde{C}_{iju}$, $\widetilde{d}_{jkv}$, $\widetilde{s}_{1u}$, $\widetilde{s}_{2v}$, we find their membership functions $\mu_{\widetilde{C}}$, $\mu_{\widetilde{d}}$, $\mu_{\widetilde{s}_1}$, $\mu_{\widetilde{s}_2}$ respectively by extension Principle.

Step-2: Taking α-cut of each fuzzy number.

Step-3: Here all fuzzy numbers are approximated by a corresponding crisp interval.

Step-4: Apply centroid method to find the defuzzified amount.

Step-5: For a fuzzy quantity $\widetilde{a}_i$, it is converted to its nearest interval approximation.

Step-6: end.

4.5 Numerical Illustration

To illustrate the proposed transportation model, let us consider an imprecise TSSTP with two sources, two destinations and two distribution centers, we are also taken two conveyances in each system of transportation. The corresponding inputs values (in proper units) are given below:

Table-4.2: Unit Transportation Cost

	Stage-1					Stage-2			
Conv. 1	$\widetilde{C}_{111}$	$\widetilde{C}_{121}$	$\widetilde{C}_{211}$	$\widetilde{C}_{221}$		$\widetilde{d}_{111}$	$\widetilde{d}_{121}$	$\widetilde{d}_{211}$	$\widetilde{d}_{221}$
Conv. 2	$\widetilde{C}_{112}$	$\widetilde{C}_{122}$	$\widetilde{C}_{212}$	$\widetilde{C}_{222}$		$\widetilde{d}_{112}$	$\widetilde{d}_{122}$	$\widetilde{d}_{212}$	$\widetilde{d}_{222}$
	(1,3,5),	(3,5,6,7),	(5,6,7),	(1,3,4,6),		(1,3,4),	(7,8,9),	(3,5,6,7),	(1,3,4,5)
	(1,2,3,4),	(2,5,6,8),	(1,3,4,5),	(3,4,6),		(1,3,5,6),	(4,5,6),	(2,3,4,5),	(3,4,5)

Table-4.3: Input for the availability, conveyance and demand

$\widetilde{a}_i$	$\widetilde{t}_u$	$\widetilde{r}_v$	$\widetilde{b}_k$
(33,35,37)	(32,35,36,37)	(26,30,33)	(27,30,31,32)
(30,32,33,34)	(29,32,34,35)	(22,25,26,28)	(23,25,27)

other input characteristics are $\lambda = 0.01; \mu = 0.015;$ $\widetilde{s}_{11} = (34, 35, 36)$, $\widetilde{s}_{12} = (31, 32, 33)$, $\widetilde{s}_{21} = (33, 34, 35)$, $\widetilde{s}_{22} = (31, 32, 33)$.

With the above input data, the problem first is reduced to a crisp TSSTP for a known value of α, then it is optimized using the $Lingo - 11.0$ toolbox. The optimum results (in appropriate units) for differennt α-cut are presented below:

Table-4.4: Optimum Result of TSSTP

α	x_{111} x_{121} x_{211} x_{221} x_{112} x_{122} x_{212} x_{222}	y_{111} y_{121} y_{211} y_{221} y_{112} y_{122} y_{212} y_{222}	Cost for Stage-1	Cost for Stage-2	Safety Cost	Total Cost
0.1	3.0, 0.0, 24.3, 9.5, 0.0, 13.8, 0.0, 0.0,	3.2, 0.0, 0.0, 23.2 24.1 0.0, 0.0, 0.0	73.14	66.12	17.00	156.22
0.3	3.0, 0.0, 24.6, 8.5, 0.0, 15.2, 0.0, 0.0,	3.6, 0.0, 0.0, 23.6 24.3 0.0, 0.0, 0.0	93.20	93.08	19.00	205.28
0.6	18., 0.0, 10.7, 7.0, 0.0, 17.3, 0.0, 0.0,	4.2, 0.0, 0.0, 24.2 24.6 0.0, 0.0, 0.0	123.75	135.32	22.00	281.07
0.9	15.70, 0.0, 14.0, 5.5, 0.0, 19.4, 0.0, 0.0,	4.8, 0.0, 0.0, 24.8 24.9 0.0, 0.0, 0.0	154.84	179.72	25.00	359.56

The pictorial representation of the considered problem for α=0.5 is given by $Figure$ 4.2 From $Figure$ 4.2, it reveals that, the total amount of transportation in the the stage-1

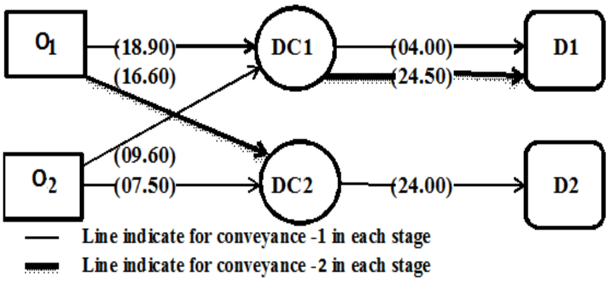

Figure 4.2: Representation of optimal transportation for α=0.5

is larger than stage-2, this is due to the breakability nature of the item. The obtained results are non-degenerate in nature in both stages. Similar results also can be obtained for other different values of x.

4.5.1 Special Case: Model with single conveyance: Two Stage Transportation Problem(TSTP)

If we consider a traditional two stage transportation problem(TSTP) with $P = 1$, $Q = 1$, the corresponding problem reduces to

$$Min\ \widetilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N} \widetilde{C}_{ij}\, x_{ij} + \sum_{j=1}^{N}\sum_{k=1}^{R} \widetilde{d}_{jk}\, y_{jk} + \sum_{i=1}^{M}\sum_{j=1}^{N} \widetilde{s}_1\, f_{ij} + \sum_{j=1}^{N}\sum_{k=1}^{R} \widetilde{s}_2\, g_{jk} \tag{4.7}$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{N} x_{ij} \le \widetilde{a}_i && i = 1, 2, 3, ..., M\\
&\sum_{j=1}^{N} y_{jk} \ge \widetilde{b}_k && k = 1, 2, 3, ..., R\\
&\sum_{i=1}^{M} (1-\lambda) x_{ij} \ge \sum_{k=1}^{R} (1-\mu) y_{jk} && j = 1, 2, 3, ..., N\\
&\sum_{i=1}^{M}\sum_{j=1}^{N} \lambda\, x_{ij} + \sum_{j=1}^{N}\sum_{k=1}^{R} \mu\, y_{jk} \le S\\
&x_{ij} \ge 0, y_{jk} \ge 0 && \forall\ i,\ j,\ k
\end{aligned} \tag{4.8}$$

After defuzzification process, it reduces to

$$Min\ Z^{\alpha} = \sum_{i=1}^{M}\sum_{j=1}^{N} C_{ij}^{\alpha}\, x_{ij} + \sum_{j=1}^{N}\sum_{k=1}^{R} d_{jk}^{\alpha}\, y_{jk} + \sum_{i=1}^{M}\sum_{j=1}^{N} s_1^{\alpha}\, f_{ij} + \sum_{j=1}^{N}\sum_{k=1}^{R} s_2^{\alpha}\, g_{jk} \tag{4.9}$$

$$
\begin{aligned}
& a_{iL}^{\alpha} \leq \sum_{j=1}^{N} x_{ij} \leq a_{iU}^{\alpha} && i = 1, 2, 3, ..., M \\
& b_{kL}^{\alpha} \leq \sum_{j=1}^{N} y_{jk} \leq b_{kU}^{\alpha} && k = 1, 2, 3, ..., R \\
& \sum_{i=1}^{M} (1-\lambda) x_{ij} \geq \sum_{k=1}^{R} (1-\mu) y_{jk} && j = 1, 2, 3, ..., N \qquad (4.10) \\
& \sum_{i=1}^{M} \sum_{j=1}^{N} \lambda\, x_{ij} \;+ \sum_{j=1}^{N} \sum_{k=1}^{R} \mu\, y_{jk} \leq S \\
& x_{ij} \geq 0, y_{jk} \geq 0 && \forall\ i,\ j,\ k
\end{aligned}
$$

For numerical example, let us consider the following input parameters.

Table-4.5: Unit Transportation Cost (in suitable units) of TSTP

Stage-1				Stage-2			
$\widetilde{C}_{11}$	$\widetilde{C}_{12}$	$\widetilde{C}_{21}$	$\widetilde{C}_{22}$	$\widetilde{d}_{11}$	$\widetilde{d}_{12}$	$\widetilde{d}_{21}$	$\widetilde{d}_{22}$
(1,3,4,5)	(3,5,6,7)	(3,5,6,7)	(1,3,4,6),	(1,2,3,4)	(6,7,8,9)	(3,5,6,7)	(1,3,4,5)

The corresponding solution (in suitable units) are in the following $Table - 4.6$

Table-4.6: Result of TSTP

α	x_{11} x_{12} x_{21} x_{22}	y_{11} y_{12} y_{21} y_{22}	Cost for Stage-1	Cost for Stage-2	Safety Cost	Total Cost
0.0	27.1, 9.9, 13.1, 0.0,	0.0, 27.0, 0.0, 23.0	109.20	77.00	10.00	150.20
0.3	28.0, 8.4, 15.2, 0.0,	0.0, 27.9, 0.0, 23.6	144.96	110.30	12.10	220.10
0.6	28.9, 6.9, 17.3, 0.0,	0.0, 28.8, 0.0, 24.2	182.54	145.50	14.20	293.84
0.9	29.8, 5.4, 19.4, 0.0,	0.0, 29.7, 0.0, 24.8	221.91	182.30	16.30	370.91

4.6 Sensitivity Analysis

The effect of the breakability of the item with the possibility level $\alpha = 0.5$ for the model-4.3 is shown in the following $Table - 4.7$

Table-4.7: Sensitivity Analysis

Rate of Breakability(λ) Change in Stage-1(%)	Rate of Breakability(μ) Change in Stage-2(%)	Change of Cost for Stage-1(%)	Change of Cost for Stage-2(%)	Change of Total Cost(%)
-25	-25	-6.3	-3.2	-9.5
-10	-10	-2.4	-1.7	-4.1
0.0	0.0	0.0	0.0	0.0
10	10	2.9	2.4	5.3
25	25	6.0	5.4	11.4

Here it is seen that, when percentage of breakability increases, the percentage of cost in each stage also increases and the rate of increasing is quite greater for the first stage due to larger amount of cost of those route in which transportation is available.

4.7 Discussion

For the same input data, $Table - 4.4$ & $Table - 4.6$ show the optimal result of the proposed two staged solid transportation problem and two staged transportation problem respectively. From both tables, it is seen that all cost (like safety cost, cost in stage-1 and stage-2) increase with the value of α, this is due to the increase of impreciseness of the fuzzy parameters. Another important features of the results are when number of conveyance in route of transportation is more than one, then the safety cost is high due to the several paths.

4.8 Conclusion

In the literature of solid transportation problem, there is neither two staged problem nor any provision of cost for the safety factor. More over, due to breakability, an additional constraints be appeared related to each stage separately. The model is formulated for imprecise environment also.